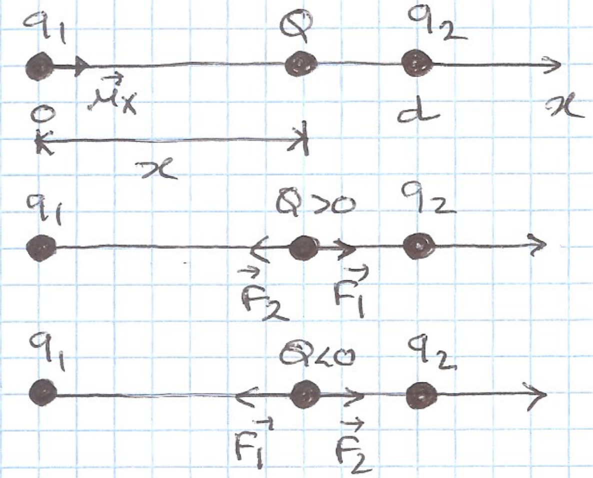
Série 1 - Campo elétrico

1.



$q_1 = +3q$

$q_2 = +q$

a) $\vec{F}_1 = k_e \dfrac{q_1 Q}{x^2}\,\vec{u}_x \quad , \quad \vec{F}_2 = -k_e \dfrac{q_2 Q}{(d-x)^2}\,\vec{u}_x$

Estas expressões para $\vec{F}_1$ e $\vec{F}_2$ são válidas para $Q>0$ ou $Q<0$:

- se $Q>0$, $q_1 Q > 0$ e $q_2 Q > 0$, $\vec{F}_1$ aponta no sentido de $\vec{u}_x$ e $\vec{F}_2$ aponta no sentido contrário de $\vec{u}_x$
- se $Q<0$, $q_1 Q < 0$ e $q_2 Q < 0$, $\vec{F}_1$ aponta no sentido contrário de $\vec{u}_x$ e $\vec{F}_2$ aponta no sentido de $\vec{u}_x$

Usando módulo:

- Se $Q>0$:

$$\vec{F}_1 = k_e \frac{|q_1||Q|}{x^2}\,\vec{u}_x \quad , \quad \vec{F}_2 = -k_e \frac{|q_2||Q|}{(d-x)^2}\,\vec{u}_x$$

- Se $Q<0$:

$$\vec{F}_1 = -k_e \frac{|q_1||Q|}{x^2}\,\vec{u}_x \quad , \quad \vec{F}_2 = k_e \frac{|q_2||Q|}{(d-x)^2}\,\vec{u}_x$$

b)

Equilíbrio:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = \vec{0}$$

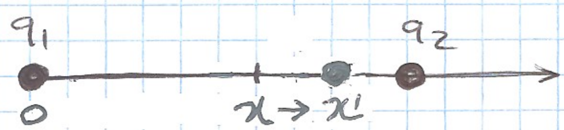
$$\vec{F} = k_e \frac{q_1 Q}{x^2} \vec{u}_x - k_e \frac{q_2 Q}{(d-x)^2} \vec{u}_x = \vec{0} \Rightarrow$$

$$\Rightarrow \quad \frac{3}{x^2} = \frac{1}{(d-x)^2}$$

Como $(d-x) > 0$ escolhe-se a raiz positiva

$$\frac{3}{x} = \frac{1}{d-x} \Rightarrow x = \frac{d}{\left(1 + \frac{1}{\sqrt{3}}\right)} = 0.634d$$

Equilíbrio estável ou instável?



