CFTC Centro de Física Teórica e Computacional

Ciências ULisboa

Percolation theory

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Contents

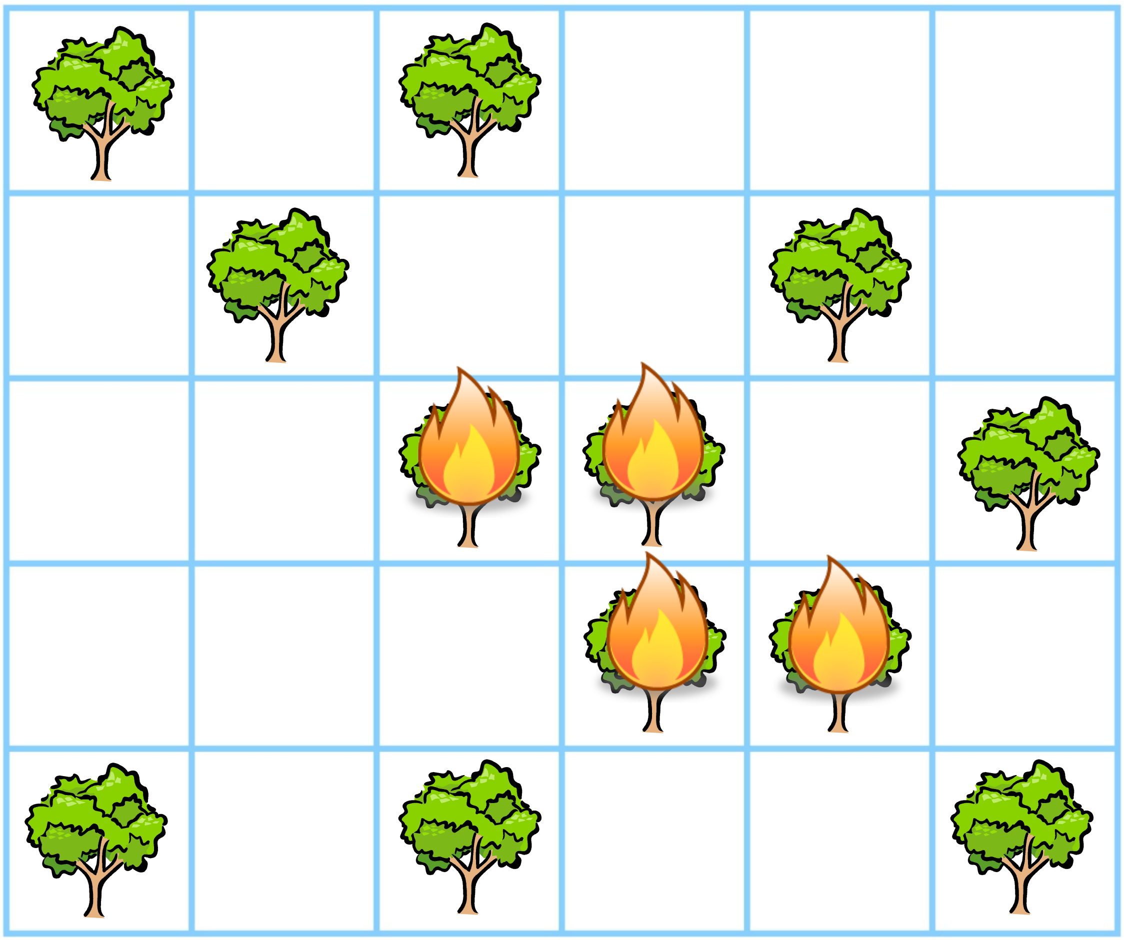
- Classical examples of *percolation*
- The *percolation* model
- What is going on...
- Schramm-Loewner evolution

Books on percolation

- D. Stauffer and A. Aharony, *Introduction to percolation theory*. CRC Press (2000).
- M. Sahimi, *Applications of percolation theory*. Taylor & Francis (1994).
- K. Christensen and N. R. Moloney, *Complexity and criticality*. Imperial College Press (2005).

