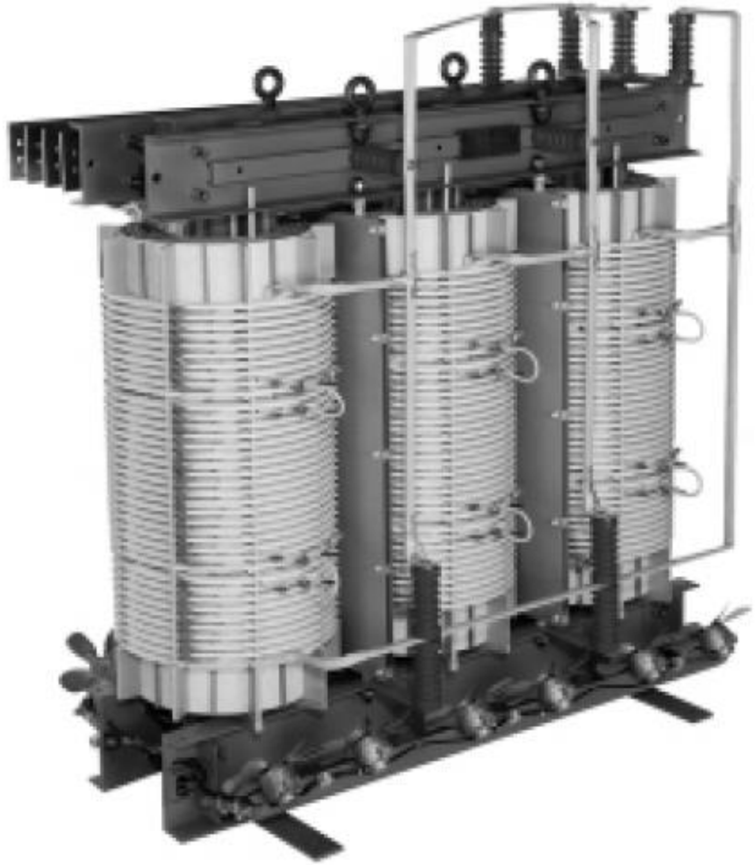
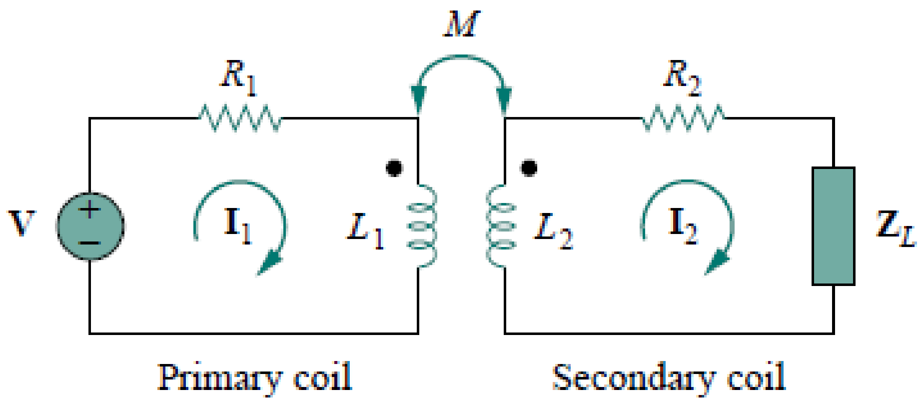
Sistemas magnéticamente acoplados

Transformadores

Um transformadore é uma máquina eléctrica que utiliza a indutância mútua





Qual a impedância vista do gerador?

$$\mathbf{V} = (R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M \mathbf{I}_2$$

