Sistemas magnéticamente acoplados

Transformadores

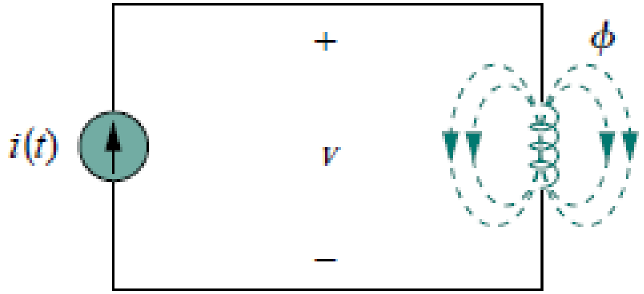
Até agora vimos circuitos electricamente ligados

Vamos agora tratar de sistemas magneticamente acoplados

Um exemplo é o transformador

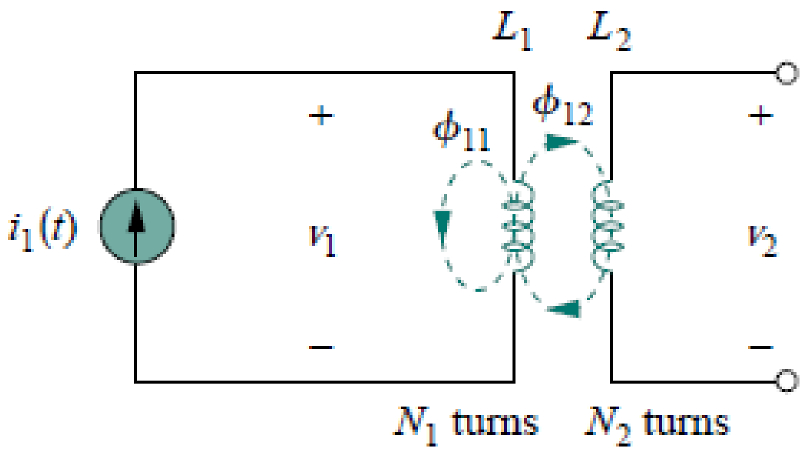
Lei da Faraday $$v = N\frac{d\phi}{dt}$$