$x' = x + \delta$

No equilíbrio: $\delta = 0$

[Notação: $F = |\vec{F}|$]

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = \vec{0}$$

$Q > 0$: $\vec{F} = (F_1 - F_2)\vec{u}_x = \vec{0}$

$Q < 0$: $\vec{F} = (-F_1 + F_2)\vec{u}_x = \vec{0}$

$F_1 = F_2$

Desviando a carga Q do equilíbrio: $\delta > 0$

$$\delta > 0: \begin{cases} F'_1 < F_1 & \text{porque } x' > x \\ F'_2 > F_2 & \text{porque } (d-x') < (d-x) \end{cases}$$

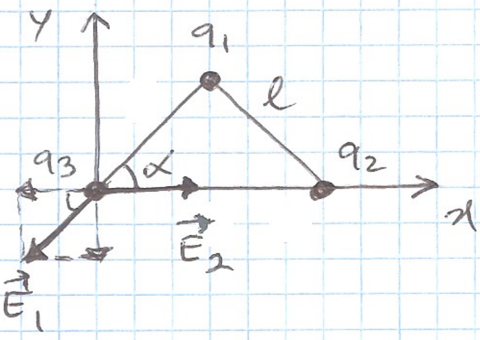
$\underline{Q > 0}$: $\vec{F}' = (F'_1 - F'_2)\vec{u}_x = -F'\vec{u}_x$

<u>equilíbrio estável</u> porque a carga Q é atuada por uma força F' que a leva para a posição de equilíbrio

$\underline{Q < 0}$: $\vec{F}' = (-F'_1 + F'_2)\vec{u}_x = F'\vec{u}_x$

<u>equilíbrio instável</u> porque a carga Q é atuada por uma força F' que a afasta da posição de equilíbrio

2.



$q_1 = 7.00\,\mu C$

$q_2 = -4.00\,\mu C$

$q_3 = 2.00\,\mu C$

$l = 0.50\,m$

$\alpha = 60^\circ$

a) $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$\begin{cases} E_1 = k_e \dfrac{|q_1|}{l} = 8.99 \times 10^9 \times \dfrac{7.00 \times 10^{-6}}{0.50^2} = 2.52 \times 10^5\ N/C \\[2ex] E_2 = k_e \dfrac{|q_2|}{l} = 8.99 \times 10^9 \times \dfrac{4.00 \times 10^{-6}}{0.50^2} = 1.44 \times 10^5\ N/C \end{cases}$$

$$\vec{E} = (-E_1 \cos\alpha\, \vec{u}_x - E_1 \sin\alpha\, \vec{u}_y) + E_2\, \vec{u}_x$$

$$= \left(-2.52 \times 10^5 \times \frac{1}{2} + 1.44 \times 10^5\right) \vec{u}_x +$$

$$+ \left(-2.52 \times 10^5 \times \frac{\sqrt{3}}{2}\right) \vec{u}_y$$

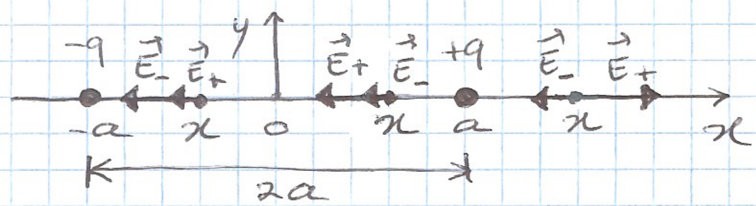
$$= (18.0\, \vec{u}_x - 218\, \vec{u}_y) \times 10^3\ N/C$$

b) $\vec{F} = q_3 \vec{E}$

$$= 2.00 \times 10^{-6} \times (18.0\, \vec{u}_x - 218\, \vec{u}_y)$$

$$= (36.0\, \vec{u}_x - 463\, \vec{u}_y) \times 10^{-3}\ N$$

3. Dipolo elétrico



a) $$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$0 < x < a$:

$$\vec{E} = -K_e \frac{q}{\underbrace{(a-x)^2}_{a-|x|}} \vec{u}_x - K_e \frac{q}{\underbrace{(a+x)^2}_{a+|x|}} \vec{u}_x$$

$-a < x < 0$:

$$\vec{E} = -K_e \frac{q}{\underbrace{(a-x)^2}_{a+|x|}} \vec{u}_x - K_e \frac{q}{\underbrace{(a+x)^2}_{a-|x|}} \vec{u}_x$$

