Instabilities

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Overview

- Fluid instabilities show up in everyday life, nature and engineering applications. A seemingly stable system may give rise to the development of an instability, which can cascade into turbulence.
- When the system is exposed to a perturbation, some wavelengths will grow, while others will not, governed by the parameters of the flow. This selectivity of specific structure sizes can be determined using linear stability analysis and then accounting for viscosity.
- Once these unstable wavelengths have grown to a substantial degree, the system becomes nonlinear before turbulence eventually sets in.
- Looking at buoyancy-driven instabilities, one can clearly see how certain wavelengths are selected. This can be extended to shear-driven instabilities and to other systems.
- For some flows, simplifications can be made to analyze the specific fluid structures, while for others, only broad conclusions can be drawn about the stability criteria.

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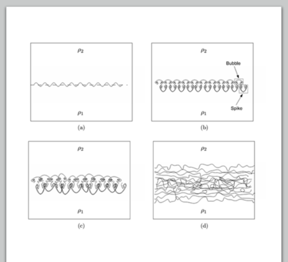
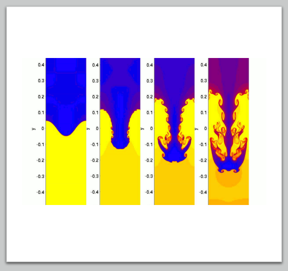

Rayleigh-Taylor

- Instability of an interface between two fluids of different densities, which occurs when the lighter fluid ρ_1 is pushing the heavier fluid ρ_2.

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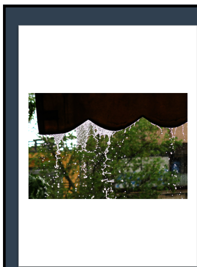
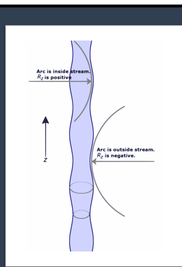
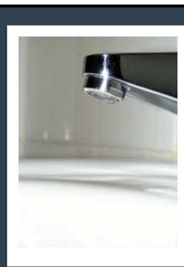

Rayleigh-Plateau

- Instability of a falling stream of fluid that breaks up into smaller packets with the same volume but less surface area.

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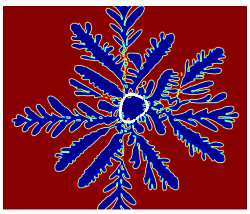
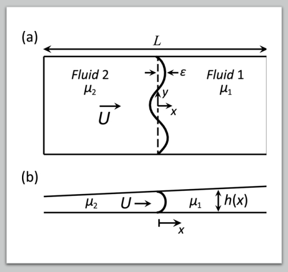

Saffman-Taylor

- Instability that occurs when a more viscous fluid μ_2, is pushed through a less viscous one μ_1 .

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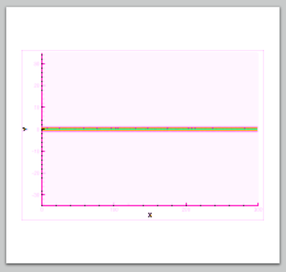


Kelvin-Helmhotz

- Instability that occurs when there is velocity shear in a single continuous fluid, or when there is a velocity difference across the interface between two fluids.

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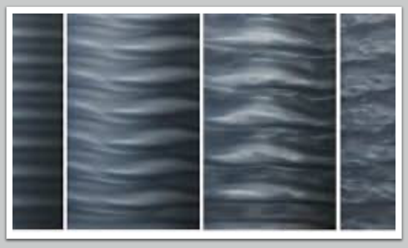

Taylor-Couette

Taylor showed that when the angular velocity of the inner cylinder is increased above a certain threshold, Couette flow becomes unstable and a secondary steady state characterized by axisymmetric toroidal vortices, known as **Taylor vortex** flow, emerges. Subsequently, upon increasing the angular speed of the cylinder the system undergoes a progression of instabilities which lead to states with greater spatio-temporal complexity, with the next state being called **wavy vortex flow.** Beyond a certain Reynolds number there is the onset of turbulence.