$$v = N\frac{d\phi}{di}\frac{di}{dt}$$



$$v = L\frac{di}{dt}$$

$$L = N\frac{d\phi}{di}$$ Auto-indutância



$$v_1 = N_1 \frac{d\phi_1}{dt} \qquad v_2 = N_2 \frac{d\phi_{12}}{dt}$$

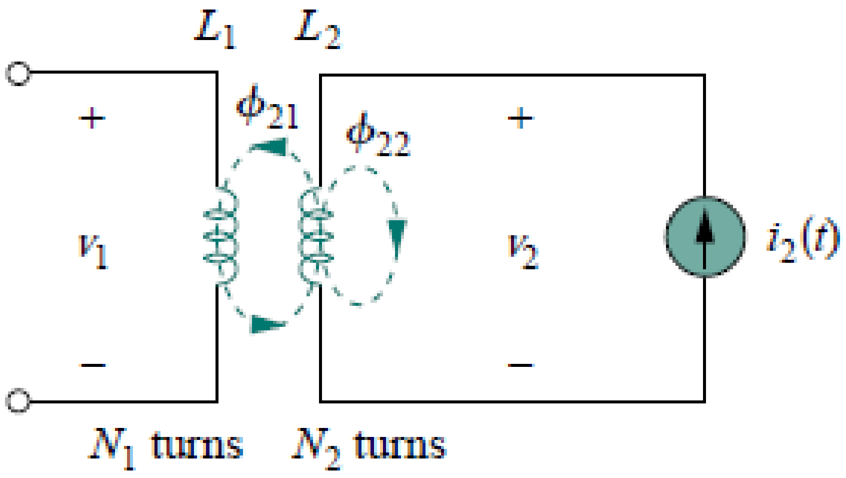
$$\phi_1 = \phi_{11} + \phi_{12}$$

$$v_1 = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

$$M_{21} = N_2 \frac{d\phi_{12}}{di_1}$$

Indutância mútua



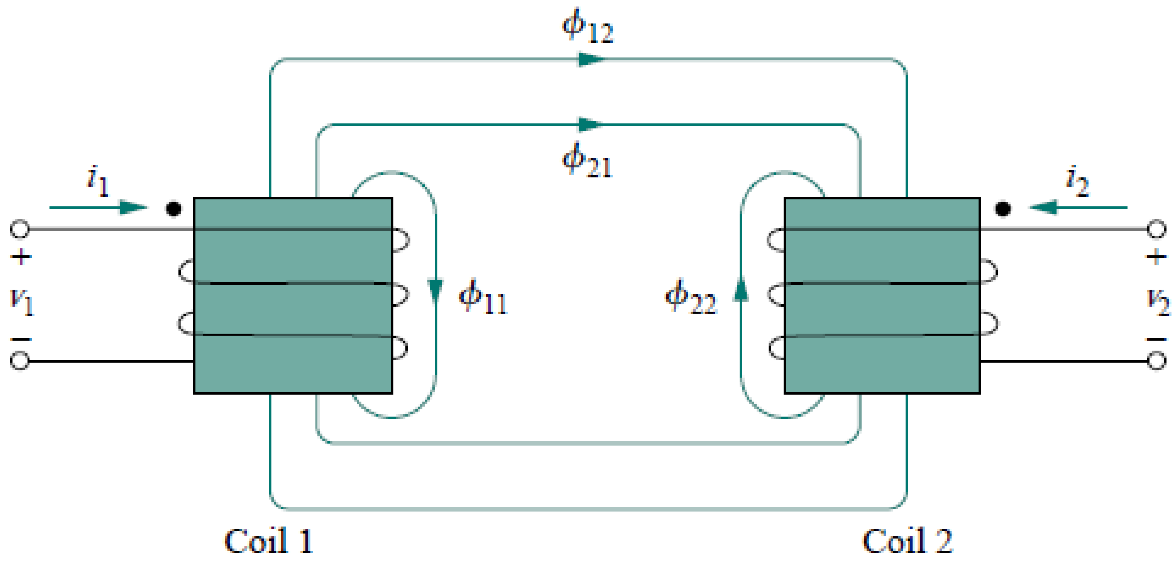
$$\phi_2 = \phi_{21} + \phi_{22}$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_2}{di_2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

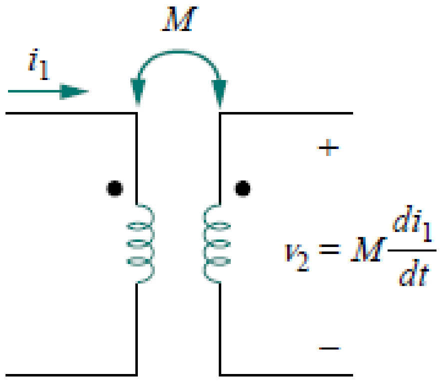
$$v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2}\frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