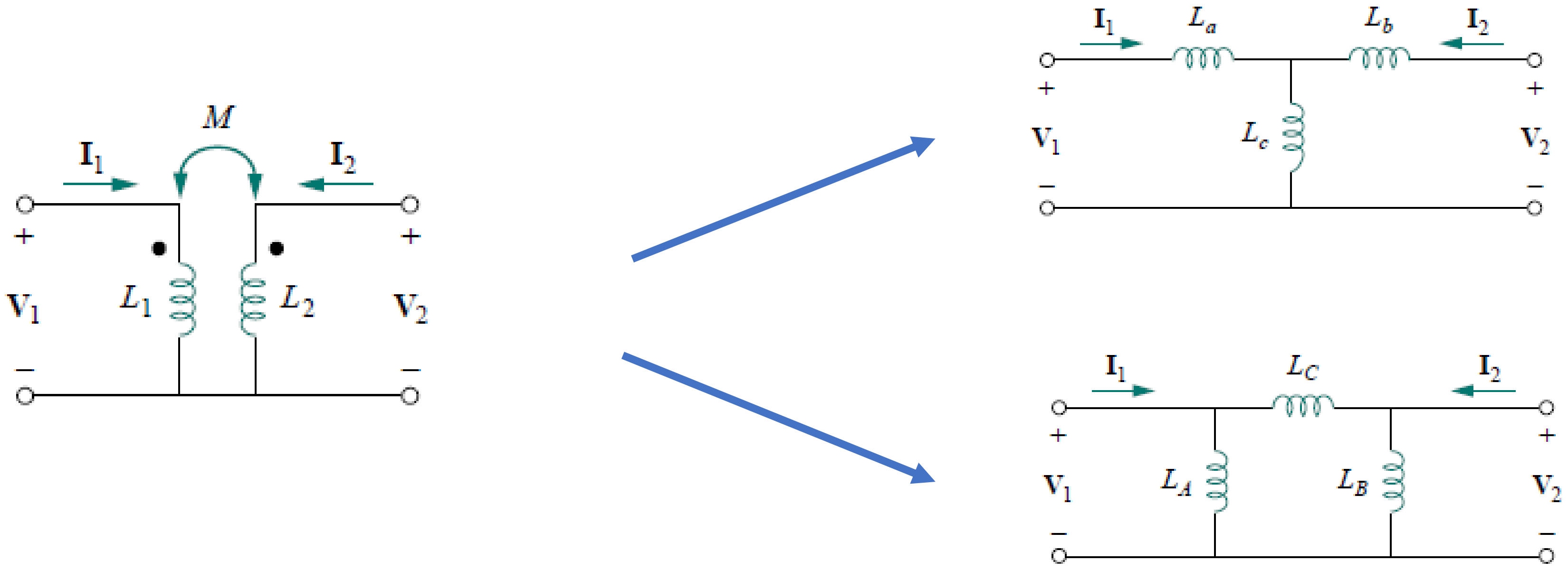
$$0 = -j\omega M \mathbf{I}_1 + (R_2 + j\omega L_2 + \mathbf{Z}_L)\mathbf{I}_2$$

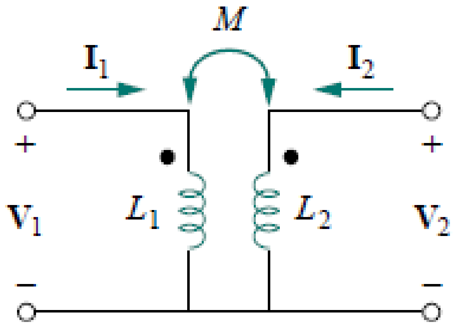
$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

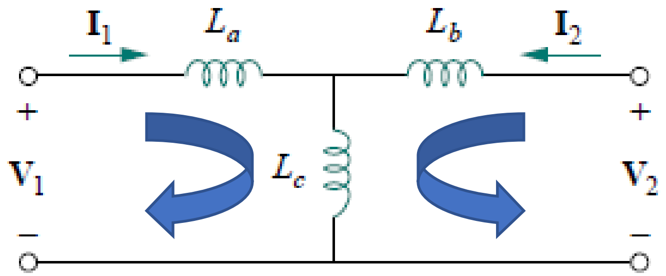
primário

Impedância do secundário reflectida no primário

Vemos que analisar circuitos com indutâncias mútuas é mais díficil do que quando estas não existem. Vampos por isso tentar transformar o circuito num circuito sem indutâncias acopladas







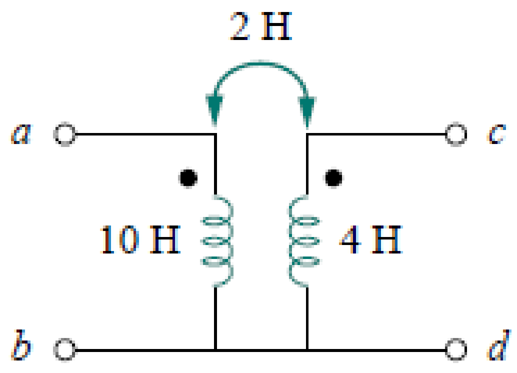
$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

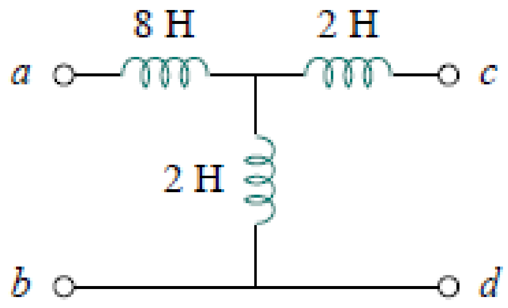
$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