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Marangoni effect

Since a liquid with a high surface tension pulls more strongly on the surrounding liquid than one with a low surface tension, the presence of a gradient in surface tension will cause the liquid to flow away from regions of low surface tension.

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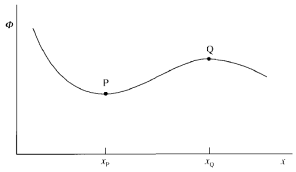

Figure 8.1 Positions of stable (P) and unstable (Q) equilibrium for a part whose potential energy Φ varies with x in the manner shown.

Stability, instability and overstability

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As every physicist knows, a dynamical system which is in equilbrium may be *stable* or *unstable*. The simplest case of the distinction is that of a particle of mass m which can move only in one dimension, in circumstances where the particle's potential energy Φ varies with its position x in the manner suggested by fig. 8.1. The particle experiences no force when it is situated at the minimum, P, or at the maximum, Q, and in principle it can remain at rest indefinitely in either of these positions. However, if it is slightly displaced from P it accelerates towards P, whereas if it is slightly displaced from Q it accelerates away from Q; in the first position the particle is stable and in the second it is unstable. Near any minimum such as P the restoring force $\partial\Phi/\partial x$ can normally be expanded as a Taylor series in powers of displacement $\xi_P = x - x_P$. Since it is zero at P itself, an adequate approximation for small values of ξ_P is

$$\frac{\partial \Phi}{\partial x} \approx \left(\frac{\partial^2 \Phi}{\partial x^2}\right)_P \xi_P,$$

in which case the equation of motion of the particle is linear in ξ_P,

$$m\,\frac{\partial^2 \xi_P}{\partial t^2} = -\left(\frac{\partial^2 \Phi}{\partial x^2}\right)_P \xi_P.$$

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The oscillations which it describes are then simple harmonic, with angular frequency ω_P such that

$$\omega_P^2 = \frac{1}{m}\left(\frac{\partial^2 \Phi}{\partial x^2}\right)_P.$$

An equation of motion similar to (8.1) applies in the neighbourhood of Q, but since $(\partial^2\Phi/\partial x^2)_Q$ is negative the roots for ω are necessarily imaginary, $\omega_Q = \pm i s_Q$ with s_Q real. Hence the displacement $\xi_Q = x - x_Q$ of a particle which starts at rest at $t = 0$ from a position such that $\xi_Q = \xi_o$ is given at later times by

$$\xi_Q \approx \frac{1}{2}\,\xi_o\,(e^{s_Q t} + e^{-s_Q t}),$$

as long as it remains small. If ξ_o is infinitesimal, then by the time the displacement becomes apparent $\exp(s_Q t)$ must be very much greater than unity, in which case $\exp(-s_Q t)$ must be negligible. When a particle leaves a position of unstable equilibrium, therefore, its displacement normally grows in an exponential fashion.

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Suppose now that $(\partial\Phi/\partial x)_P$ is necessarily always zero – perhaps because of some symmetry requirement – while $(\partial^2\Phi/\partial x^2)_P$ can be reduced in magnitude and ultimately reversed in sign by altering the external constraints which determine Φ. In that case P is always an equilibrium position, but the equilibrium is stable in one range of the constraints and unstable in an adjacent range. Where the changeover occurs one has

$$\omega_P^2 = \frac{1}{m}\left(\frac{\partial^2 \Phi}{\partial x^2}\right)_P = 0,$$

and this is the condition for what is called *marginal stability*. When it is satisfied, the force experienced by a particle near P is normally determined by $(\partial^3\Phi/\partial x^3)_P$ or, if $(\partial^3\Phi/\partial x^3)_P$ is zero for symmetry reasons, by $(\partial^4\Phi/\partial x^4)_P$; it is then proportional to ξ_P^2 or ξ_P^3 rather than to ξ_P.

