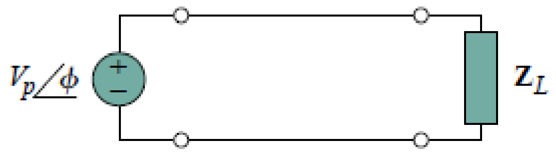
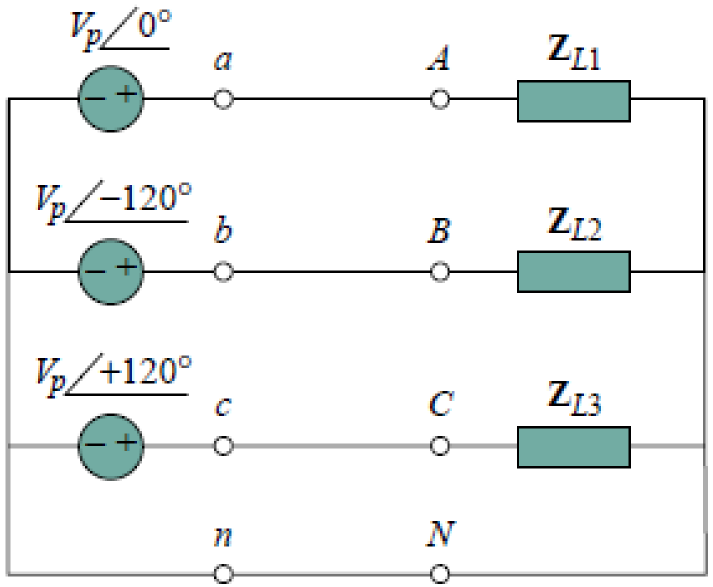
Sistemas trifásicos

Até agora temos lidado com sistemas alternados em que apenas existe uma tensão variável



(a)



Vantagens

Quase toda a energia eléctrica produzida é de cariz trifásico

Se necessário podemos utilizar apenas uma fase

A potência instantânea pode ser constante

Para a mesma potência, os sistemas trifásicos são mais baratos do que os monofásicos