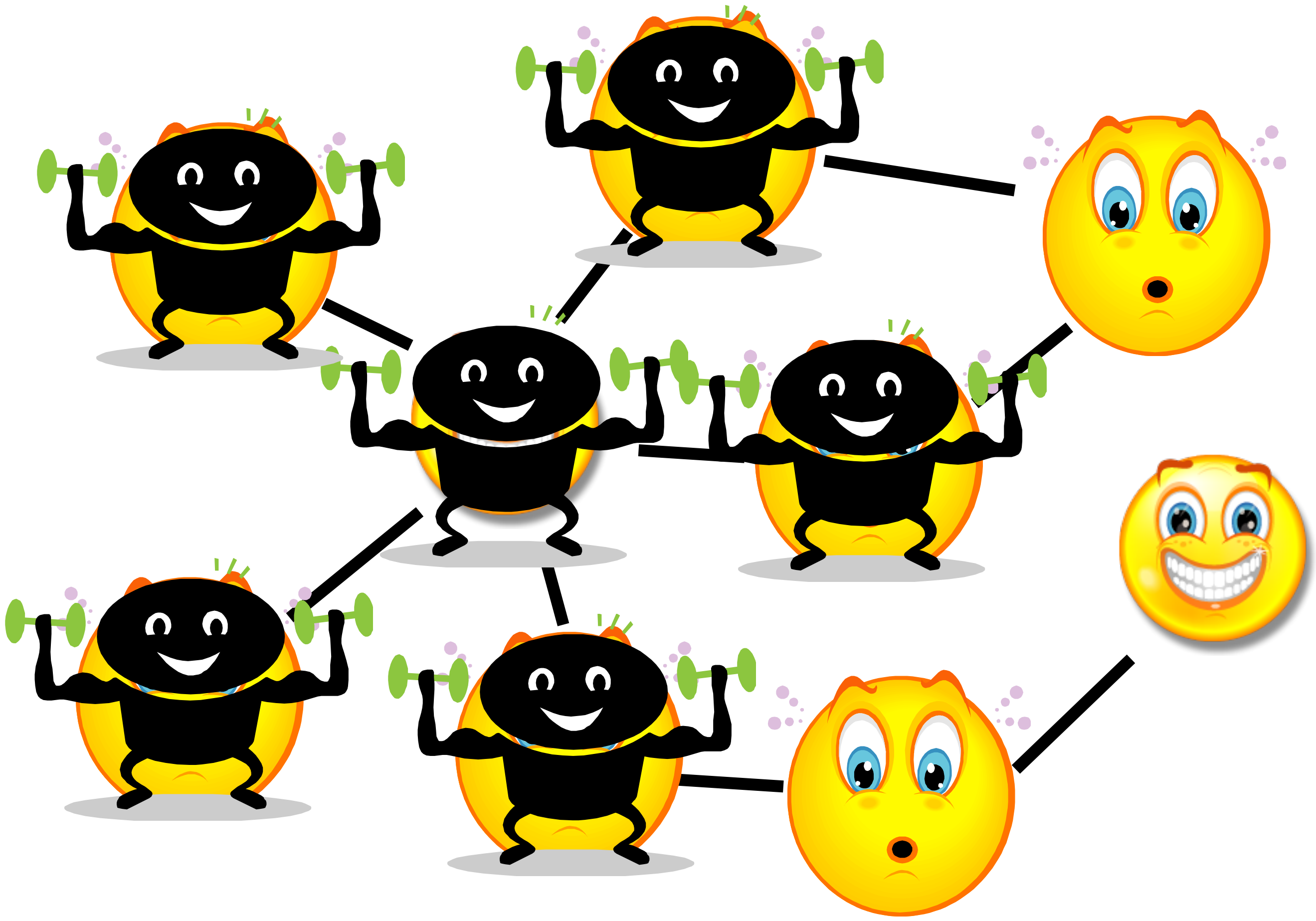
Forest fire

Forest fire



Spreading of epidemics

Spreading of epidemics