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Similar results apply, of course, to any mechanical system for which energy is conserved. If the system is a complicated one, a full description of its state requires specification of a great many different coordinates of position. There always exists a set of normal coordinates ζ_n, however, such that for small ζ_n the potential energy Φ and kinetic energy T of the system may be expressed in the form

$$\Phi = \Phi_0 + \sum_n \frac{1}{2} m_n \omega_n^2 \zeta_n^2,$$

$$T = \sum_n \frac{1}{2} m_n \left(\frac{\partial \zeta_n}{\partial t}\right)^2,$$

where Φ_0 is the potential energy of the equilibrium state for which all ζ_n are zero, and the equilibrium is stable if and only if $m_n\omega_n^2 > 0$ for *all* values of n. In continuous systems the normal coordinates often describe periodic modes of distortion of the system as a whole, rather than displacements of isolated parts of the system.

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Strut under compression (Euler)

Consider, as an example, the well-known problem first solved by Euler of a uniform elastic strut of length L which is straight in equilibrium but free to bend, and which is compressed by longitudinal forces $\pm F$ applied to its ends [fig. 8.2]. When the ends are pinned in such a way that they cannot move except towards one another, the normal coordinates describe the amplitude of flexural modes in which the transverse displacement varies sinusoidally, like $\sin(n\pi x/L)$ where x is distance measured along the strut from one end and where $n = 1, 2, 3$ etc. The potential energy stored in the elastic deformation of the strut associated with the nth mode is proportional for small ζ_n to $n^4\zeta_n^2$, while the corresponding potential energy stored in whatever system supplies the compressive force is proportional to $-Fn^2\zeta_n^2$. When $F = 0$ the strut in its undeformed state is completely stable, because the second derivatives of the total potential energy, $\partial^2\Phi/\partial\zeta_n^2$, are positive for all n. As F is increased from zero, the marginal stability condition, $\partial^2\Phi/\partial\zeta_n^2 = 0$, is reached first for $n = 1$. Once the load which corresponds to that condition is exceeded the strut becomes unstable and is bound to buckle. Because $\partial^4\Phi/\partial\zeta_1^4$ is positive it stabilises after buckling at a finite value of ζ_1 (which may have either sign), but that is an aspect of the problem which need not concern us here.

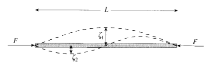

Figure 8.2 A strut under compression. The broken curves suggest two normal modes of flexural distortion, corresponding to $n = 1$ and $n = 2$.

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Dissipative systems

In so far as the above remarks apply to conservative systems they may seem to have little relevance to viscous fluids, which are inherently dissipative. If, however, a particle moving in the potential of fig. 8.1 is subject to a dissipative retarding force proportional to its velocity, the principal effect of this is merely to damp – and perhaps overdamp – oscillations in ξ_P and to slow down the exponential rate of growth of ξ_Q. That does not invalidate the conclusion that P and Q represent states of stable and unstable equilibrium respectively. Indeed, the fluctuations which always accompany dissipation in thermal equilibrium now make it impossible in principle, as well as in practice, for a particle to remain indefinitely at Q. Nor does the existence of dissipation invalidate the conclusion that when, as a result of a continuous change in the form of $\Phi\{x\}$, the equilibrium at P changes from being stable to being unstable, this equilibrium passes through a state of marginal stability.

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The general procedure for investigating the stability or otherwise of patterns of fluid flow involves perturbing the pattern in various ways and calculating whether the amplitude – say $\zeta_n\{t\}$ – of each perturbation mode decreases or increases with time; the amplitude may well describe a velocity rather than a displacement, but that is a rather trivial distinction in this context. The modes must be consistent with the boundary conditions to which the fluid is subject, and they should form, like the periodic normal modes of the Euler strut, a complete set in terms of which any possible perturbation may be expanded. The exact equations of motion of the fluid are always non-linear in ζ_n, and one cannot achieve a detailed understanding of what happens once an instability has developed without taking non-linear terms into account. As a first step, however, it may suffice to establish the condition for a state of marginal stability to exist; having done that, one may confidently assert that true stability lies on one side of this condition and instability on the other.