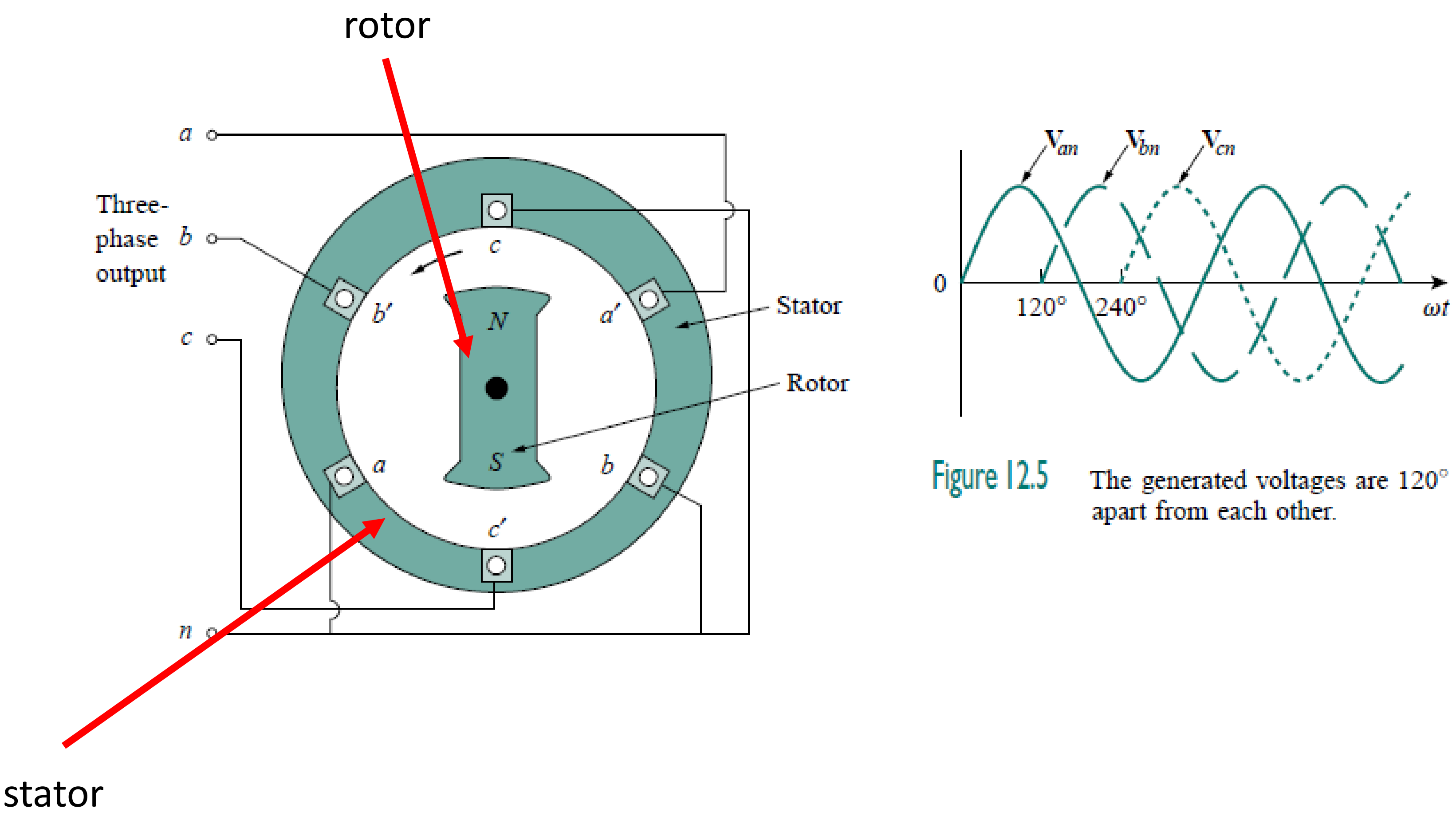
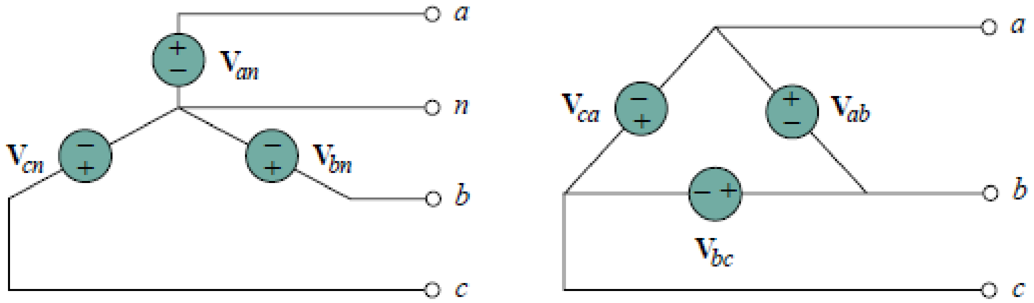


Figure 12.5 The generated voltages are 120° apart from each other.



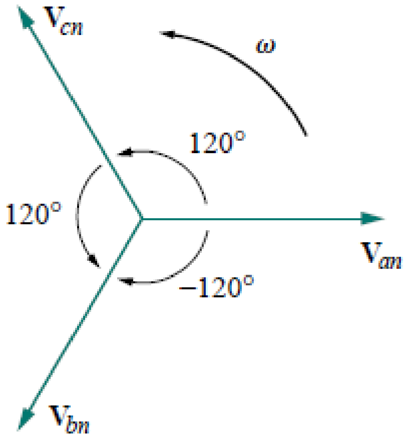
(a) (b)

Num sistema equilibrado as tensões são iguais nas 3 fases e estão desfasadas de 120º

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

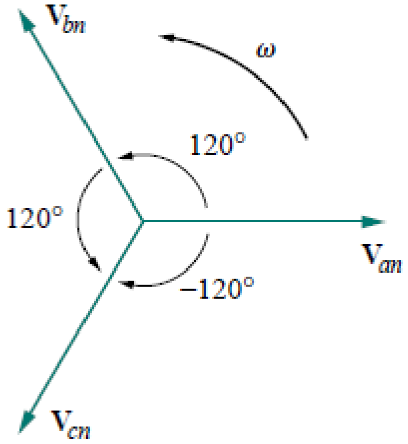
$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

Como as 3 fases estão desfasadas de 120º existem duas possiblidades



$$\mathbf{V}_{an} = V_p \angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p \angle -120^\circ$$
$$\mathbf{V}_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

