Ciências ULisboa

Diffusion of vorticity & Boundary layers

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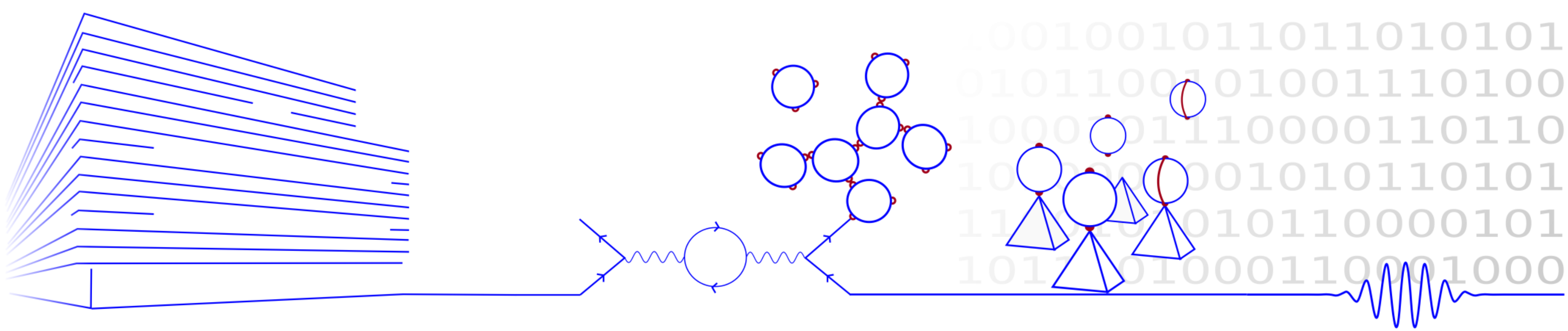
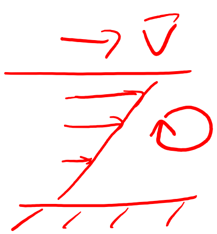


Overview

$R_e = \frac{L \cdot V}{\nu}$ → $R_e > 1$ Inercial; $R_e < 1$ Viscoso

- There are at least two flow situations in which the viscous term in the Navier–Stokes equation can be neglected.
- The first occurs in high Reynolds number regions of flow where net viscous forces are known to be negligible compared to inertial and/or pressure forces; we call these inviscid regions of flow.
- The second situation occurs when the vorticity is negligibly small; we call these irrotational or potential regions of flow. $\vec{\omega} = \nabla \times \vec{u}$
- In either case, removal of the viscous terms from the Navier–Stokes equation yields the Euler equation.
- There are some serious deficiencies associated with application of the Euler equation to practical flow problems. High on the list of deficiencies is the inability to specify the no-slip condition at solid walls.



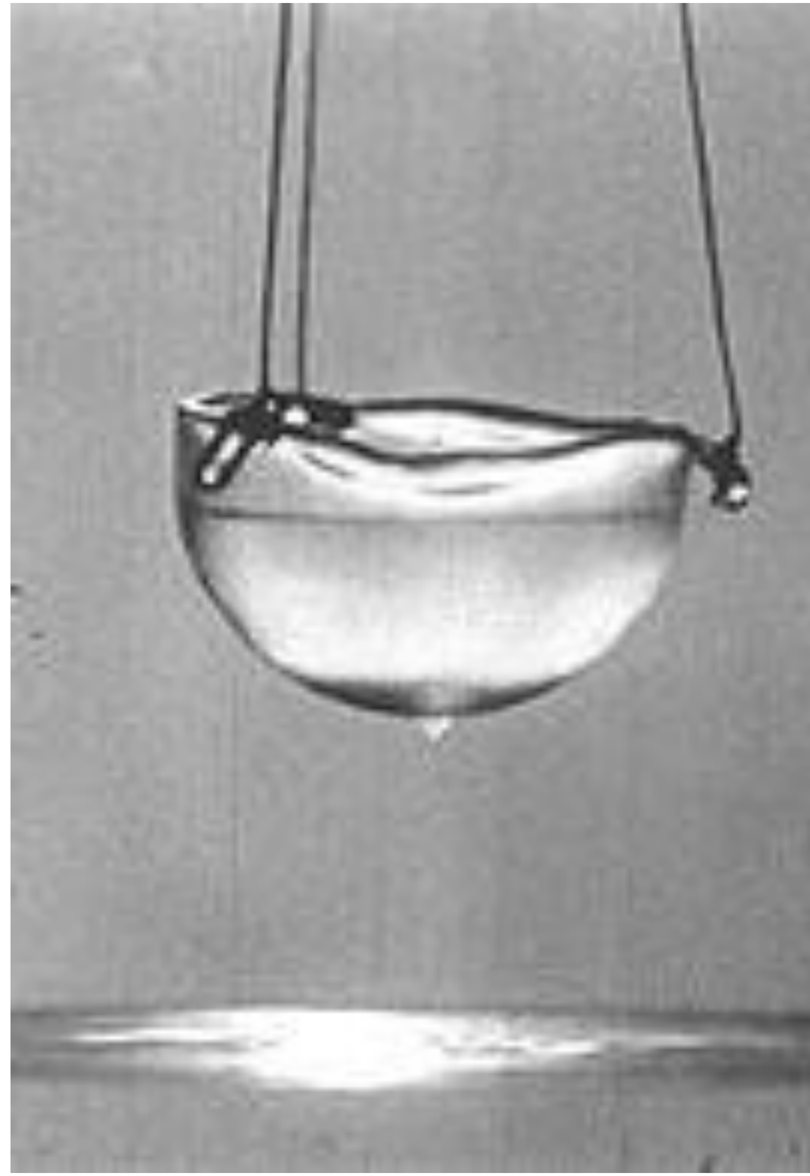
$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Çengel 10.6