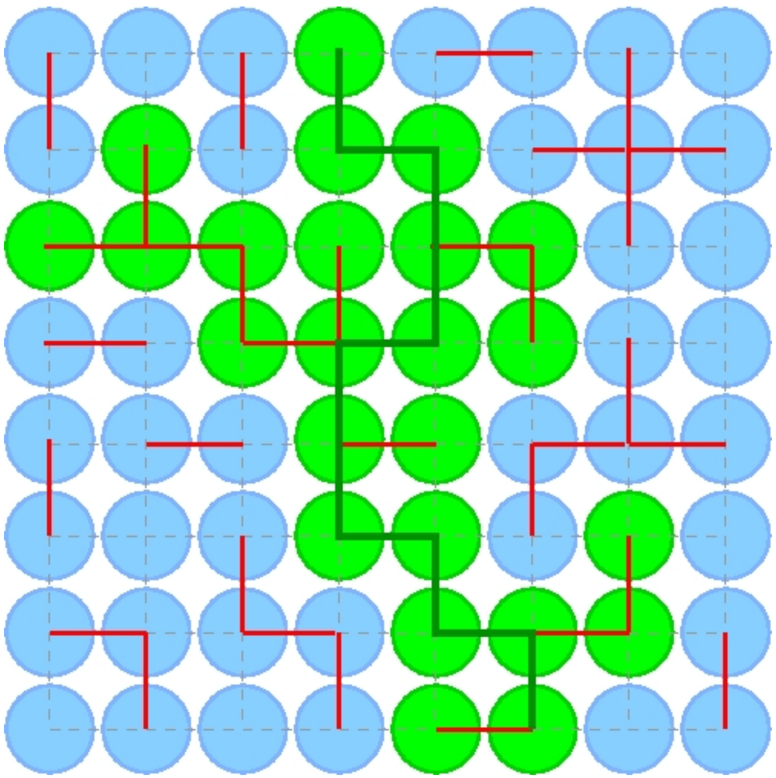
Oil fields

at *Barrancabermeja (Colombia)*, photo by *Melissa Jiménez*.