Expressão geral:

$$-a<x<a: \quad \vec{E} = -K_e\left[\frac{q}{(a-x)^2} + \frac{q}{(a+x)^2}\right]\vec{u}_x$$

$$\vec{E} = -2K_e q\left[\frac{(a^2+x^2)}{(a^2-x^2)^2}\right]\vec{u}_x$$

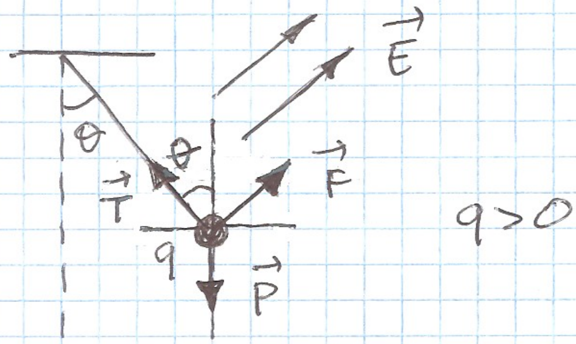
b) $$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$x>a: \quad \vec{E} = K_e \frac{q}{(x-a)^2}\vec{u}_x - K_e \frac{q}{(x+a)^2}\vec{u}_x$$

$$= K_e q \frac{4ax}{(x^2-a^2)^2}\vec{u}_x$$

$$x \gg a: \quad E \simeq 4K_e q \frac{x}{x^3}$$

6.



$m = 1.00g = 1.00 \times 10^{-3} Kg$, $\theta = 37.0°$

$\vec{E} = (3.00 \vec{u}_x + 5.00 \vec{u}_y) \times 10^5 \ NC^{-1}$

$E_x = 3.00 \times 10^{-5}$, $E_y = 5.00 \times 10^5$

$\vec{F} = q\vec{E}$

$\vec{P} + \vec{T} + \vec{F} = \vec{0}$

$-mg u_y + (-T \sin\theta \vec{u}_x + T\cos\theta \vec{u}_y) + (qE_x \vec{u}_x + qE_y \vec{u}_y) = \vec{0}$

$$\begin{aligned} \vec{u}_y &: \left\{ \begin{array}{l} -mg + T\cos\theta + qE_y = 0 \\ -T\sin\theta + qE_x = 0 \end{array} \right. \Rightarrow \\ \vec{u}_x &: \end{aligned}$$

$$\Rightarrow \left\{ \begin{array}{l} q = \dfrac{mg}{\left(E_y + E_x \dfrac{\cos\theta}{\sin\theta}\right)} \\ T = \dfrac{qE_x}{\sin\theta} \end{array} \right.$$

$$\left\{ \begin{array}{l} q = \dfrac{1.00 \times 10^{-3} \times 9.8}{\left(5.00 + 3.00 \dfrac{\cos 37.0°}{\sin 37.0°}\right) \times 10^5} = 10.9 \times 10^{-9} C \\ T = \dfrac{10.9 \times 10^{-9} \times 3.00 \times 10^5}{\sin 37.0°} = 5.44 \times 10^{-3} N \end{array} \right.$$

$q = 10.9 \ nC$

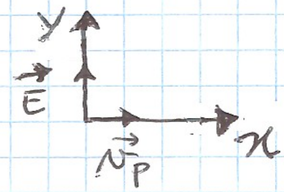
$T = 5.44 \ mN$

7.

PROTÃO: $\begin{cases} q = +e = 1.60 \times 10^{-19}\ C \\ m = 1.67 \times 10^{-27}\ kg \end{cases}$

$v_p = 4.50 \times 10^{5}\ m/s$

$E = 9.60 \times 10^{3}\ NC^{-1}$



$\vec{F} = q\vec{E} = eE\,\vec{u}_y$

$\vec{F} = m\vec{a} \Rightarrow \vec{a} = \frac{eE}{m}\,\vec{u}_y$

a) $a_x = 0 \Rightarrow v_x = v_p$

$x = v_x t \Rightarrow t^* = \frac{x^*}{v_p} = \frac{5.00 \times 10^{-2}}{4.50 \times 10^{5}} = 1.11 \times 10^{-7}\ s$

b) $a_y = \frac{eE}{m} = \frac{1.60 \times 10^{-19} \times 9.60 \times 10^{3}}{1.67 \times 10^{-27}} = 9.21 \times 10^{11}\ m/s^2$