$$M_{12} = M_{21} = M$$

M indutância mútua vem em Henry

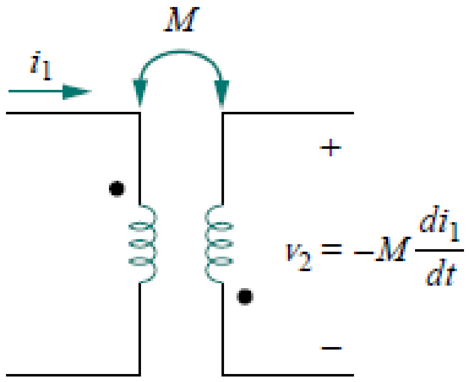


ϕ12
ϕ21
i1
i2
ϕ11
ϕ22
+
v1
−
+
v2
−
Coil 1
Coil 2



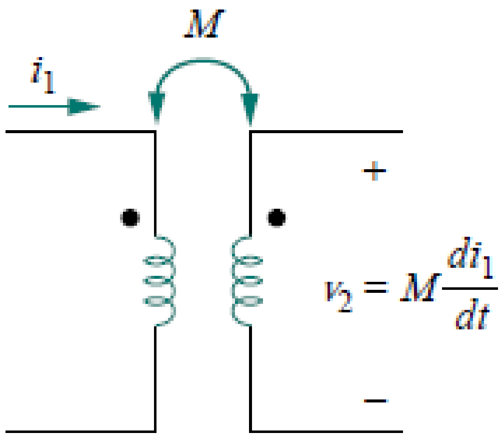
M
i1
+
−
v2 = M di1/dt

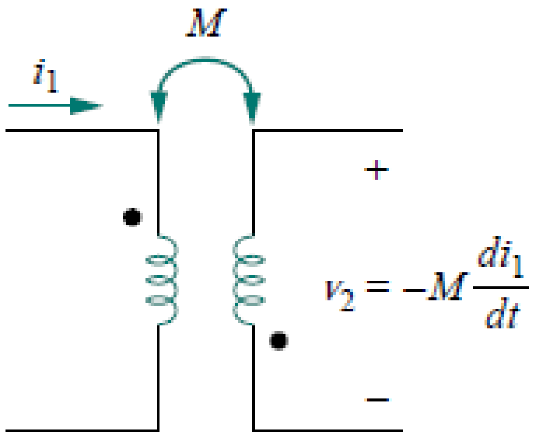
(a)

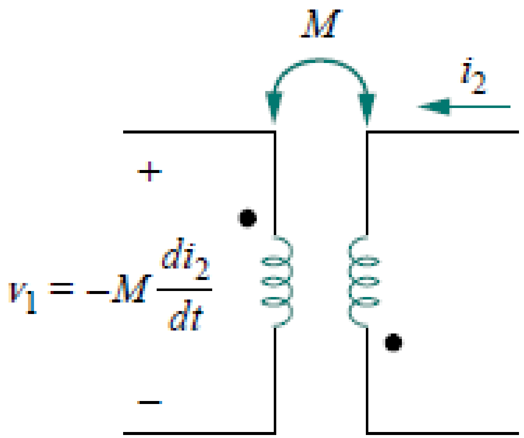


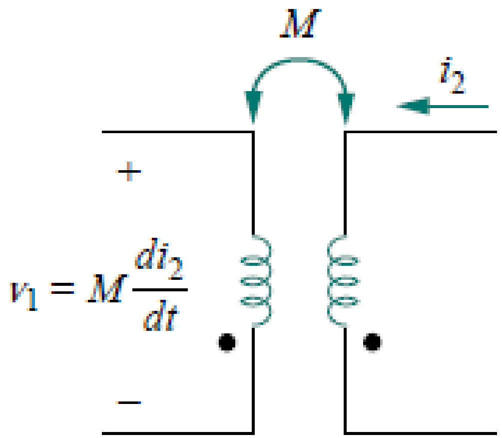
M
i1
+
−
v2 = −M di1/dt

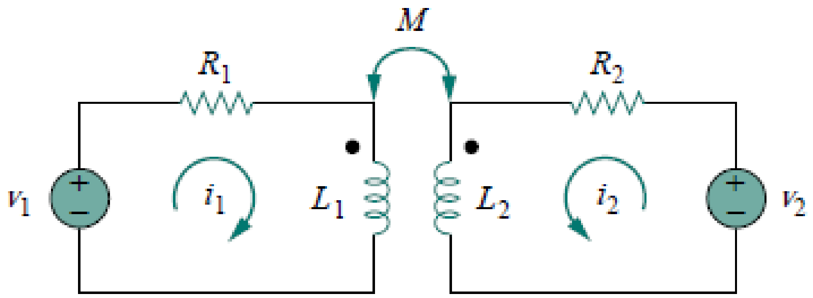
(b)

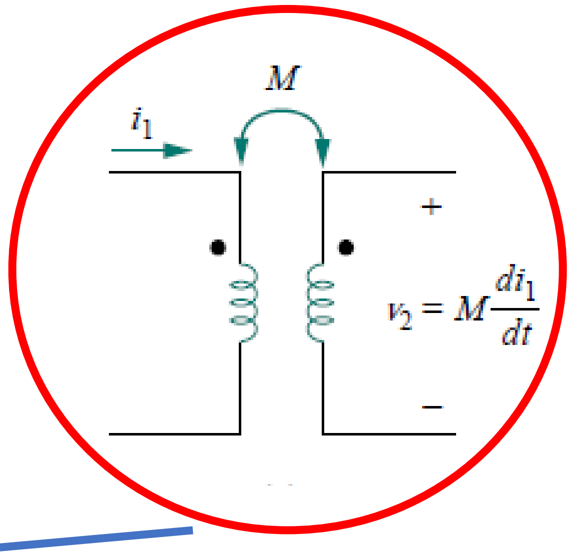

M
i_1
+
$v_2 = M\frac{di_1}{dt}$
–
(a)