Se $L_a = L_1 - M, \quad L_b = L_2 - M, \quad L_c = M$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{L_2}{j\omega(L_1L_2 - M^2)} & \dfrac{-M}{j\omega(L_1L_2 - M^2)} \\ \dfrac{-M}{j\omega(L_1L_2 - M^2)} & \dfrac{L_1}{j\omega(L_1L_2 - M^2)} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Os circuitos são equivalentes





$$L_a = L_1 - M = 10 - 2 = 8 \text{ H}$$

$$L_b = L_2 - M = 4 - 2 = 2 \text{ H}, \qquad L_c = M = 2 \text{ H}$$

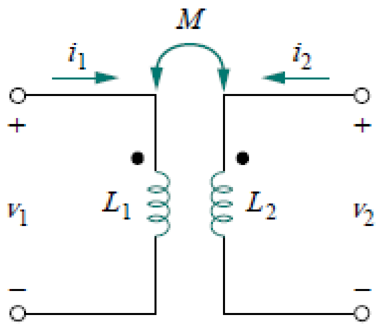
Transformadores ideais

$$\mathbf{V}_1 = j\omega L_1\mathbf{I}_1 + j\omega M\mathbf{I}_2$$
$$\mathbf{V}_2 = j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2$$

$$\mathbf{I}_1 = (\mathbf{V}_1 - j\omega M\mathbf{I}_2)/j\omega L_1$$

$$\mathbf{V}_2 = j\omega L_2\mathbf{I}_2 + \frac{M\mathbf{V}_1}{L_1} - \frac{j\omega M^2\mathbf{I}_2}{L_1}$$

$$M = \sqrt{L_1L_2}$$ K=1



$$\mathbf{V}_2 = j\omega L_2\mathbf{I}_2 + \frac{\sqrt{L_1L_2}\mathbf{V}_1}{L_1} - \frac{j\omega L_1L_2\mathbf{I}_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\mathbf{V}_1 = n\mathbf{V}_1$$

$$(L_1, L_2, M \to \infty)$$

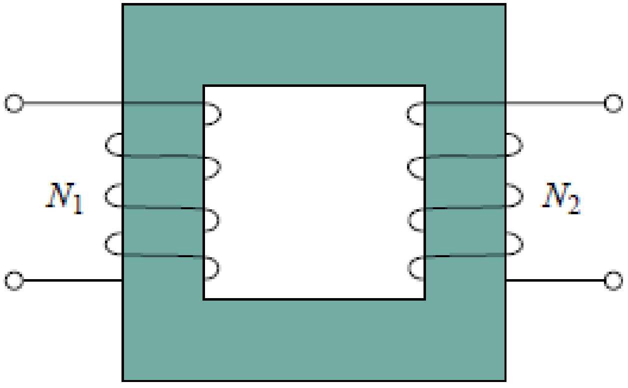
K=1

$$n = \sqrt{L_2/L_1}$$

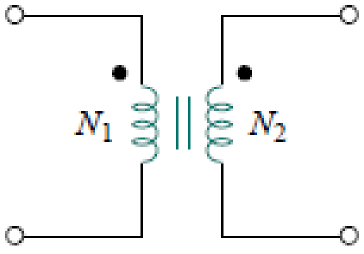
R1, R2 =0

n é designado por razão de voltas do transformador

O transformador é ideal



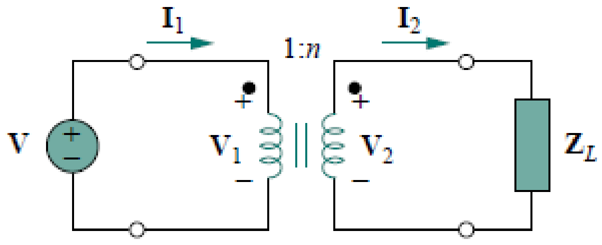
(a)



$$v_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n$$



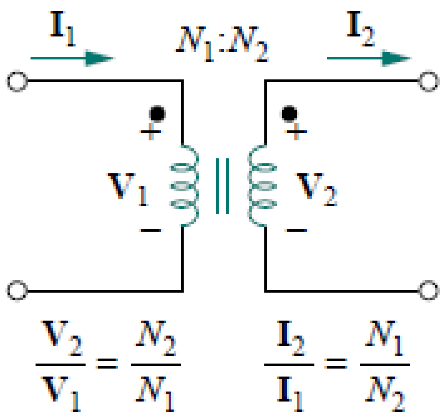
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n$$