$y = \frac{1}{2} a_y t^2 \Rightarrow y^* = \frac{1}{2} a_y t^{*2}$

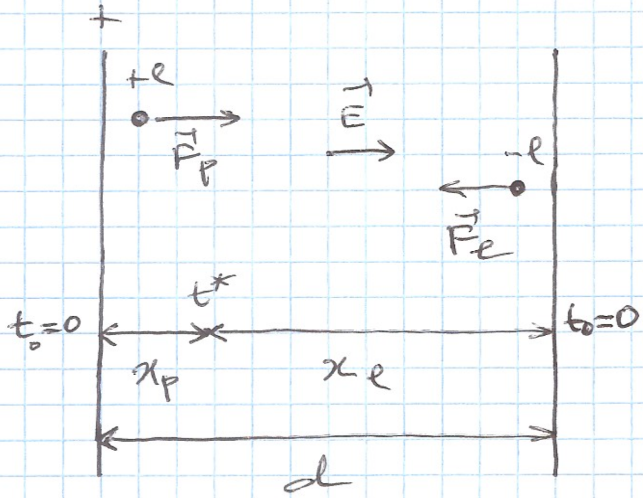
$y^* = \frac{1}{2} \times 9.21 \times 10^{11} \times (1.11 \times 10^{-7})^2 = 5.68 \times 10^{-3}\ m$

c) $v_x = v_p = 4.50 \times 10^{5}\ m/s$

$v_y = a_y t \Rightarrow v_y^* = a_y t^*$

$v_y^* = 9.21 \times 10^{11} \times 1.11 \times 10^{-7} = 1.02 \times 10^{5}\ m/s$

8.



$E = 640\,N/C$

$d = 4.00\ cm$

$m_p = 1.673 \times 10^{-27}\ kg$

$m_e = 9.110 \times 10^{-31}\ kg$

protão : $q = +e$; eletrão : $q = -e$

$$\vec{F} = q\vec{E} \Rightarrow \begin{cases} \vec{F}_p = e\vec{E} \\ \vec{F}_e = -e\vec{E} \end{cases}$$

$$\vec{F} = m\vec{a} \Rightarrow \begin{cases} F_p = eE = m_p a_p \Rightarrow a_p = \frac{eE}{m_p} \\ F_e = eE = m_e a_e \Rightarrow a_e = \frac{eE}{m_e} \end{cases}$$

$$a_e / a_p = m_p / m_e = 1836$$

$$t = t^* : \begin{cases} x_p = \frac{1}{2} a_p t^{*2} \\ x_e = \frac{1}{2} a_e t^{*2} \end{cases}$$

$$x_p + x_e = d \Rightarrow$$

$$\Rightarrow \frac{1}{2} a_p t^{*2} + \frac{1}{2} a_e t^{*2} = d$$

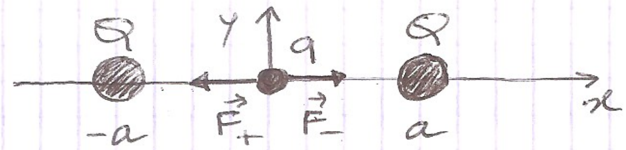
$$\Rightarrow t^{*2} = \frac{2d}{(a_p + a_e)}$$

$$x_p = \frac{1}{2} a_p t^{*2} = \frac{1}{2} a_p \frac{2d}{(a_p + a_e)} = \frac{d}{(1 + a_e/a_p)}$$

$$x_p = \frac{4.00 \times 10^{-2}}{(1 + 1836)} = 21.8 \times 10^{-6}\, m = 21.8\ \mu m$$

Série 3 - Potencial Elétrico

1.