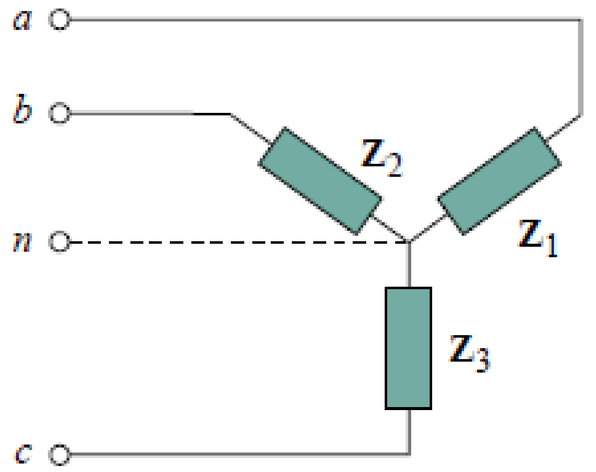
Sequência positiva; sentido anti-ponteiros do relógio



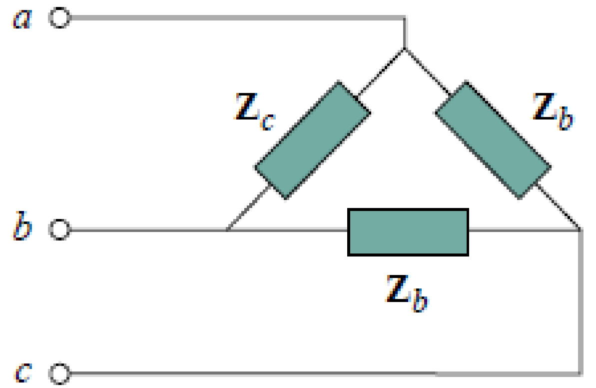
$$\mathbf{V}_{an} = V_p \angle 0^\circ$$
$$\mathbf{V}_{cn} = V_p \angle -120^\circ$$
$$\mathbf{V}_{bn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

Sequência negativa; sentido ponteiros do relógio

A sequência de fase é importante porque determina a direcção de rotação de um motor trifásico ligado a esse gerador

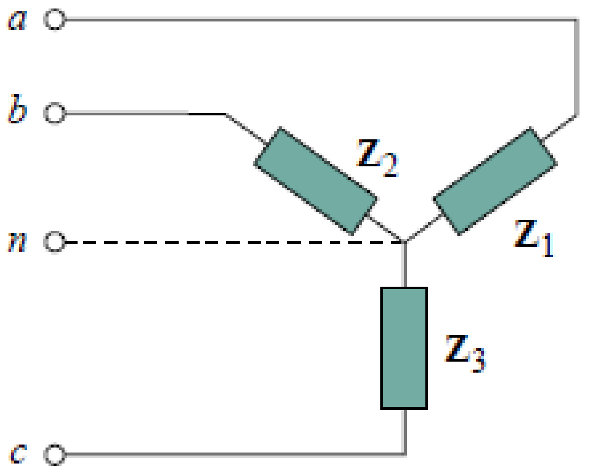


(a)

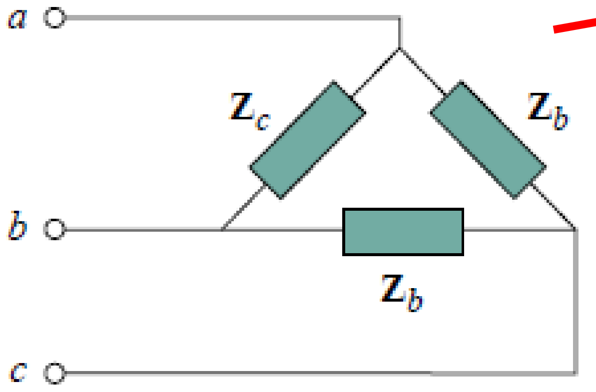


Tal como acontece com a forma de ligação de um gerador trifásico, as cargas trifásicas também podem ser ligas em duas configurações: estrela e triângulo

Uma carga trifásica diz-se equilibrada se as impedâncias são iguais em amplitude e fase



(a)