Since marginal stability requires

$$\frac{\partial \zeta_n}{\partial t} = 0 \qquad (8.2)$$

to first order only in ζ_n, the condition for its existence may be established using approximate equations of motion from which all terms which are non-linear in ζ_n have been deleted. If, as is often the case, there are several competing modes of instability, the first to develop once the condition for marginal stability has been exceeded is normally the one for which s_n ($= \zeta_n^{-1}\partial\zeta_n/\partial t$) is largest. Linearised equations of motion suffice to settle this question as well.

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Oscillatory instabilities

A characteristic of dissipative systems, which cannot readily be illustrated by reference to simple mechanical models of point masses moving in variable potentials but which is familiar in the context of electrical circuits, is that they may spontaneously oscillate. To do so they must incorporate a source of power, of course, and some feedback mechanism which selectively amplifies an oscillatory component in the thermal fluctuations of the system. At the onset of an oscillatory instability a system is said to become *overstable* (a potentially misleading term for which Eddington was responsible). If the amplitude $\zeta_n\{t\}$ of the overstable mode is taken to vary with time like $\exp\{-i(\omega_n + is_n)t\}$ for small ζ_n, where ω_n ($\neq 0$) and s_n are both real, overstability requires s_n to be positive whereas stability requires it to be negative. The condition for marginal overstability is $s_n = 0$, i.e.

$$\frac{\partial \zeta_n}{\partial t} = -i\omega_n\zeta_n$$

to first order in ζ_n.

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Rayleigh-Taylor instability

The Rayleigh–Taylor instability arises when a vessel which contains two fluids separated by a horizontal interface – one at least of the fluids must of course be a liquid – is suddenly inverted so that the heavier fluid lies above the lighter one. The gravitational potential energy of the system, which was at its minimum value before inversion, is now at its maximum, and although the system is still in equilibrium while the interface remains horizontal the equilibrium is clearly liable to be unstable. Whether or not it is actually unstable with respect to any particular perturbation depends upon whether the gravitational energy which this releases is greater or less than the increase in surface free energy. The system is marginally stable with respect to the perturbation when the two are equal.

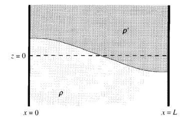

Figure 8.3 A layer of one fluid with a denser fluid above it, in a container of width L, is stabilised by surface tension against the perturbation suggested here provided that (8.6) is satisfied.

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All possible small perturbations of the surface may be expressed in terms of their Fourier components, a typical Fourier component involving a vertical displacement of the interface

$$\zeta = \zeta_k\{t\}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}},$$

where $\boldsymbol{r}$ is a vector which lies in the $z = 0$ plane, i.e. the plane of the undisturbed interface. Per unit area of the interface, the reduction in gravitational potential energy associated with a single wave of this form, averaged over any integral number of wavelengths, is [(5.29)]

$$\frac{1}{4}(\rho' - \rho)g\zeta_k^2,$$

where ρ' and ρ are the densities of the heavier and lighter fluids respectively. The increase of the surface free energy, similarly averaged, is

$$\sigma\left\langle\left\{1 + \left(\frac{\partial\zeta}{\partial x}\right)^2 + \left(\frac{\partial\zeta}{\partial y}\right)^2\right\}^{1/2} - 1\right\rangle \approx \frac{1}{4}\sigma k^2\zeta_k^2$$

to second order in ζ_k, where σ is the interfacial surface tension. Marginal stability is therefore only possible for one wavevector k_c, such that

$$(\rho' - \rho)g = \sigma k_c^2. \qquad (8.4)$$

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In order to find the rate at which modes for which $k < k_c$ grow in amplitude, one needs to know how the velocities of each fluid depend upon $\partial\zeta_k/\partial t$. With that information at one's disposal, one may follow the routine procedure of evaluating the mean kinetic energy per unit area and hence the total energy, a sum of gravitational and surface terms proportional to ζ_k^2 and kinetic terms proportional to $(\partial\zeta_k/\partial t)^2$; by equating the time derivative of the total energy to zero one may then obtain, after cancellation of a factor $\partial\zeta_k/\partial t$, a linear equation of motion relating ζ_k to $\partial^2\zeta_k/\partial t^2$ which provides the required answer. We, however, can make use of a result already available as (5.40), which tells us, in the notation of §8.1, that the dispersion relation for waves on the interface is