$Q = 2.00\,\mu C = 2.00 \times 10^{-6}\,C$, $a = 0.800\,m$

$q = 1.28 \times 10^{-18}\,C$

a) $\vec{F} = \vec{F}_+ + \vec{F}_- = -k_e \dfrac{Qq}{a^2}\,\vec{u}_x + k_e \dfrac{Qq}{a^2}\,\vec{u}_x = \vec{0}\ N$

b) $\vec{F} = q\vec{E} \Rightarrow \vec{E} = \dfrac{\vec{F}}{q} = \vec{0}\ N/C$

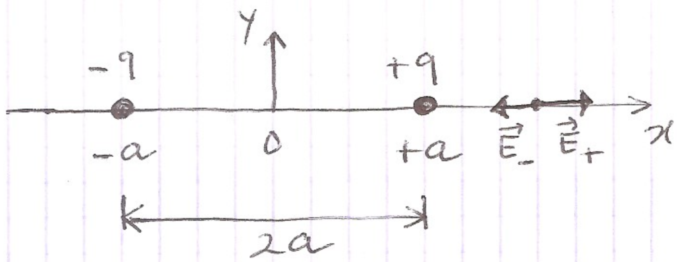
c) $V = V_+ + V_- = k_e \dfrac{Q}{a} + k_e \dfrac{Q}{a} = 2k_e \dfrac{Q}{a}$

$V = 2 \times 8.99 \times 10^{9} \times \dfrac{2.00 \times 10^{-6}}{0.800} = 45.0 \times 10^{3}\,V$

d) $U = qV$

$U = 1.28 \times 10^{-18} \times 45.0 \times 10^{3} = 5.76 \times 10^{-14}\,J$

2.



a) $0 < x < a$:

$$V = V_+ + V_- = k_e \left(\frac{+q}{a-x} + \frac{-q}{a+x} \right)$$

$-a < x < 0$:

$$V = V_+ + V_- = k_e \left(\frac{+q}{a-x} + \frac{-q}{a+x} \right)$$

$-a < x < a$:

$$V = V_+ + V_- = k_e \left(\frac{q}{a-x} - \frac{q}{a+x} \right)$$

$$= k_e q \frac{2x}{(a^2 - x^2)}$$

b) $x > a$:

$$V = V_+ + V_- = k_e \left(\frac{+q}{x-a} + \frac{-q}{x+a} \right)$$

$$= k_e q \frac{2a}{(x^2 - a^2)}$$

$x \gg a$:

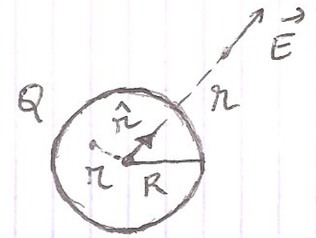
$$V = k_e q \frac{2a}{x^2}$$

c) $\vec{E} = E_x \vec{u}_x$

$$E_x = -\frac{dV}{dx} \Rightarrow E_x = k_e q \frac{4a}{x^3}$$

Comparar com o problema 3 da 1ª Série

4.



$Q = 26.0\,\mu C = 26.0 \times 10^{-6}\,C$

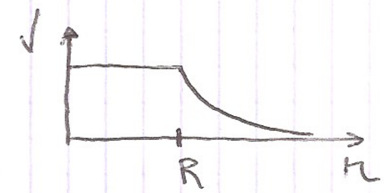
$R = 14.0\,cm = 14.0 \times 10^{-2}\,m$

$$\vec{E} = E\,\hat{r} \quad , \quad E = -\frac{dV}{dr}$$

$r < R$:

$$V = k_e \frac{Q}{R}$$

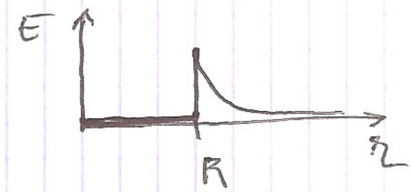
$$E = 0$$



$r > R$:

$$V = k_e \frac{Q}{r}$$

$$E = k_e \frac{Q}{r^2}$$



a) $r = 10.0\,cm$

$$V = k_e \frac{Q}{R} = \frac{8.99 \times 10^{9} \times 26.0 \times 10^{-6}}{14.0 \times 10^{-2}} = 1.67 \times 10^{6}\,V$$