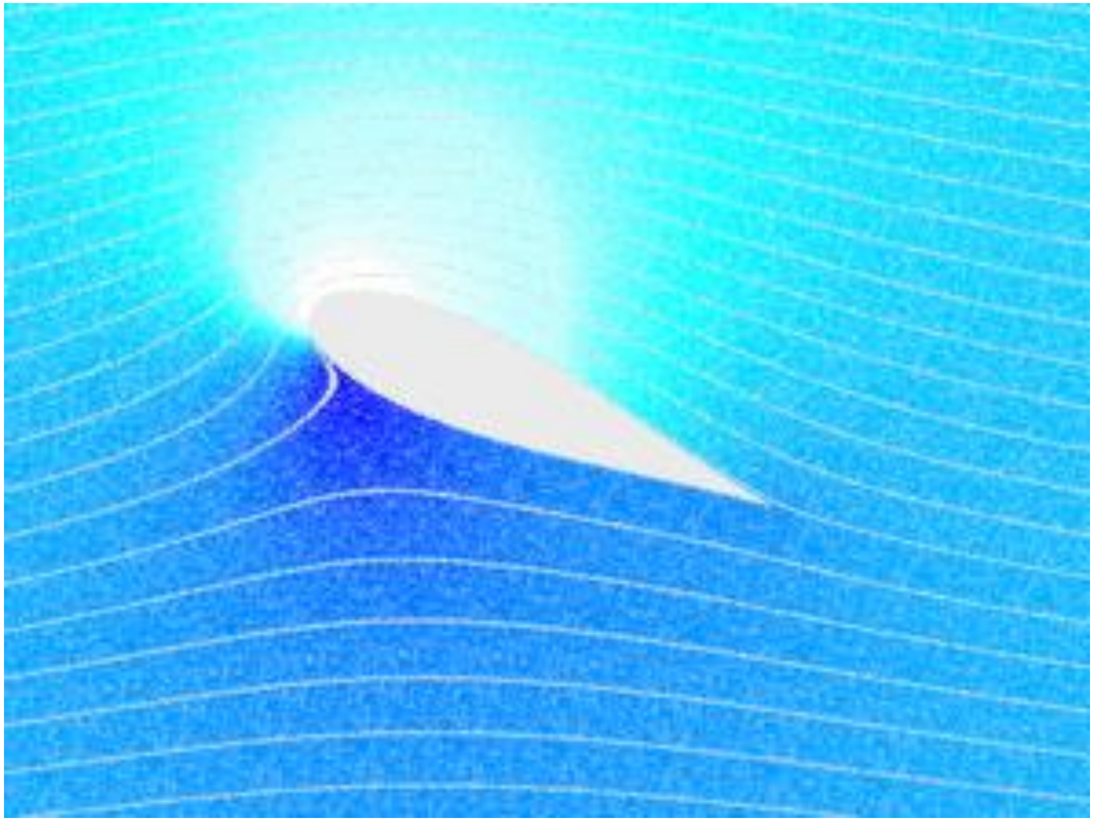
Inviscid flow

- Superfluid He^4 has zero viscosity and it flows without any resistance. Persistent currents.
- Since the viscosity is nearly zero, the Reynolds number approaches infinity.