$$\omega_k^2 = -\frac{\rho' - \rho}{\rho' + \rho}gk + \frac{\sigma k^3}{\rho' + \rho}, \quad \text{with } s_k = 0,$$

as long as k is greater than the critical wavevector which (8.4) describes, which implies that when $k < k_c$ we have

$$s_k^2 = \frac{\rho' - \rho}{\rho' + \rho}gk - \frac{\sigma k^3}{\rho' + \rho}, \quad \text{with } \omega_k = 0.$$

The value of k, say k_{max}, which maximises s_k and hence the rate of growth is clearly such that

$$(\rho' - \rho)g = 3\sigma k_{max}^2,$$

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so

$$k_{max} = \frac{k_c}{\sqrt{3}}. \qquad (8.5)$$

We may infer from the above results that if it were possible to invert almost instantaneously a *large* vessel containing two fluids, so large that the boundary conditions imposed virtually no limitations on the allowed values of $\boldsymbol{k}$, the contents would be inherently unstable. The interface would inevitably develop corrugations whose periodicity would be the wavelength associated with k_{max}, i.e. $2\pi\sqrt{3\sigma/(\rho' - \rho)g}$, which amounts to about 3 cm when the heavier fluid is water and the lighter one is air. In practice, however, rapid inversion is possible only with small vessels, and the fact that liquid inside an inverted bottle is stabilised by surface tension if the opening of the bottle is small enough must be familiar to every reader. For simplicity, suppose the vessel to be a rectangular one, with vertical sides and a cross-section in the $z = 0$ plane of which the larger dimension is L. The smallest non-zero value of k consistent with the boundary conditions [fig. 8.3 and some remarks about the boundary conditions applicable to water waves at the start of §5.8] is then π/L. In that case the inverted contents are stable provided that $\pi/L > k_c$, i.e. provided that

$$L < \pi\sqrt{\frac{\sigma}{(\rho' - \rho)g}}. \qquad (8.6)$$

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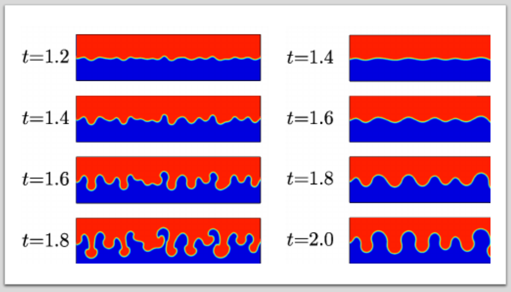

Rayleigh-Taylor (MFM 372)

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Rayleigh-Plateau

A free jet of water, emerging from a circular orifice, is liable to break up into a regular succession of drops, and according to Plateau's analysis of some observations by Savart the drops are separated by a distance λ which is about 8·8 times the radius a of the jet before it disintegrates. If a stationary cylinder of water could be obtained it would break up in the same way, and indeed the droplets of water which are to be seen on spiders' webs after a damp cold night are probably formed by accretion from layers of dew which are cylindrical when first deposited. The explanation lies in the fact that, volume for volume, spheres have smaller surface areas than cylinders.