Devido à conservação da energia

$$v_1 i_1 = v_2 i_2$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = n$$

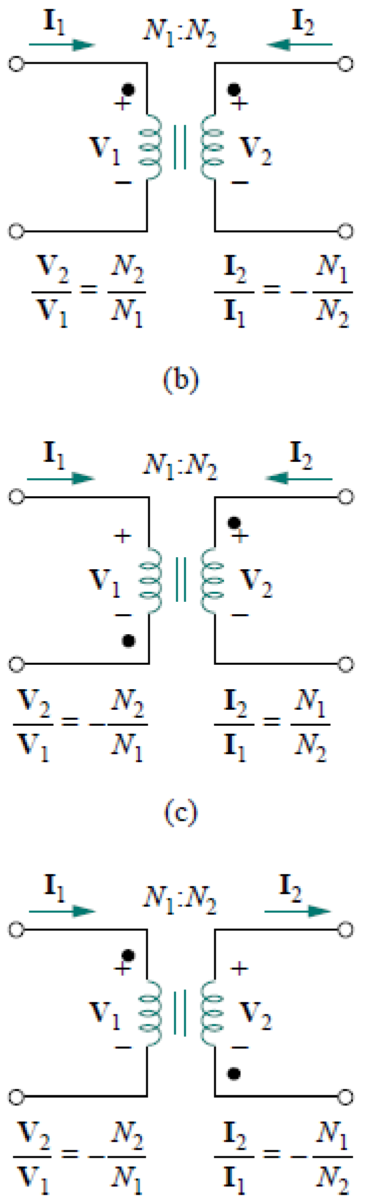
$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

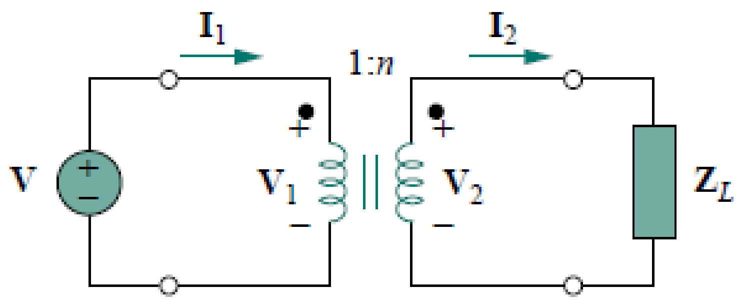


Se $V_2 < V_1$ transformador de redução

Se $V_2 > V_1$ transformador de elevação

Se $n=1$ transformador de isolamento ($V_2=V_1$)





$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{n^2}\frac{\mathbf{V}_2}{\mathbf{I}_2}$$

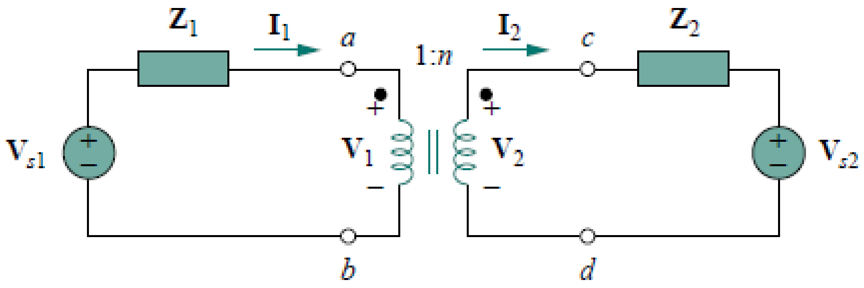
ZL

$$\mathbf{Z}_{in} = \frac{\mathbf{Z}_L}{n^2}$$

A impedância escrita desta forma é uma impedância reflectida (secundário aparece no primário)

Isto significa que podemos fazer adaptação de impedâncias para maximizar a transferência de energia

Uma metodologia de análise envolvendo transformadores é rfefelctir a impedÂncia do secundário no primário

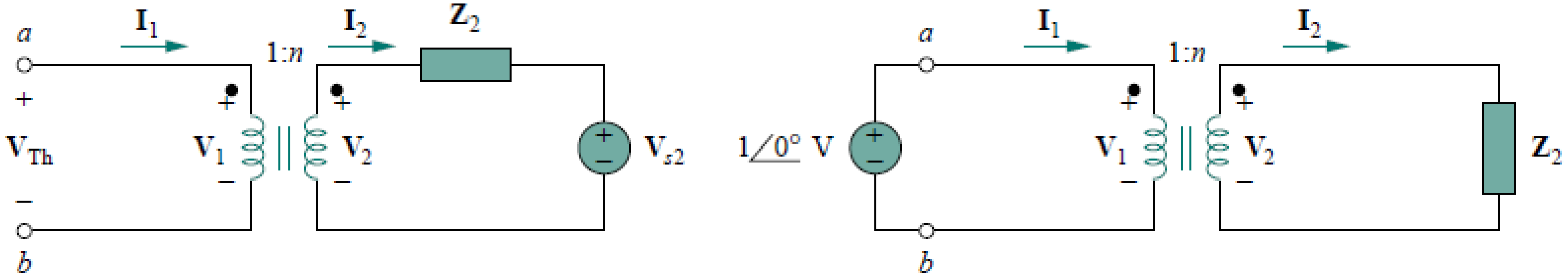


$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{n} \quad \text{or} \quad \mathbf{V}_2 = n\mathbf{V}_1$$