- Superfluids can flow out of containers by climbing the walls.
- Supercurrents persist without drive.



$\nabla^2 \phi = 0 \qquad \nabla \cdot \vec{u} = 0 \;, \; \nabla \times \vec{u} = 0$



$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = - \frac{\nabla p}{\rho} + \vec{g} + \nu \nabla^2 \vec{u}$$

Irrotational flow

- Irrotational flow around a wing.
- The solution may be obtained from potential flow theory.
- This particular solution is obtained as the sum of three elementary solutions: free stream, line source and line vortex.
- Lift force proportional to the circulation and free stream velocity.
- Drag force zero.

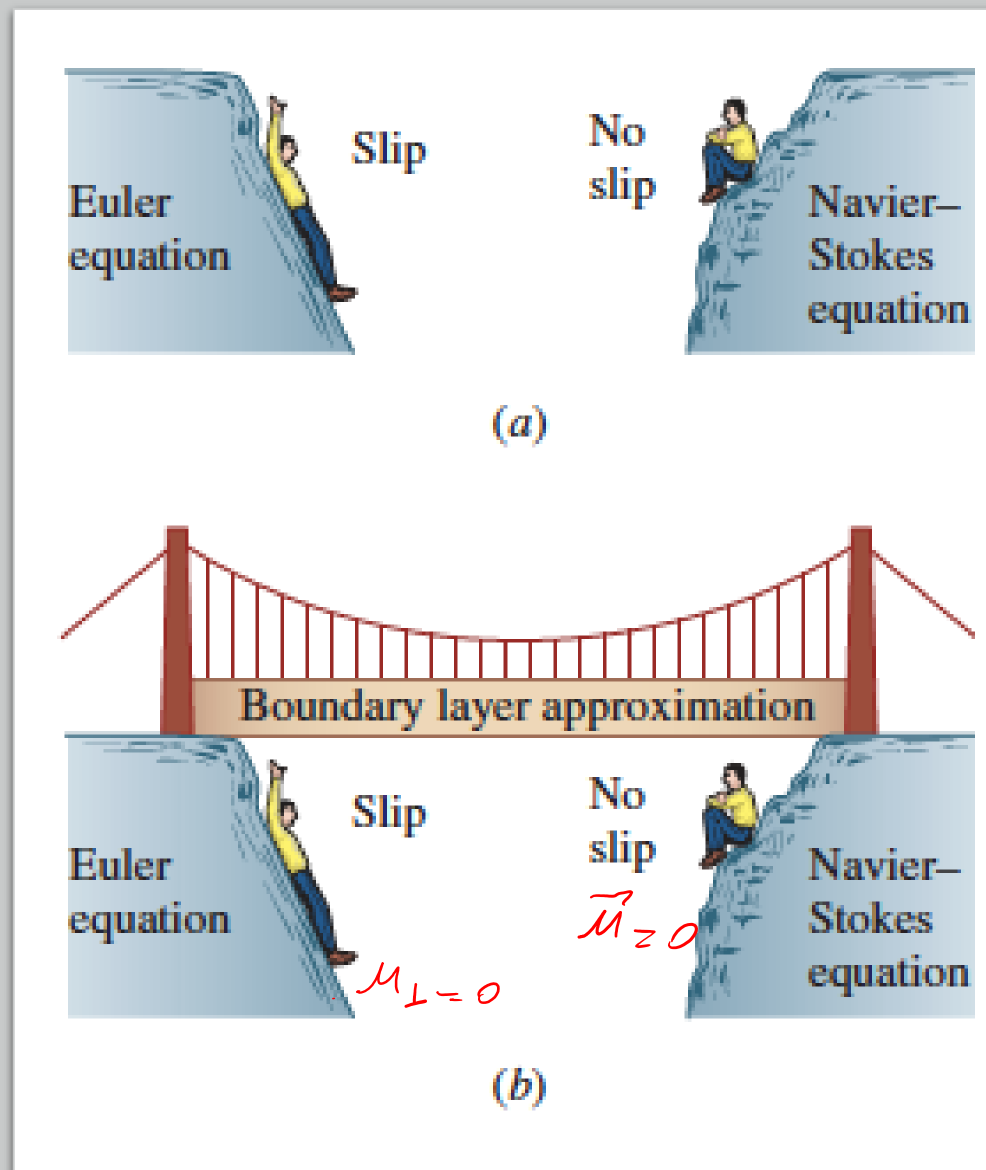
- By the mid-1800s, the Navier–Stokes equation was known, but couldn't be solved except for flows of very simple geometries.
- Meanwhile, mathematicians were able to obtain beautiful analytical solutions of the Euler equation and of the potential flow equations for flows of complex geometry, but their results were often physically meaningless.