Percolation model

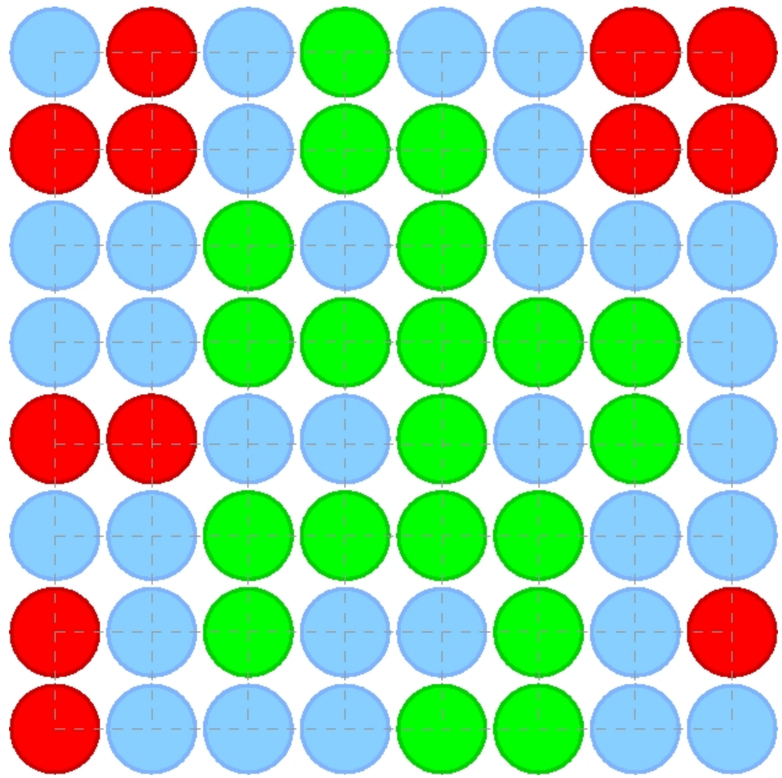
$$p^{O}(1-p)^{E}$$

Bonds



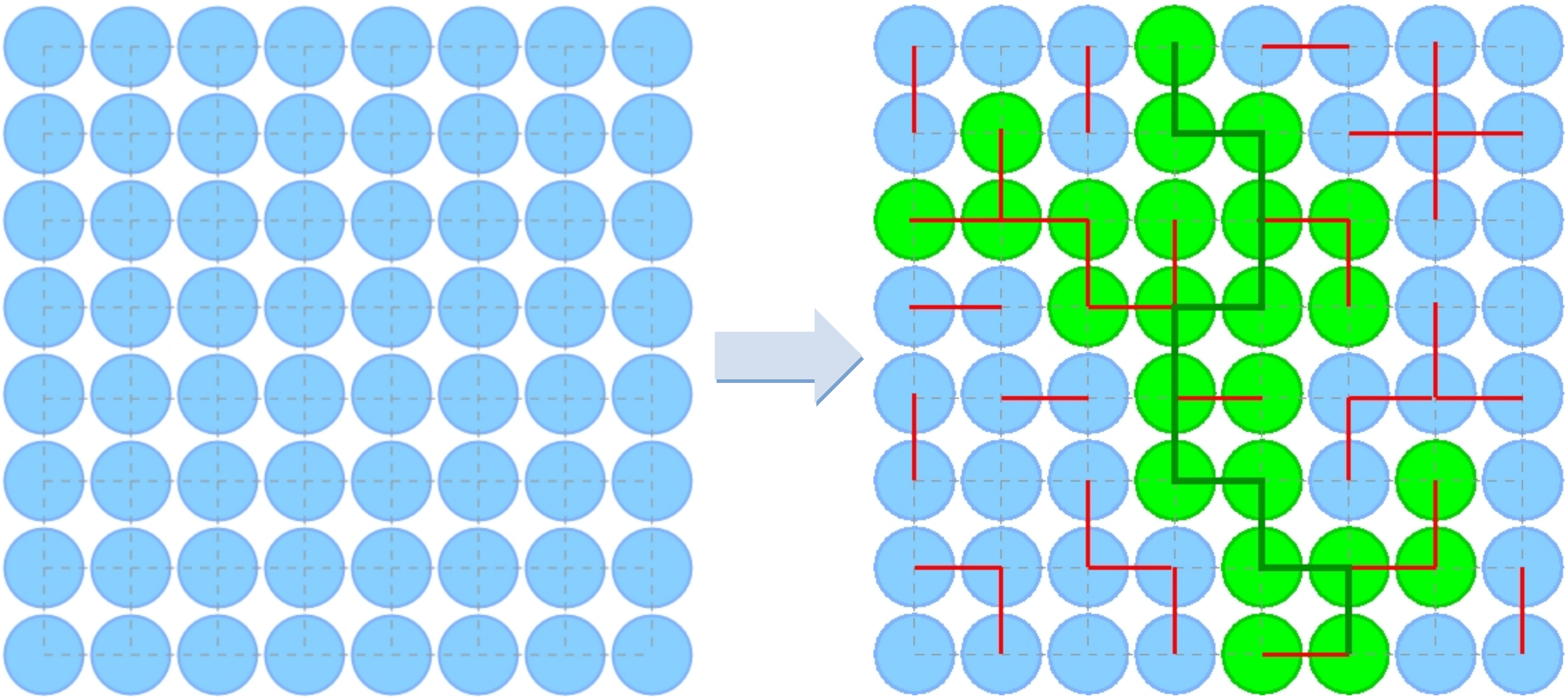
$$2^{N_{Bonds}}$$

Sites