M
i_1
+
$v_2 = -M\frac{di_1}{dt}$
–
(b)


M
i_2
+
$v_1 = -M\frac{di_2}{dt}$
–
(c)


M
i_2
+
$v_1 = M\frac{di_2}{dt}$
–



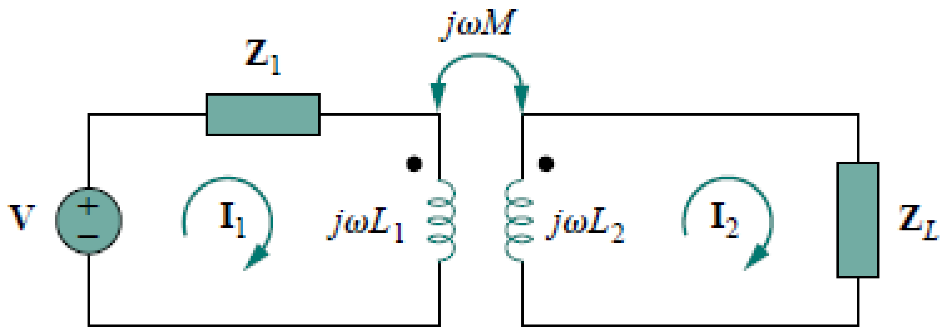


Para o coil 1 $$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Para o coil 2 $$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Domínio da frequência

$$\mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 + j\omega M \mathbf{I}_2$$
$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + (R_2 + j\omega L_2)\mathbf{I}_2$$

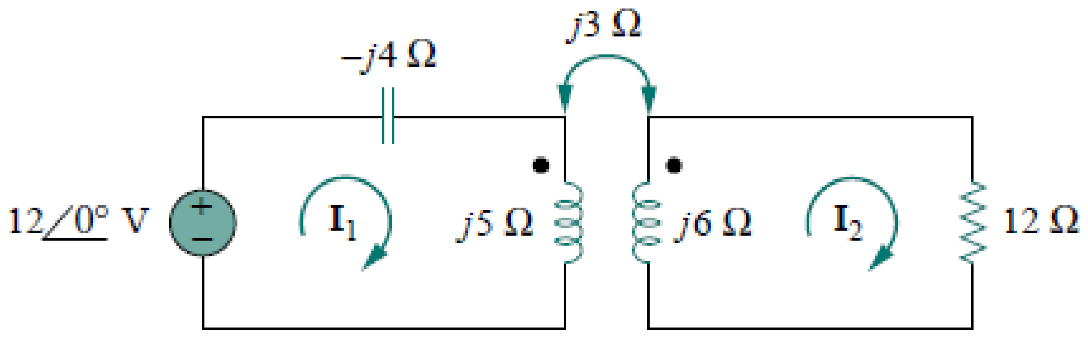


$$\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$

$$0 = -j\omega M\mathbf{I}_1 + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2$$

No contexto deste curso não estamos interessados em saber como se calcula por exemplo uma indutância mútua, com base nos objectos físicos

Exemplo: tomemos o circuito e calculemos os fasors das correntes



For coil 1, KVL gives

$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$$

or

$$j\mathbf{I}_1 - j3\mathbf{I}_2 = 12$$

For coil 2, KVL gives

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_1 = \frac{(12 + j6)\mathbf{I}_2}{j3} = (2 - j4)\mathbf{I}_2$$

Substituting this in Eq. (13.1.1), we get

$$(j2 + 4 - j3)\mathbf{I}_2 = (4 - j)\mathbf{I}_2 = 12$$

or

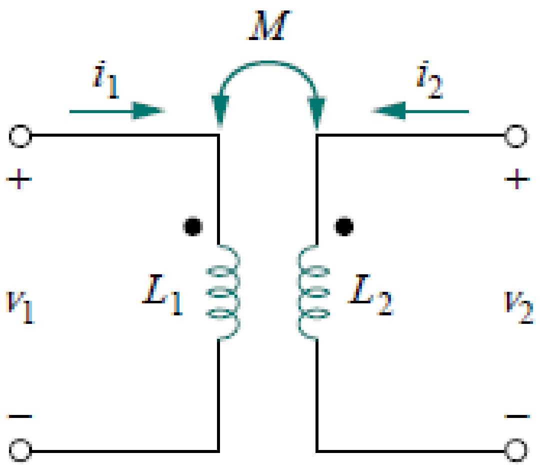
$$\mathbf{I}_2 = \frac{12}{4 - j} = 2.91\underline{/14.04^\circ}\ \text{A}$$