$$P_r = \frac{L \cdot V}{D}$$

- A major breakthrough in fluid mechanics occurred in 1904 when Ludwig Prandtl (1875–1953) introduced the boundary layer approximation.
- Prandtl's idea was to divide the flow into two regions: an outer flow region that is inviscid and/or irrotational, and an inner flow region called a boundary layer—a very thin region of flow near a solid wall where viscous forces and rotationality cannot be ignored.
- In the outer flow region, the continuity and Euler equations apply to obtain the outer flow velocity field, and the Bernoulli equation to obtain the pressure field. Alternatively, if the outer flow region is irrotational, we may use potential flow techniques.

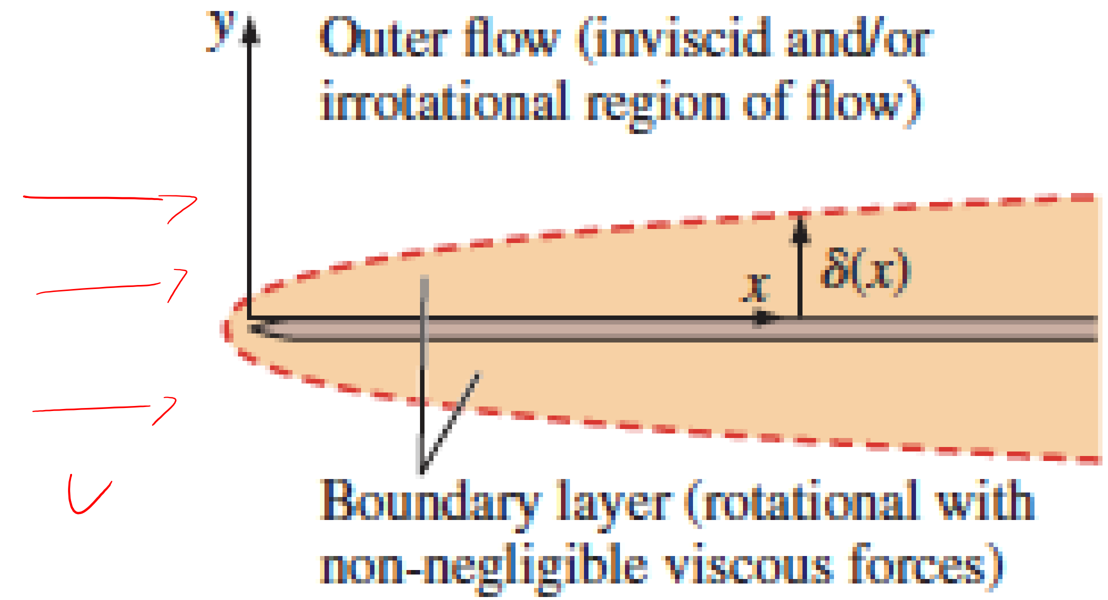
The boundary layer

Prandtl introduced the boundary layer approximation to bridge the gap between the Euler equation and the Navier–Stokes equation, and between the slip condition and the no-slip condition at solid walls.