Para obter Vth, sabemos que temos de ter o circuito aberto entre a e b

$$\mathbf{I}_1 = 0 = \mathbf{I}_2 \qquad \mathbf{V}_2 = \mathbf{V}_{s2}.$$

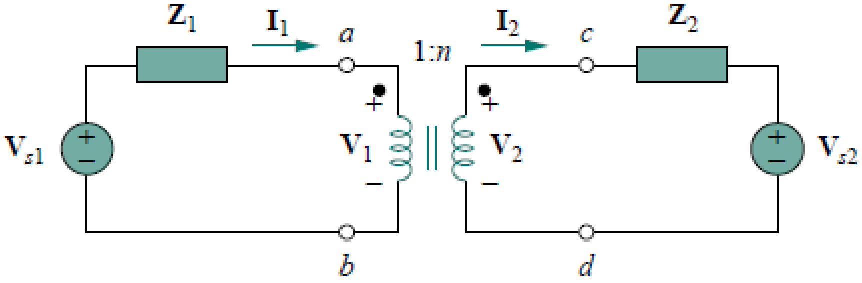
$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_1 = \frac{\mathbf{V}_2}{n} = \frac{\mathbf{V}_{s2}}{n}$$

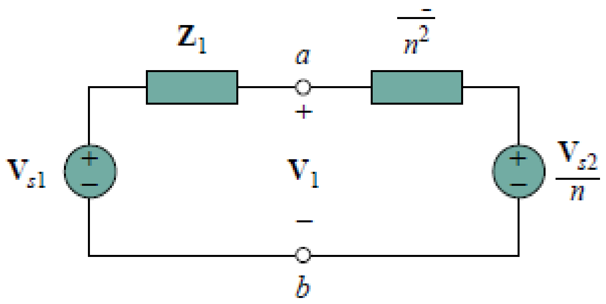


(a) (b)

$$\mathbf{I}_1 = n\mathbf{I}_2 \text{ and } \mathbf{V}_1 = \mathbf{V}_2/n,$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{V}_2/n}{n\mathbf{I}_2} = \frac{\mathbf{Z}_2}{n^2}, \qquad \mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}_2$$





The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side is: divide the secondary impedance by n^2, divide the secondary voltage by n, and multiply the secondary current by n.

The rule for eliminating the transformer and reflecting the primary circuit to the secondary side is: multiply the primary impedance by n^2, multiply the primary voltage by n, and divide the primary current by n.

Considerems um transformador

2400/120 V, 9.6 kVA E com 50 voltas no secundário

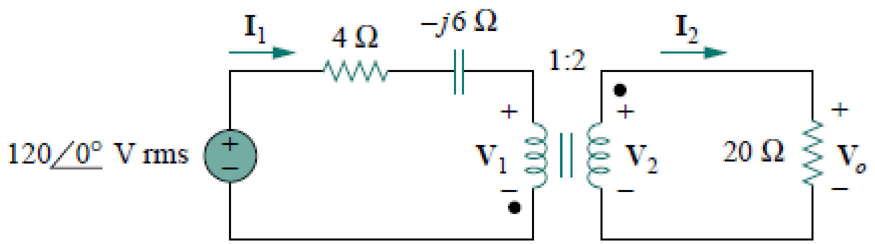
Trata-se de um transformador de redução $n = \frac{V_2}{V_1} = \frac{120}{2400} = 0.05$

$n = \frac{N_2}{N_1}$ $N_1 = \frac{50}{0.05} = 1000$voltas

$$S = V_1 I_1 = V_2 I_2 = 9.6\ \text{kVA}$$

$$I_1 = \frac{9600}{V_1} = \frac{9600}{2400} = 4\ \text{A}$$

$$I_2 = \frac{I_1}{n} = \frac{4}{0.05} = 80\ \text{A}$$



Calcular I1 e Vo

A impedância no secundário pode ser reflectida no primário

$$\mathbf{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5\ \Omega$$

$$\mathbf{Z}_{in} = 4 - j6 + \mathbf{Z}_R = 9 - j6 = 10.82\angle -33.69^\circ\ \Omega$$

$$\mathbf{I}_1 = \frac{120\angle 0^\circ}{\mathbf{Z}_{in}} = \frac{120\angle 0^\circ}{10.82\angle -33.69^\circ} = 11.09\angle 33.69^\circ\ \text{A}$$

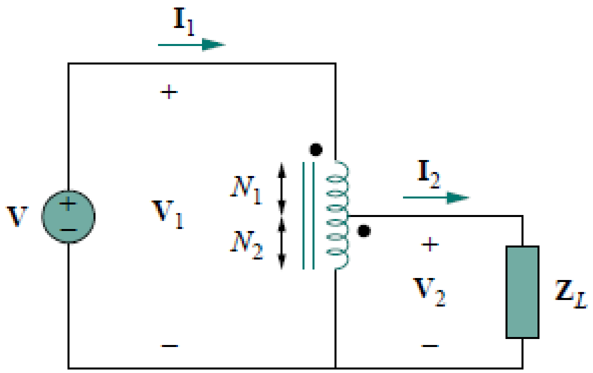
Como ambas as correntes saiem dos terminais com ponto