$$E = 0\ V/m$$

b) $r = 20.0\,cm$

$$V = k_e \frac{Q}{r} = \frac{8.99 \times 10^{9} \times 26.0 \times 10^{-6}}{20.0 \times 10^{-2}} = 1.17 \times 10^{6}\,V$$

$$E = k_e \frac{Q}{r^2} = \frac{8.99 \times 10^{9} \times 26.0 \times 10^{-6}}{(20.0 \times 10^{-2})^2} = 5.84 \times 10^{6}\,V/m$$

c) $r = 14.0\,cm$

$$V = k_e \frac{Q}{R} = \frac{8.99 \times 10^{9} \times 26.0 \times 10^{-6}}{14.0 \times 10^{-2}} = 1.67 \times 10^{6}\,V$$

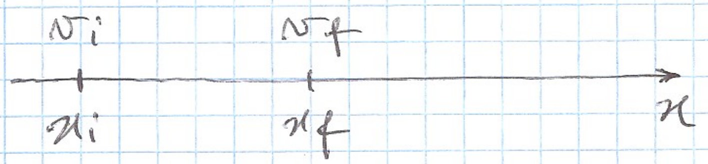
$r \to R^{+}$: $$E = k_e \frac{Q}{R^2} = \frac{8.99 \times 10^{9} \times 26.0 \times 10^{-6}}{(14.0 \times 10^{-2})^2} = 11.9 \times 10^{6}\,V/m$$

$r \to R^{-}$: $E = 0$

5. Eletrão:

$q = -e = -1.60 \times 10^{-19}\,C$

$m = 9.11 \times 10^{-31}\,Kg$



$x_i = 0.00\,cm \qquad v_i = 3.70 \times 10^6\,m/s$

$x_f = 2.00\,cm \qquad v_f = 1.40 \times 10^5\,m/s$

Variação de energia cinética:

$$\Delta E_c = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= \frac{1}{2} 9.11 \times 10^{-31} \left((1.40 \times 10^5)^2 - (3.70 \times 10^6)^2 \right)$$

$$= -6.23 \times 10^{-18}\,J$$

Variação de energia potencial elétrica:

$$-\Delta U = \Delta E_c \qquad \text{(conservação de energia)}$$

$$\Delta U = +6.23 \times 10^{-18}\,J$$

Diferença de potencial:

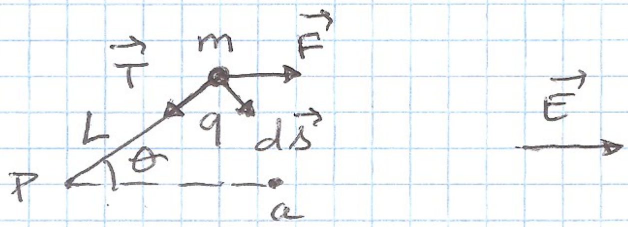
$$\Delta V = \frac{\Delta U}{q}$$

$$\Delta V = \frac{6.23 \times 10^{-18}}{-1.60 \times 10^{-19}} = -38.9\,V$$

$$\Delta V = V_f - V_i < 0 \quad \Rightarrow \quad V_i > V_f$$

a origem está a um potencial mais elevado

6.



$q = +2.00\,\mu C = +2.00 \times 10^{-6}\,C$

$m = 0.0100\ kg$

$L = 1.50\ m$

$\theta = 60.0°$, $v_i = 0\ m/s$

$E = 300\,V/m$

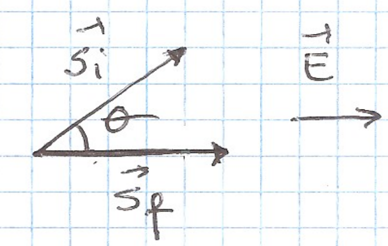
Força do campo elétrico: $\vec{F} = q\vec{E}$