$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta$$

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$$

Determinar a sequência das tensões

$$v_{an} = 200\cos(\omega t + 10^\circ)$$

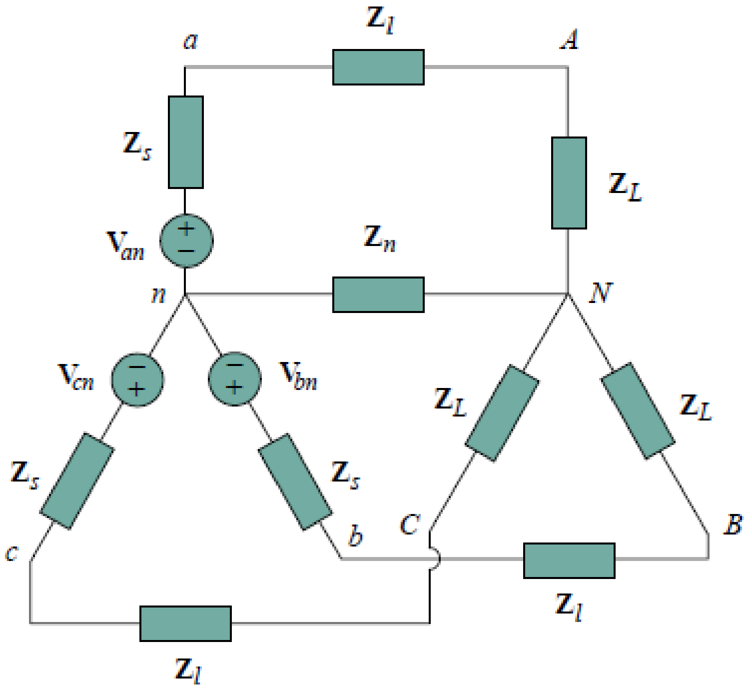
$$v_{bn} = 200\cos(\omega t - 230^\circ), \qquad v_{cn} = 200\cos(\omega t - 110^\circ)$$

$$\mathbf{V}_{an} = 200\angle 10^\circ, \qquad \mathbf{V}_{bn} = 200\angle -230^\circ, \qquad \mathbf{V}_{cn} = 200\angle -110^\circ$$

Sequência acb

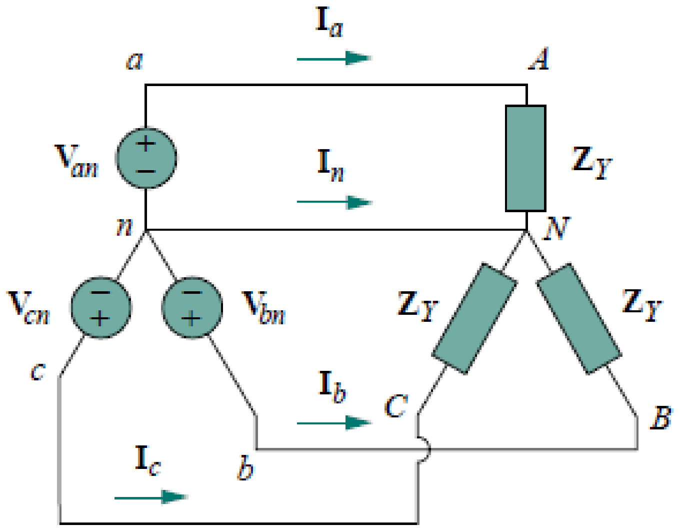
Uma vez que tanto o gerador como a carga podem estra configurados em estrela ou triângulo temos ao todo 6 configurações

A ligação estrela-estrela equilibrada é a mais importante porque todas as outras podem ser reduzidas a esta



$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$

$$\mathbf{Z}_Y = \mathbf{Z}_L$$ Se Zs e Zl pequenas



Assuming the positive sequence, the *phase* voltages (or line-to-neutral voltages) are

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ \tag{12.10}$$

The *line-to-line* voltages or simply *line* voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$ are related to the phase voltages. For example,

$$\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0^\circ - V_p\angle -120^\circ$$
$$= V_p\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_p\angle 30^\circ \tag{12.11a}$$

Similarly, we can obtain

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle -90^\circ \tag{12.11b}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle -210^\circ \tag{12.11c}$$