Suppose an initially uniform cylinder of liquid to be subject to a small *varicose deformation*, which preserves rotational symmetry about the x axis (the axis of the cylinder) but alters its radius in a periodic fashion from a to

$$b = \langle b\rangle + \zeta_k \cos kx \ (\zeta_k \ll a).$$

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The volume of the cylinder per unit length, averaged over an integral number of wavelengths, is

$$V = \langle\pi b^2\rangle = \pi\langle b\rangle^2 + \frac{1}{2}\pi\zeta_k^2,$$

and since this must equal the initial volume per unit length, πa^2, we have

$$\langle b\rangle = \sqrt{a^2 - \tfrac{1}{2}\zeta_k^2} \approx a - \frac{\zeta_k^2}{4a}.$$

Thus the surface area of the cylinder per unit length, similarly averaged, is

$$A = \left\langle 2\pi b\sqrt{1 + \left(\frac{\mathrm{d}b}{\mathrm{d}x}\right)^2}\right\rangle$$

$$\approx 2\pi a + \frac{\pi\zeta_k^2}{2a}\{(ka)^2 - 1\}.$$

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In this problem there is no gravitational term to consider, and it is the surface free energy per unit length, σA, which plays the role of the potential energy Φ of §8.1. The condition for marginal stability is $\partial^2 A/\partial\zeta_k^2 = 0$, equivalent to

$$k = k_c = \frac{1}{a}\cdot$$

The cylinder is inherently unstable, as Plateau was the first to note, to any periodic deformation for which k is less than k_c, i.e. for which the wavelength λ is greater than $2\pi a$.

To find the rate of growth of a mode for which $k < k_c$ one may follow the routine procedure outlined in §8.2. Provided that the viscosity of the liquid may be neglected, i.e. provided that potential theory may be employed, it is not difficult to calculate the fluid velocity $\boldsymbol{u}\{x, r\}$ associated with rate of change of ζ_k. It is described by a flow potential ϕ which is a solution of Laplace's equation proportional to $\cos(kx)f\{r\}(\partial\zeta_k/\partial t)$; the function $f\{r\}$ involves Bessel functions. Hence the constant of proportionality relating the fluid's mean kinetic energy per unit length to $(\partial\zeta_k/\partial t)^2$ may be found, and the equation of motion relating $\partial^2\zeta_k/\partial t^2$ to ζ_k follows immediately. According to Rayleigh, s_k, which is zero where $k = k_c$, reaches a maximum where $k = 0{\cdot}697k_c$ or where $\lambda = 9{\cdot}02a$, in reasonable agreement with Savart's observations. The 2% discrepancy, in the wrong direction to be due to viscosity, is attributable to experimental error.

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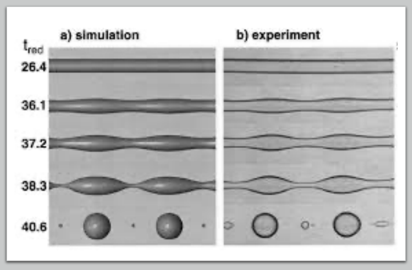

Rayleigh-Plateau (MFM 373, 374,375)

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Saffman-Taylor instability

The Saffman–Taylor instability arises, or may arise, when two fluids of different viscosity are pushed by a pressure gradient through a Hele Shaw cell [§6.8] or allowed to drain through such a cell under their own weight. It would be of little practical importance were it not for the fact that creeping flow in a Hele Shaw cell is the two-dimensional analogue of creeping flow through a porous medium [§6.13]. Something very like the Saffman–Taylor instability frustrates attempts to extract, by pushing it out with pressurised water, the last traces of oil from oil wells. Theoretically, the instability has features in common with the Rayleigh–Taylor instability discussed in §8.2; it differs in that the equibrium state is a dynamic one, in which the interface between the two fluids is moving rather than stationary, but the analysis required is nevertheless distinctly similar.