From Eqs. (13.1.2) and (13.1.3),

$$\mathbf{I}_1 = (2 - j4)\mathbf{I}_2 = (4.472\underline{/-63.43^\circ})(2.91\underline{/14.04^\circ})$$

$$= 13.01\underline{/-49.39^\circ}\ \text{A}$$

Energia num circuito magneticamente acoplado



$$w = \frac{1}{2}Li^2 \qquad p_1(t) = v_1 i_1 = i_1 L_1 \frac{di_1}{dt}$$

i_1 increase from zero to I_1 while maintaining $i_2 = 0$

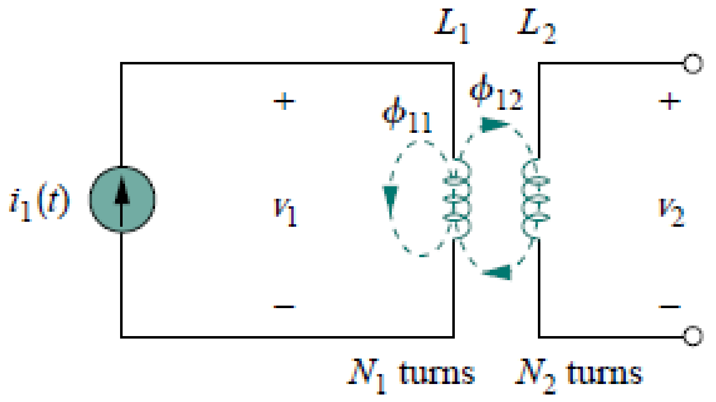
$$w_1 = \int p_1\,dt = L_1 \int_0^{I_1} i_1\,di_1 = \frac{1}{2}L_1 I_1^2$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

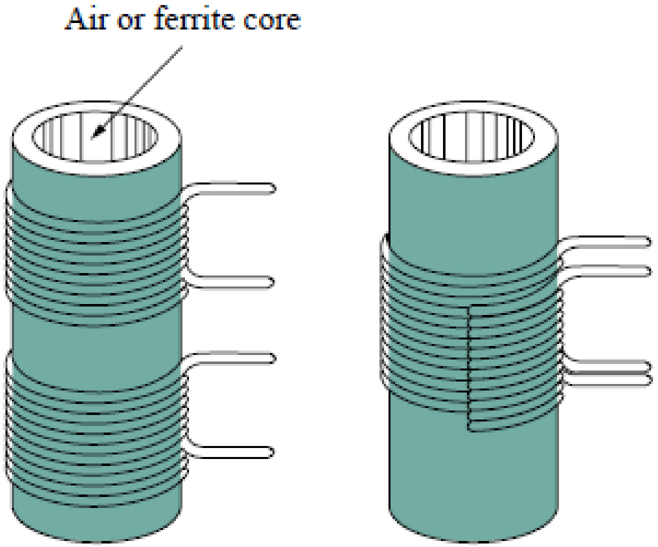
$$M = k\sqrt{L_1 L_2} \qquad 0 \leq k \leq 1$$

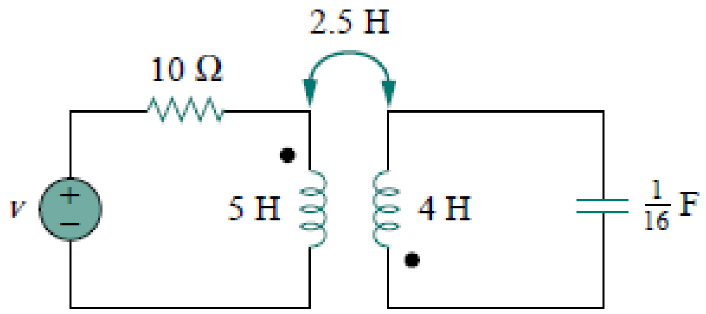
Coeficiente de acoplamento

Este coeficiente diz-nos qual a fracção do fluxo gerado por um indutor é apanhado pelo outro indutor