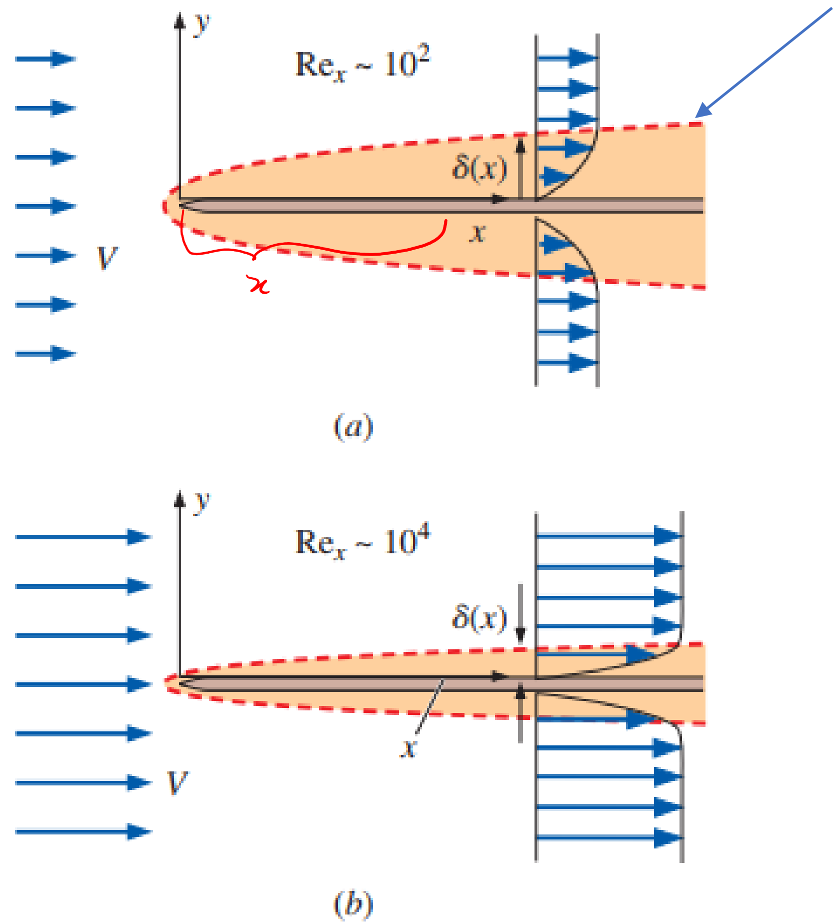


The idea

We solve for the outer flow region first, and then fit in a thin boundary layer in regions where vorticity and viscous forces cannot be neglected.



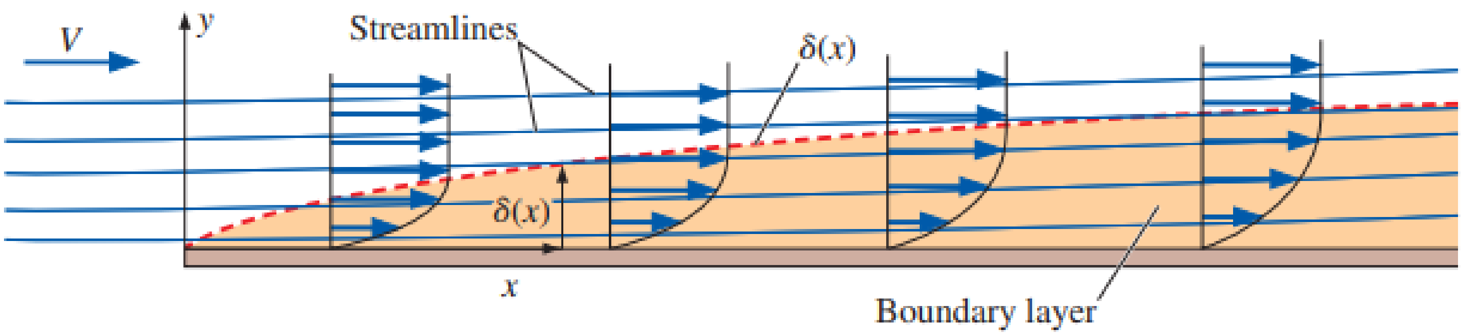
The larger the Reynolds number, the thinner the boundary layer along the plate at a given *x*-location



Note: this is not a streamline.

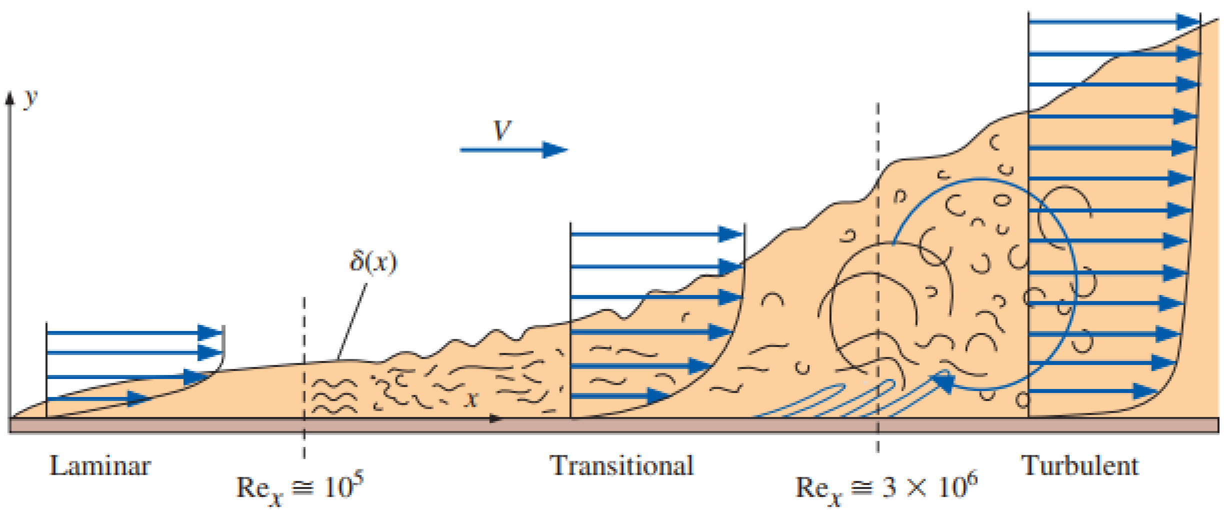
Reynolds number along a flat plate: $\mathrm{Re}_x = \dfrac{\rho V x}{\mu} = \dfrac{V x}{\nu}$