Suppose the cell to be horizontal, in which case the effects of gravity may be ignored. Suppose it to be bounded by straight edges at $y = \pm\frac{1}{2}L$, and suppose there to be pressure gradients which are driving the fluid contents in the $+x$ direction with some uniform velocity U. In the equilibrium state whose stability we are to investigate, the interface between the two fluids is the straight line $x = Ut$. Where $x < Ut$, the viscosity is η'; where $x > Ut$, the viscosity is η. According to (6.47), the pressure gradients needed to maintain this motion are given in the two regions by

$$\frac{\partial p'}{\partial x} = -\frac{12\eta' U}{d^2}, \quad \frac{\partial p}{\partial x} = -\frac{12\eta U}{d^2},$$

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where d is the thickness of the cell. The pressures p' and p are not necessarily equal at the interface, because the interface is liable to be curved in the vertical (z) direction. Provided that this curvature is constant, however, it does not affect the results of the analysis, so we may as well ignore it and write

$$p' = -\frac{12\eta' U}{d^2}(x - Ut) + p_o, \quad p = -\frac{12\eta U}{d^2}(x - Ut) + p_o,$$

for the equilibrium state, where p_o does not depend upon x.

Now suppose that the interface is perturbed, in such a way that at time t it lies at $x = X$, where

$$X = Ut + \zeta_k\,\mathrm{e}^{iky}.$$

There must be some corresponding perturbation in p' and p, and it must have the same periodicity in the y direction. However, p' and p obey Laplace's equation in two dimensions [§6.8], so any perturbing term which varies like $\exp(iky)$ must vary like $\exp(\pm kx)$ [(5.12)]. Since the perturbation cannot affect the pressure at large distances from the interface, the perturbed pressures presumably have the form

$$p' = -\frac{12\eta' U}{d^2}(x - Ut) + p_o + A'\mathrm{e}^{k(x-Ut)}\,\mathrm{e}^{iky}$$

$$p = -\frac{12\eta U}{d^2}(x - Ut) + p_o + A\mathrm{e}^{-k(x-Ut)}\,\mathrm{e}^{iky}$$

when k is positive, where the coefficients A' and A are to be determined by reference to the boundary conditions at the interface.

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These boundary conditions, applicable in each case at $x = X$, and linearised by omission of terms which are of higher than first order in A or ζ_k are as follows.

(i) $$\langle u'\rangle_x = \langle u\rangle_x = \frac{\partial X}{\partial t},$$

where $\langle \boldsymbol{u}\rangle$ is the mean velocity described by (6.47), or

$$-\frac{d^2}{12\eta'}\frac{\partial p'}{\partial x} = -\frac{d^2}{12\eta}\frac{\partial p}{\partial x} = U + \frac{\partial\zeta_k}{\partial t}\,\mathrm{e}^{iky}.$$

To first order this corresponds to

$$-\frac{d^2k}{12\eta'}A' = -\frac{d^2k}{12\eta}A = \frac{\partial\zeta_k}{\partial t}\,\mathrm{e}^{iky}. \qquad (8.7)$$

(ii) $$p' - p = -\sigma\frac{\partial^2 X}{\partial y^2} = \sigma k^2\zeta_k\,\mathrm{e}^{iky},$$

where σ is the interfacial surface tension. To first order this corresponds to

$$A' - A = \left\{\frac{12U}{d^2}(\eta' - \eta) + \sigma k^2\right\}\zeta_k\exp(iky). \qquad (8.8)$$

It is a trivial exercise to eliminate A' and A from (8.7) and (8.8), and so to obtain the result

$$s_k = \frac{1}{\zeta_k}\frac{\partial\zeta_k}{\partial t} = \frac{1}{\eta' + \eta}\left\{-U(\eta' - \eta)k - \frac{\sigma d^2k^3}{12}\right\}. \qquad (8.9)$$

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Thus if $\eta < \eta'$ the interface is stable for all k. When $\eta > \eta'$, however, i.e. when a viscous fluid is being displaced by a less viscous one, it is marginally stable with respect to a perturbation for which $k = k_c$, where

$$k_c^2 = \frac{12U(\eta - \eta')}{\sigma d^2},$$

and it is unstable with respect to perturbations for which $0 < k < k_c$. The perturbations which grow fastest (i.e. for which s_k is a maximum) have $k = k_c/\sqrt{3}$, i.e. a wavelength