$$\mathbf{I}_2 = -\frac{1}{n}\mathbf{I}_1 = -5.545\angle 33.69^\circ\ \text{A}$$

$$\mathbf{V}_o = 20\mathbf{I}_2 = 110.9\angle 213.69^\circ\ \text{V}$$

Auto transformador

Como step-down



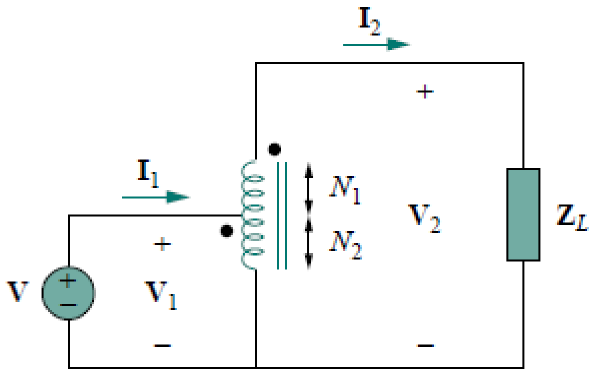
$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2}$$

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^*$$

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} \qquad \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1 + N_2}$$

Como step-up



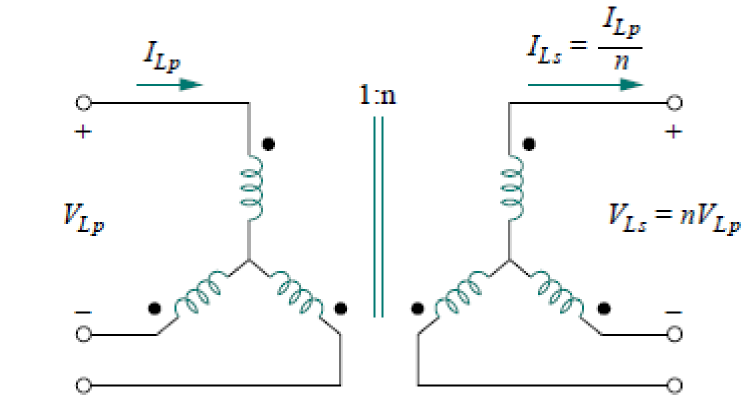
$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_1 + N_2}$$

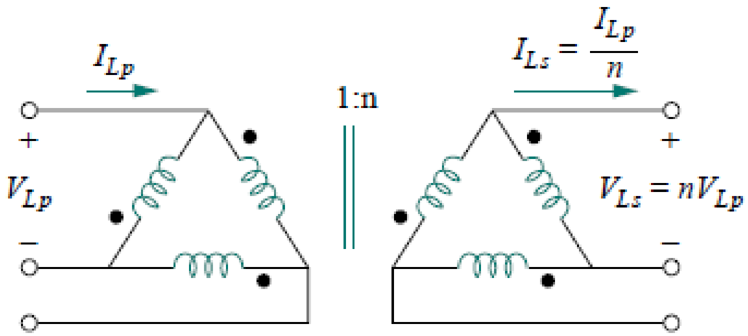
$$\boxed{\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_1 + N_2}}$$

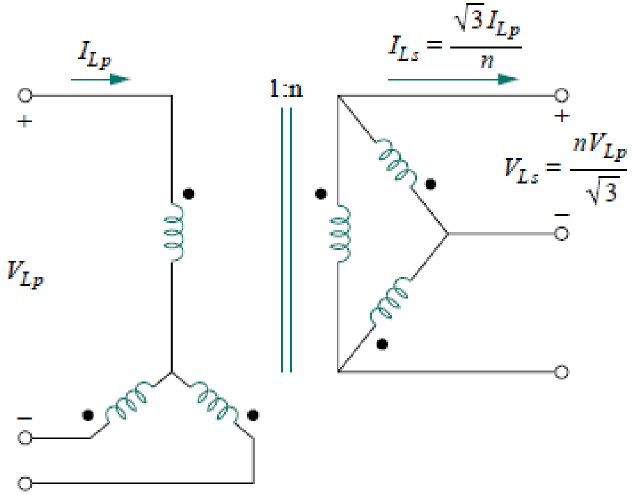
$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1}$$