Laminar to turbulent transition

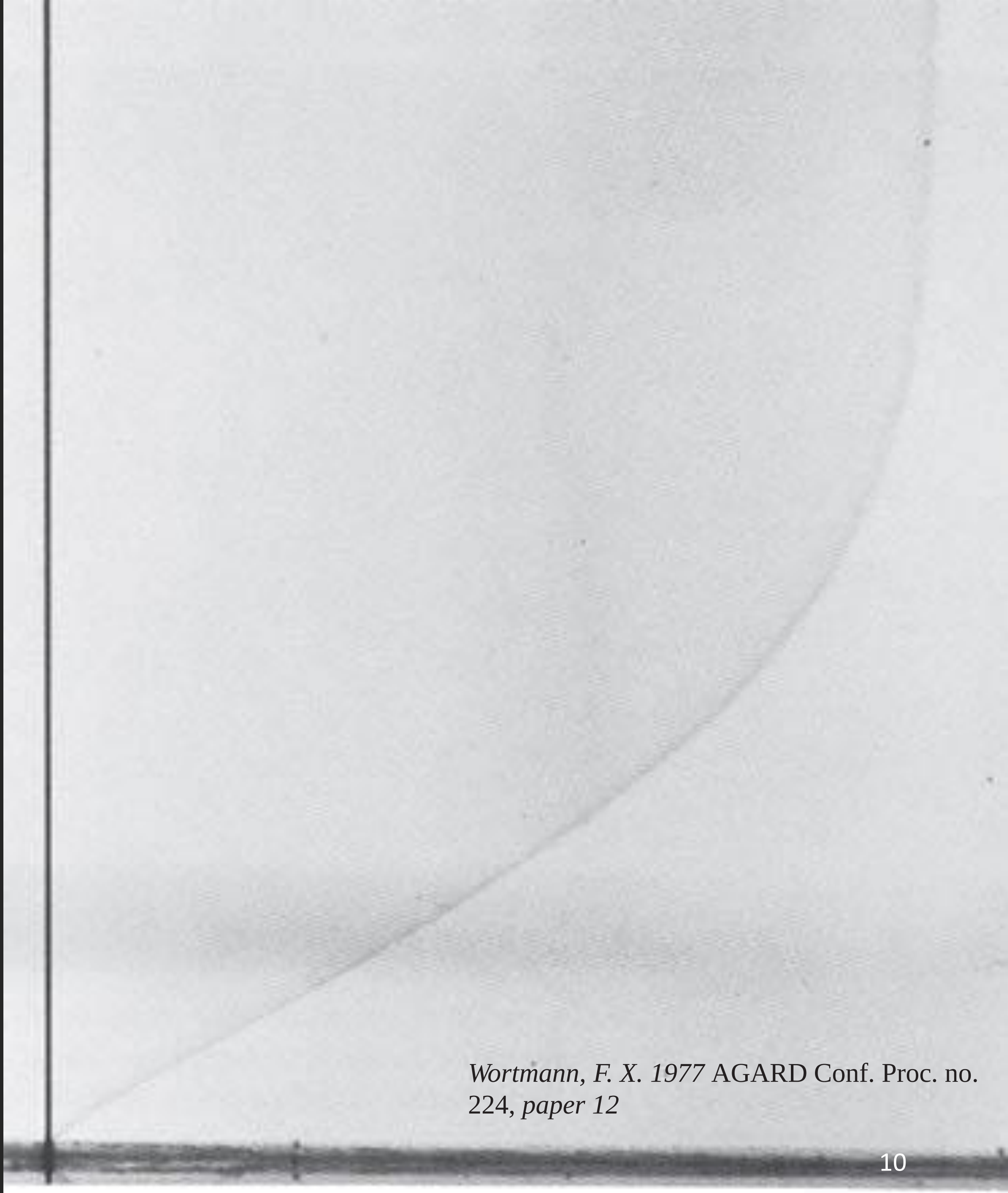


critical Reynolds number, $Re_{x,\,critical} \cong 1 \times 10^5$,

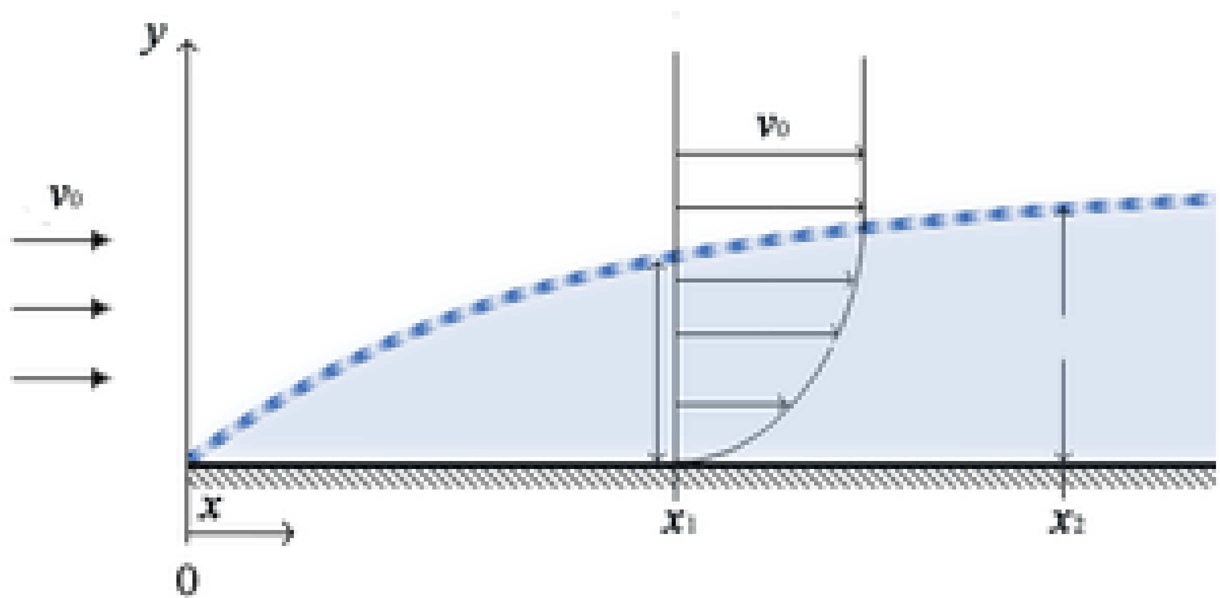
$$Re_{x,\,transition} \cong 3 \times 10^6$$



Photograph of a velocity profile of a uniform stream over a flat plate



Wortmann, F. X. 1977 AGARD Conf. Proc. no. 224, *paper 12*