$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{12}}{\phi_{11} + \phi_{12}}$$





$$v = 60\cos(4t + 30^\circ)\ \text{V}.$$

Calcular o coeficiente de acoplamento e a energia armazenada para t=1s

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$

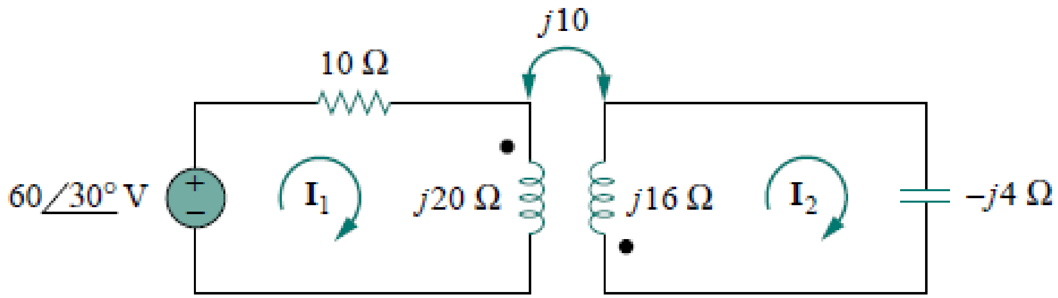
$$60\cos(4t + 30^\circ) \quad \Longrightarrow \quad 60\angle 30^\circ, \quad \omega = 4\ \text{rad/s}$$

$$5\ \text{H} \quad \Longrightarrow \quad j\omega L_1 = j20\ \Omega$$

$$2.5\ \text{H} \quad \Longrightarrow \quad j\omega M = j10\ \Omega$$

$$4\ \text{H} \quad \Longrightarrow \quad j\omega L_2 = j16\ \Omega$$

$$\frac{1}{16}\ \text{F} \quad \Longrightarrow \quad \frac{1}{j\omega C} = -j4\ \Omega$$



$$(10 + j20)\mathbf{I}_1 + j10\mathbf{I}_2 = 60\angle 30^\circ$$

Malha 1

Malha 2

$$j10\mathbf{I}_1 + (j16 - j4)\mathbf{I}_2 = 0$$

$$\mathbf{I}_1 = -1.2\mathbf{I}_2$$

$$\mathbf{I}_2 = 3.254\angle -160.6^\circ \text{ A}$$

$$\mathbf{I}_1 = -1.2\mathbf{I}_2 = 3.905\angle -19.4^\circ \text{ A}$$

$$i_1 = 3.905\cos(4t - 19.4^\circ), \qquad i_2 = 3.254\cos(4t - 199.4^\circ)$$

At time $t = 1$ s, $4t = 4$ rad $= 229.2^\circ$, and

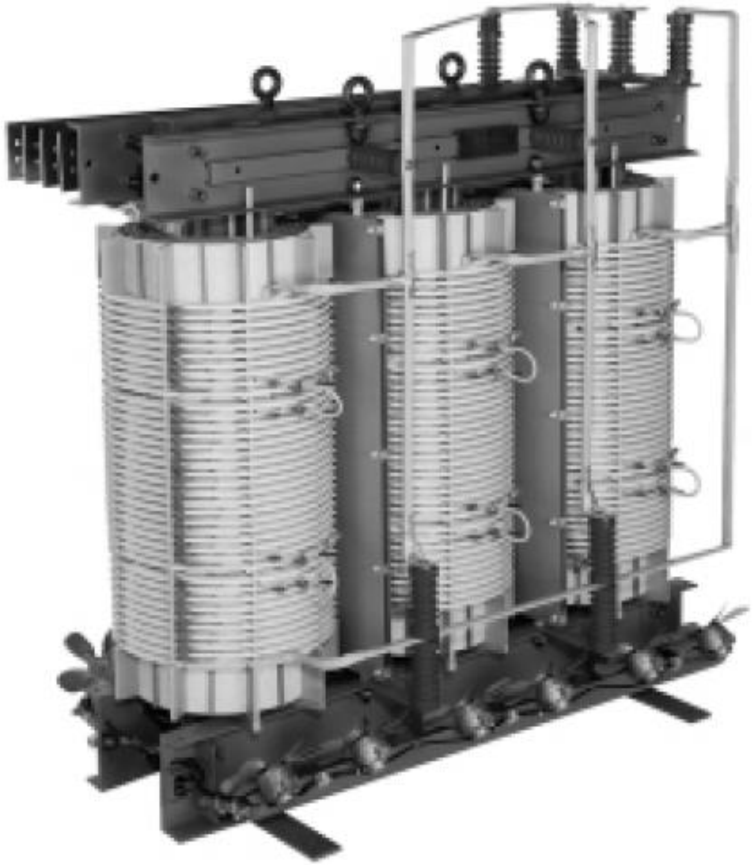
$$i_1 = 3.905\cos(229.2^\circ - 19.4^\circ) = -3.389 \text{ A}$$