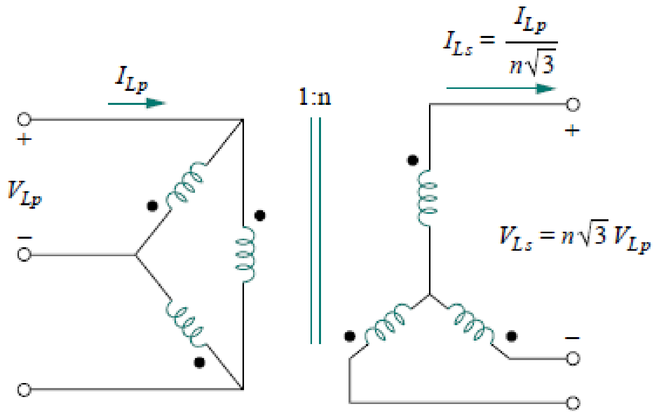
Transformadores trifásicos

Y-Y, Δ-Δ, Y-Δ, and Δ-Y

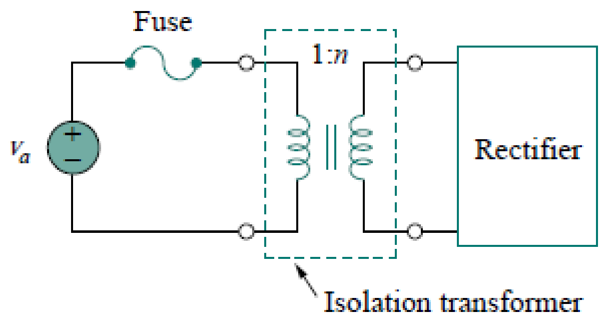


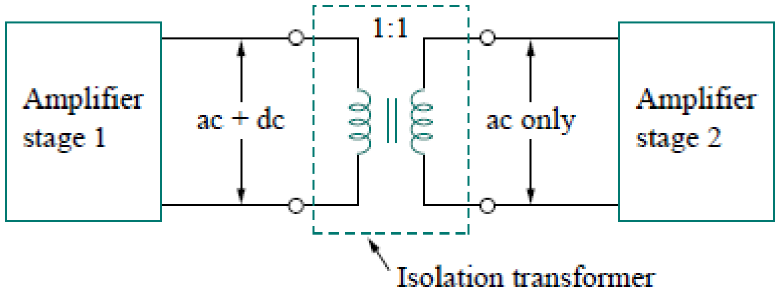


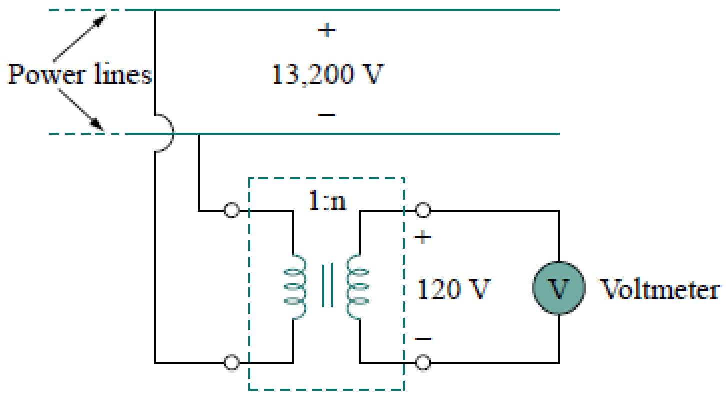


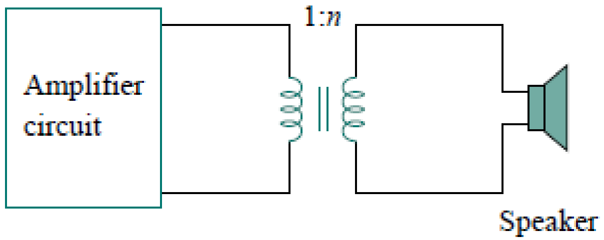


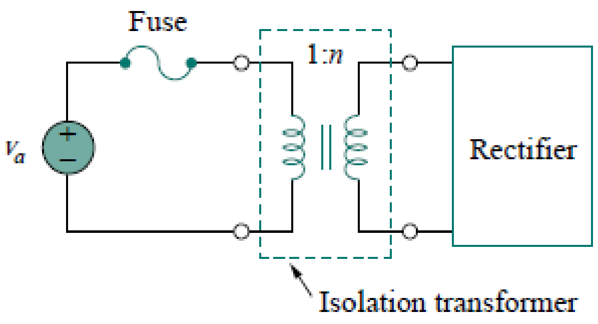
aplicações

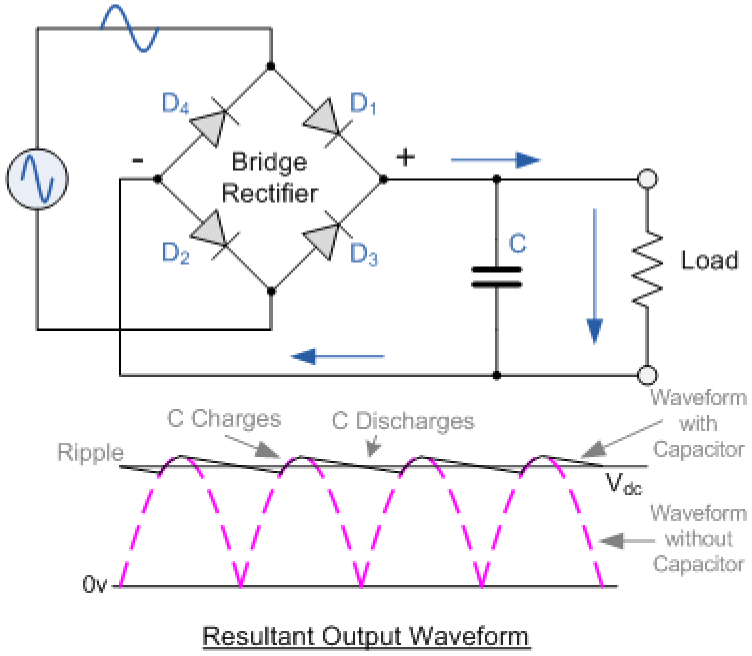

Fuse
1:n
V_a
Rectifier
Isolation transformer

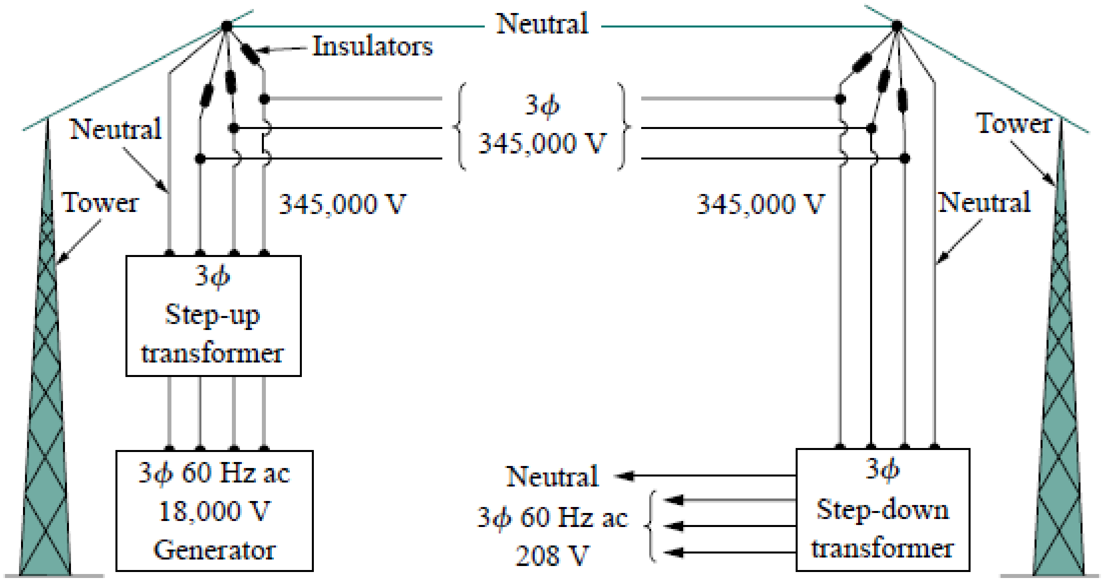

Amplifier stage 1
ac + dc
1:1
ac only
Amplifier stage 2
Isolation transformer