Negligible viscosity or irrotational flow cannot be assumed near solid boundaries, such as the case of the airplane wing.

Vorticity and lines of vorticity

- Since $\overrightarrow{\Omega} = \nabla \times \vec{V}$ its divergence is zero, i.e. $\nabla \cdot \overrightarrow{\Omega} = 0$.

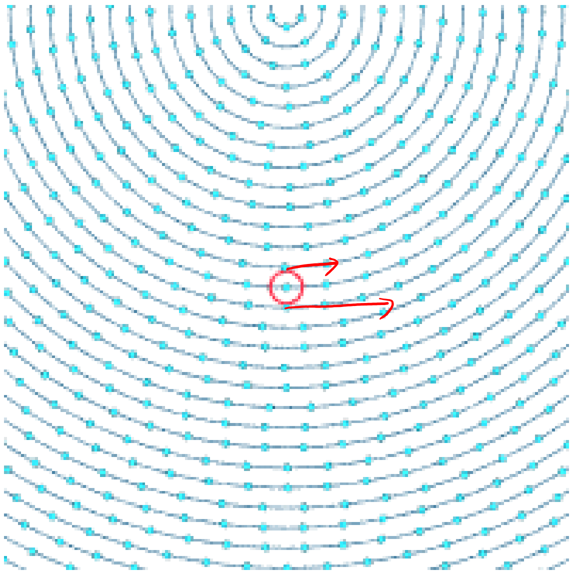
- The vorticity is a solenoidal field with lines of vorticity (like streamlines) parallel to its direction and density proportional to its magnitude.