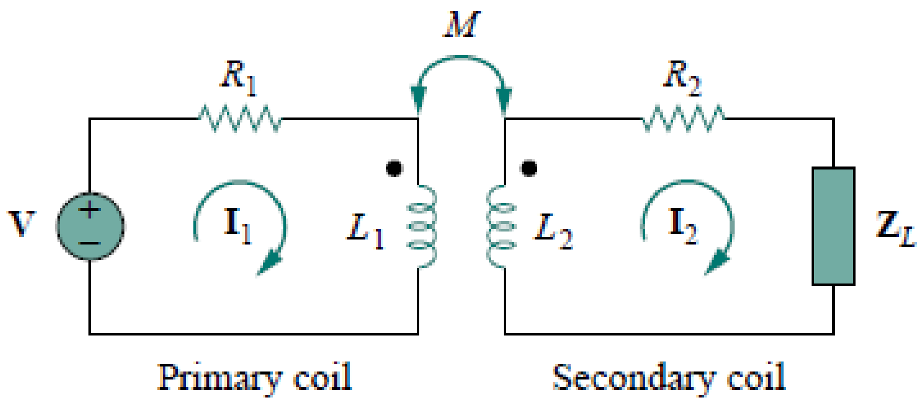
$$i_2 = 3.254\cos(229.2^\circ + 160.6^\circ) = 2.824 \text{ A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

$$= \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \text{ J}$$

Um transformadore é uma máquina eléctrica que utiliza a indutância mútua





Qual a impedância vista do gerador?

$$\mathbf{V} = (R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$

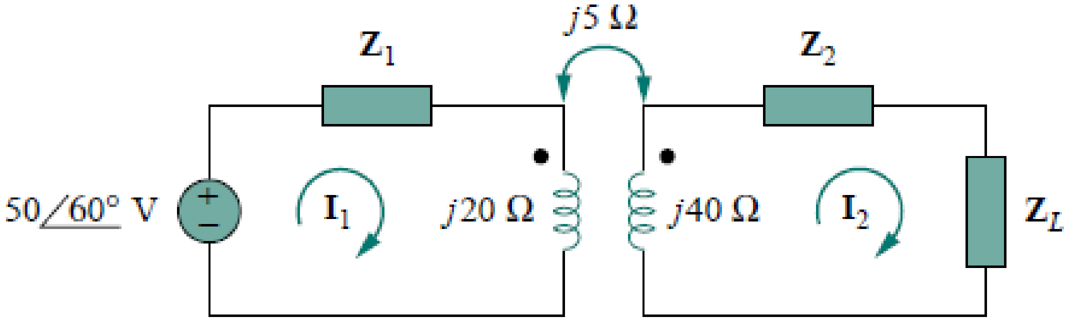
$$0 = -j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2 + \mathbf{Z}_L)\mathbf{I}_2$$

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = \underline{R_1 + j\omega L_1} + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

primário

Impedância do secundário reflectida no primário

Calcular a impedância vista pelo gerador e a currente I1



onde $\mathbf{Z}_1 = 60 - j100\ \Omega$, $\mathbf{Z}_2 = 30 + j40\ \Omega$, and $\mathbf{Z}_L = 80 + j60\ \Omega$.

$$
\begin{aligned}
\mathbf{Z}_{in} &= \mathbf{Z}_1 + j20 + \frac{(5)^2}{j40 + \mathbf{Z}_2 + \mathbf{Z}_L} \\
&= 60 - j100 + j20 + \frac{25}{110 + j140} \\
&= 60 - j80 + 0.14\underline{/-51.84^\circ} \\
&= 60.09 - j80.11 = 100.14\underline{/-53.1^\circ}\ \Omega
\end{aligned}
$$

$$
\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_{in}} = \frac{50\underline{/60^\circ}}{100.14\underline{/-53.1^\circ}} = 0.5\underline{/113.1^\circ}\ \text{A}
$$