$$\lambda = \pi d\sqrt{\frac{\sigma}{U(\eta - \eta')}}\cdot \qquad (8.10)$$

The smallest value of k which is consistent with the boundary conditions at the sides of the cell, where $y = \pm\frac{1}{2}L$, is π/L, and if the cell is so narrow, or if U is so small, that this exceeds k_c then no instabilities can be observed. In the experiments conducted by Saffman and Taylor, however, in which air was used to displace glycerine through a cell whose thickness was about 1 mm, L was 12 cm and the wavelength λ predicted by (8.10) was normally a bit less than 2 cm. Thus they expected to see, when the pressure gradient was first applied, six or seven corrugations develop in the interface over the full width of the cell, and so they did; one of their photographs is reproduced as fig. 8.4(a).

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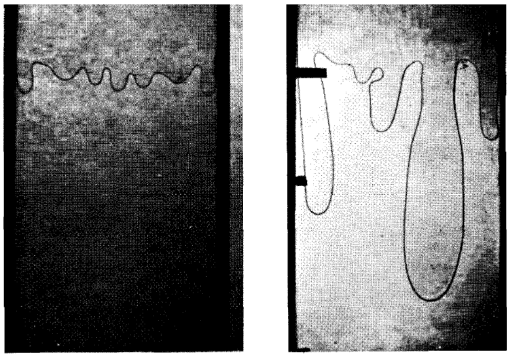

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When the corrugations are no longer very small they do not all grow at the same rate, as is shown by fig. 8.4(b). One of the advancing *fingers* of the less viscous fluid tends to get ahead, whereupon it expands sideways and, by doing so, slows down the advance of its competitors. In due course only a single finger survives. It continues to advance at its tip, but it appears to stop expanding sideways when its width reaches half the width of the cell. The tip has a characteristically rounded shape, which Saffman and Taylor were able to explain.

Are the fingers stable and, if not, how do they split up? This question has proved in recent years to be of much greater complexity and interest than Saffman and Taylor could have guessed when their paper on this subject was published in 1958. A partial answer is provided by the two remarkable photographs of fingers spreading radially from a central source which are reproduced in figs. 8.5 and 8.6. The first one shows a number of fingers which are splitting in an irregular and unsurprising way, and one finger which has developed side branches of astonishing regularity; it differs from the others by having a defect at its tip, in the shape of a small gas bubble which has accidentally entered the apparatus and become entrained in the flow. The second photograph shows an even more regular pattern

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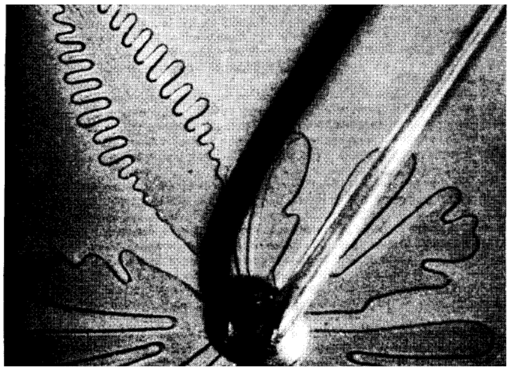
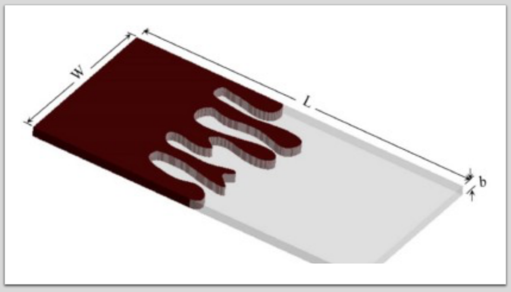


33



Saffman-Taylor (MFM 406)

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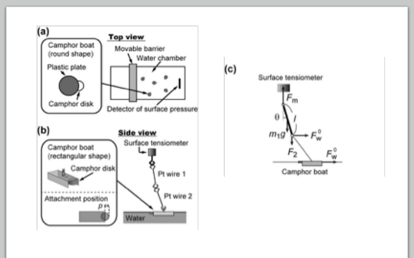

Other phenomena

Camphor boats (MFM 419)

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