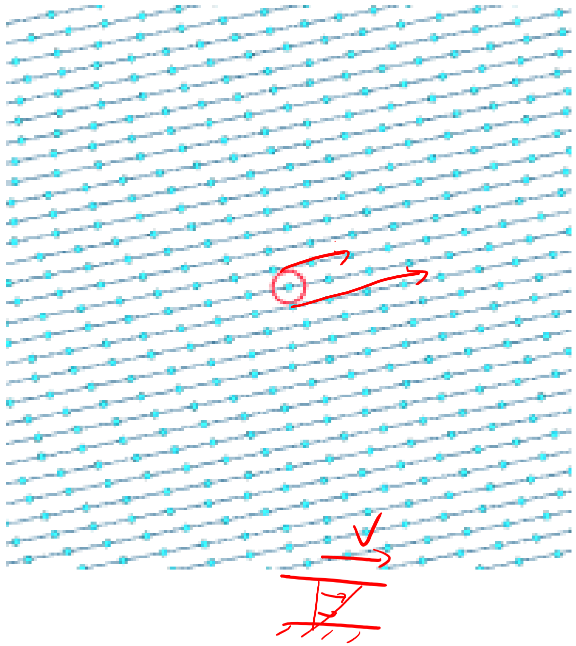
- Dynamics of the lines of vorticity differs in the Euler and Navier-Stokes equations.

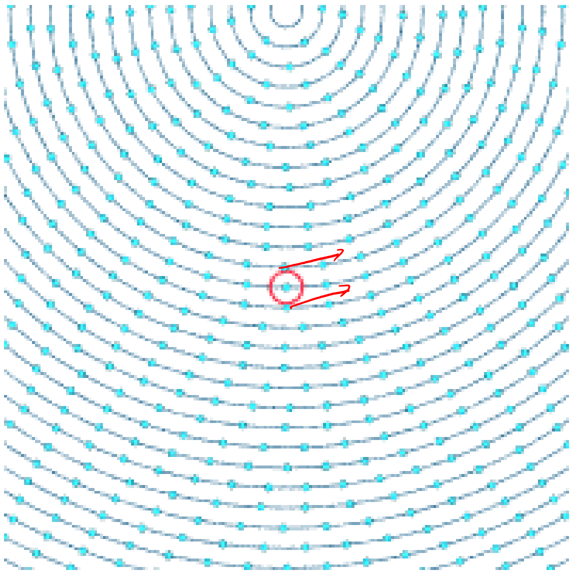
Rigid-body-like vortex
$v \propto r$

Parallel flow with shear

Irrotational vortex
$v \propto 1/r$



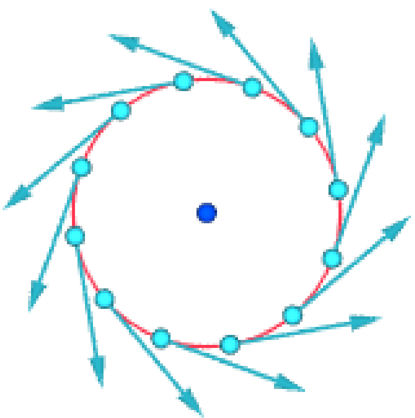




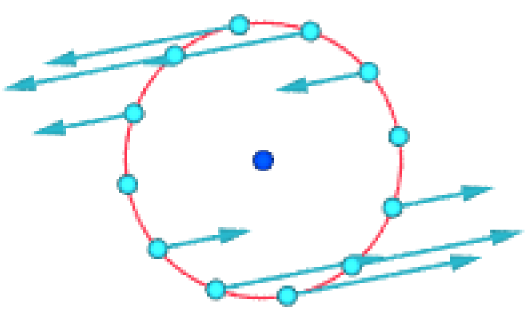
Balde

Tornado

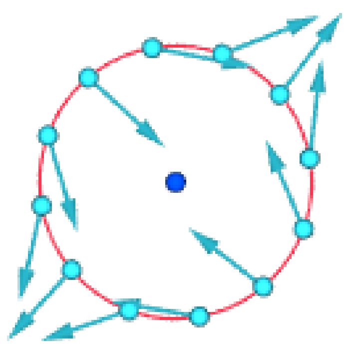
Relative velocities (magnified) around the highlighted point



Vorticity ≠ 0



Vorticity ≠ 0



Vorticity = 0