Tensão do fio: $\vec{T}$

$$W_{\vec{F}} + W_{\vec{T}} = \Delta E_c$$

$$\begin{cases} W_{\vec{F}} = -\Delta U \\ W_{\vec{T}} = 0 \text{ porque } \vec{T} \perp d\vec{s} \end{cases}$$

$$\Delta E_c = -\Delta U$$



$$\Delta U = q\,\Delta V$$

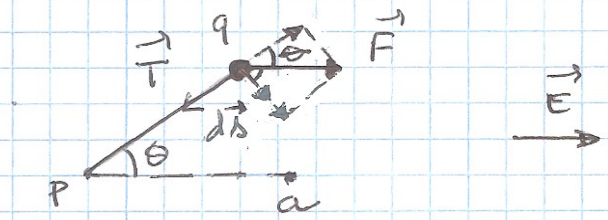
$$\begin{aligned} \Delta V &= -\vec{E}\cdot\Delta\vec{s} \\ &= -\vec{E}\cdot(\vec{s}_f - \vec{s}_i) \\ &= -E\,(L - L\cos\theta) \end{aligned}$$

Logo:

$$\Delta E_c = qEL(1-\cos\theta)$$

$$\frac{1}{2} m v_f^2 - \underbrace{\frac{1}{2} m v_i^2}_{0} = qEL(1-\cos\theta)$$

$$v_f = \sqrt{\frac{2qEL(1-\cos\theta)}{m}} = 0.300\,m/s$$



$$dV = -\vec{E} \cdot d\vec{s}$$

$$= -E\cos\left(\frac{\pi}{2} - \theta\right) ds$$

$$= -E\sin\theta \, ds \qquad , \qquad ds = -L\, d\theta$$

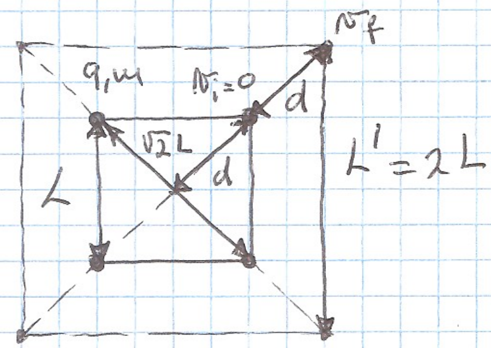
$$= E \sin\theta \, L \, d\theta$$

$$\Delta V = \int dV = \int_{\theta_i = \theta}^{\theta_f = 0} E L \sin\theta \, d\theta$$

$$= -E L \cos\theta \Big|_{\theta_i = \theta}^{\theta_f = 0}$$

$$= -E L (1 - \cos\theta)$$

8.



a) O trabalho necessário para colocar as cargas naquela posição é igual à energia potencial elétrica do sistema das quatro cargas

$$W_a = \Delta U = U - U_0$$

$$U = 4\,k_e \frac{q^2}{L} + 2\,k_e \frac{q^2}{\sqrt{2}L}$$

$$= k_e \frac{q^2}{L}\left[4 + \frac{2}{\sqrt{2}}\right] = 5{,}41\ k_e \frac{q^2}{L}$$

$U_0 = 0$ energia quando as cargas estão infinitamente afastadas umas das outras

b) Cada carga afasta-se segundo a diagonal todas as cargas têm a mesma velocidade

$$\Delta E_c = -\Delta U \quad \text{(conservação de energia)}$$

$$\Delta U = U' - U$$

$$= k_e \frac{q^2}{L'}\left[4 + \frac{2}{\sqrt{2}}\right] - k_e \frac{q^2}{L}\left[4 + \frac{2}{\sqrt{2}}\right]$$

$$= -k_e \frac{q^2}{2L}\left[4 + \frac{2}{\sqrt{2}}\right]$$

$$4 \times \frac{1}{2} m v_f^2 - 4 \times \frac{1}{2} m \underbrace{v_i^2}_{0} = k_e \frac{q^2}{L}\left[2 + \frac{1}{\sqrt{2}}\right]$$

$$v_f = \sqrt{\left(1 + \frac{1}{\sqrt{8}}\right) \frac{k_e q^2}{m L}}$$