$$\boxed{V_L = \sqrt{3}V_p}$$

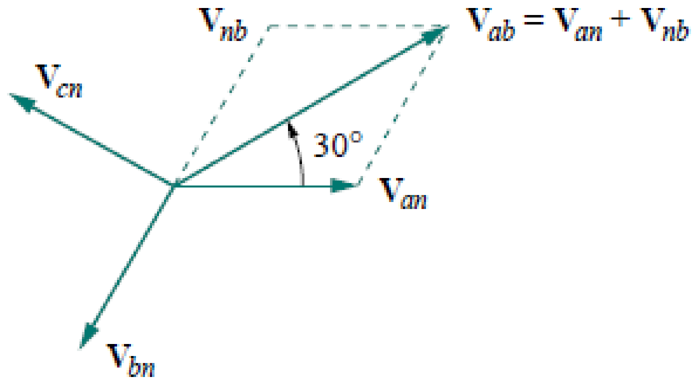
where

$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

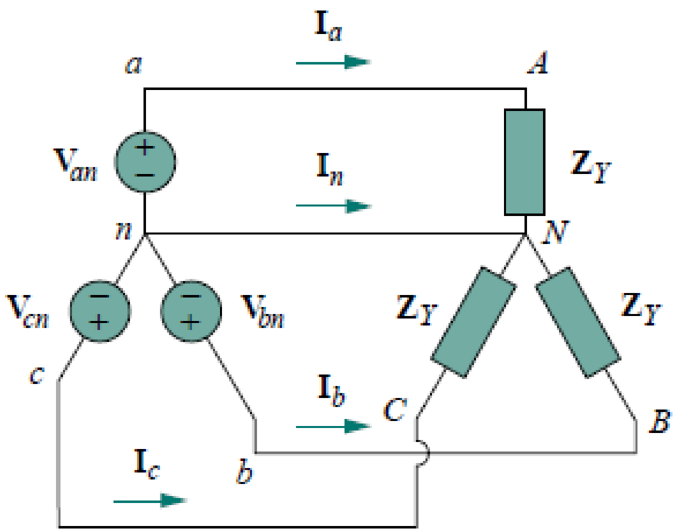
and

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$

Ou seja a tensão na carga é igual a sqrt(3)* tensão da fase



V_{nb}
$V_{ab} = V_{an} + V_{nb}$
V_{cn}
30°
V_{an}
V_{bn}



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -120^\circ$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -240^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -240^\circ$$

We can readily infer that the line currents add up to zero,

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

so that

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0$$

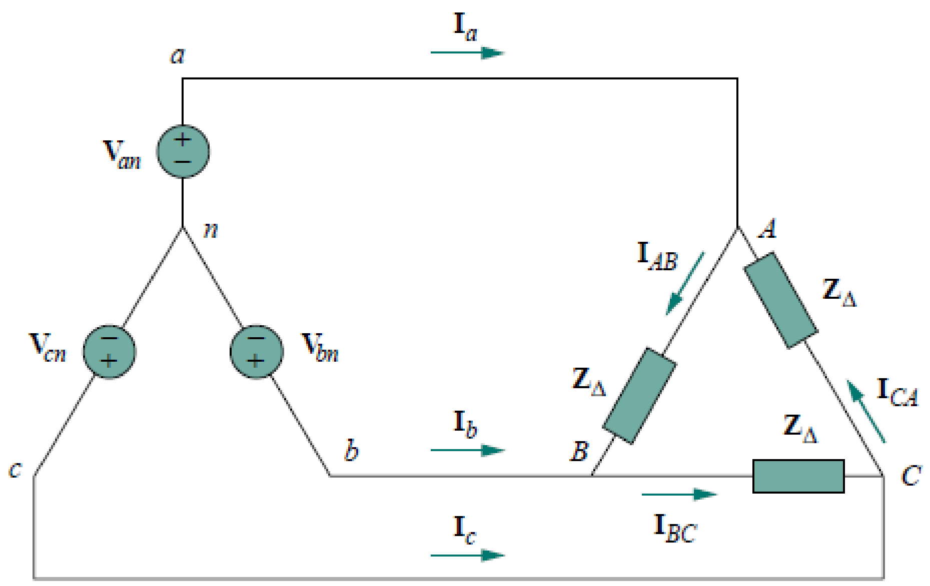
or

$$\mathbf{V}_{nN} = \mathbf{Z}_n \mathbf{I}_n = 0$$

Corrente no neutro é nula

Esta ligação pode ser omitida nas linhas de transmissão, que usam apenas 3 fios, sendo o neutro a ligação à terra

Sistemas estrela-triângulo



$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

As shown in Section 12.3, the line voltages are

$$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle -210^\circ = \mathbf{V}_{CA}$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

As correntes têm a mesma amplitude mas diferença de fase de 120º entre elas

TABLE 12.1 Summary of phase and line voltages/currents for balanced three-phase systems[1].

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

[1] *Positive or abc sequence is assumed.*

TABLE 12.1 (*continued*)

Connection	Phase voltages/currents	Line voltages/currents
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$