Cosmologia Física

Homework 2 due 22 March 2021

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Exercise 1: Friedmann equations

1.1) Consider the Friedmann equation,

$$H^2(a) = H_0^2 \left(\Omega_r(1+z)^4 + \Omega_{\rm m}(1+z)^3 + \Omega_{\rm K}(1+z)^2 + \Omega_\Lambda\right).$$

a) Is it possible for a cosmological model to have $\Omega_{\rm m} > 1$?
b) What type of curvature has a universe with $(\Omega_{\rm m} > 1, \Omega_\Lambda = 0)$?
c) Do all universes with $\Omega_{\rm m} > 1$ necessarily have the same curvature?

1.2) Consider a 2-fluid flat universe with dark matter and cosmological constant.
a) Compute the redshift of the transition to the dark energy epoch.
b) Derive the expression of the line of no acceleration in a $(\Omega_m, \Omega_\Lambda)$ plane.

1.3)
a) Show that any decelerating universe is younger than the Hubble time (or equivalently that any accelerated universe is older than the Hubble time).

Exercise 2: Universe with 5 dimensions

2.1) Consider a spacetime with 1 time and 4 spatial coordinates.
a) Write the continuity equation for this spacetime.
Hint: Use only simple arguments to write this equation, no formal derivation is needed.
b) Derive the evolution of the radiation density $\rho_r(a)$ in this spacetime.
c) Consider a cosmological fluid, in this spacetime, with a constant equation-of-state of w=0.3. Is this a tachyonic or a non-tachyonic fluid?

Exercise 3: Surprising results

3.1) Consider two identical galaxy clusters (meaning they have similar intrinsic sizes and luminosities) in the Einstein-de Sitter universe. The clusters are at different redshifts, z_1 and z_2, with $z_1 < z_2$
a) Show that the comoving distance $D_c(z)$ in this universe is

$$D_c = 2\frac{c}{H_0}\left[1 - (1+z)^{-1/2}\right].$$

b) Compute which of the 2 clusters looks the brightest.

c) Compute which of the 2 clusters looks the largest.

d) Between the luminosity and the angular diameter distances, which is the best one to parametrize the evolution of the universe?
Hint: Remember other quantities that are useful to parametrize the evolution of the universe, such as the temperature and the redshift.

3.2) Consider the concordance universe of the ΛCDM 3-fluid model, i.e. a flat cosmology with $h = 0.7$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $\Omega_r = 8 \times 10^{-5}$.

a) Even though the redshift range extends from 0 to ∞, the redshift at half of the age of the universe is a surprisingly low value. Compute it for the concordance universe
Hint: solve the integral numerically, for example using Wolfram Alpha online `Mathematica tool`.

3.3) In 1.3 you showed that a decelerating universe is younger than an accelerated one. On the other hand, the expansion is obviously slower in a decelerating universe and faster in an accelerating one. This leads to the following conclusion: if we consider two universes with the same Hubble constant, that expand with different rates $a(t) \propto t^n$, then the slowest one is also the youngest. This seems a paradox: the slowest one reaches $a = 1$ in a shorter amount of time.

a) Solve this apparent paradox, explaining that it can make sense that the slowest universe is also the youngest.
Hint: It may be helpful to compare the behaviour of the Hubble function, H(a) for the two universes, and to consider the generalized Hubble law.

b) Find out what is the crucial factor that defines when a universe reaches $a = 1$.