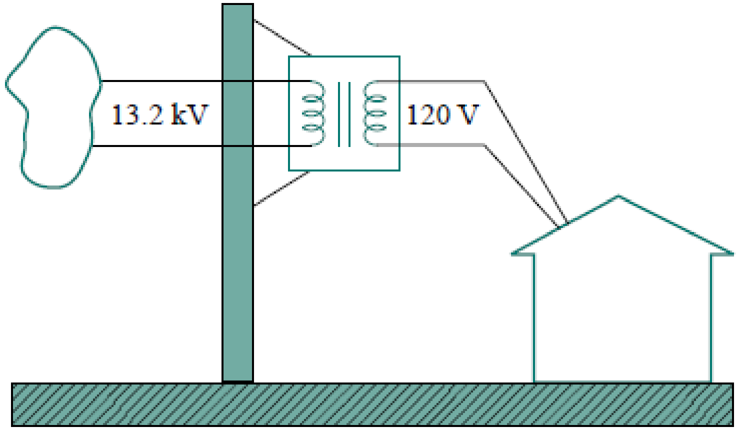

Power lines
+
13,200 V
−
1:n
+
120 V
−
V
Voltmeter


Amplifier circuit
1:n
Speaker


Fuse
1:n
v_a
Rectifier
Isolation transformer


D4
D1
Bridge
Rectifier
D2
D3
C
Load
Ripple
C Charges
C Discharges
Waveform with Capacitor
Vdc
Waveform without Capacitor
0v
Resultant Output Waveform


Neutral
Insulators
3ϕ
345,000 V
Neutral
Tower
345,000 V
345,000 V
Tower
Neutral
3ϕ
Step-up
transformer
3ϕ 60 Hz ac
18,000 V
Generator
Neutral
3ϕ 60 Hz ac
208 V
3ϕ
Step-down
transformer


13.2 kV
120 V