$$2^{N_{Sites}}$$

Percolation model

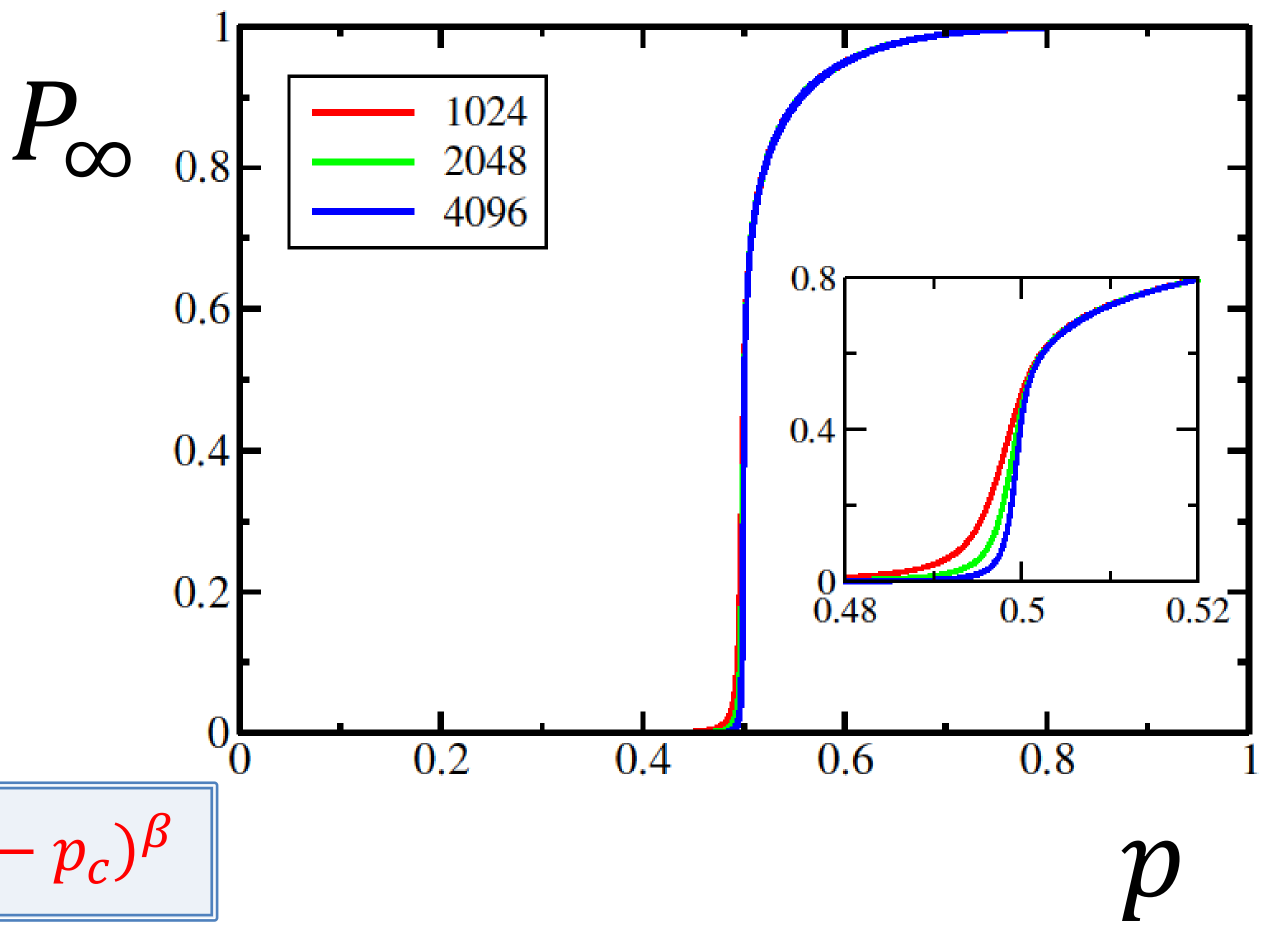


$$p^{O}(1-p)^{E}$$

Percolation model

order parameter

$$P_\infty = \frac{s_{max}}{N}$$

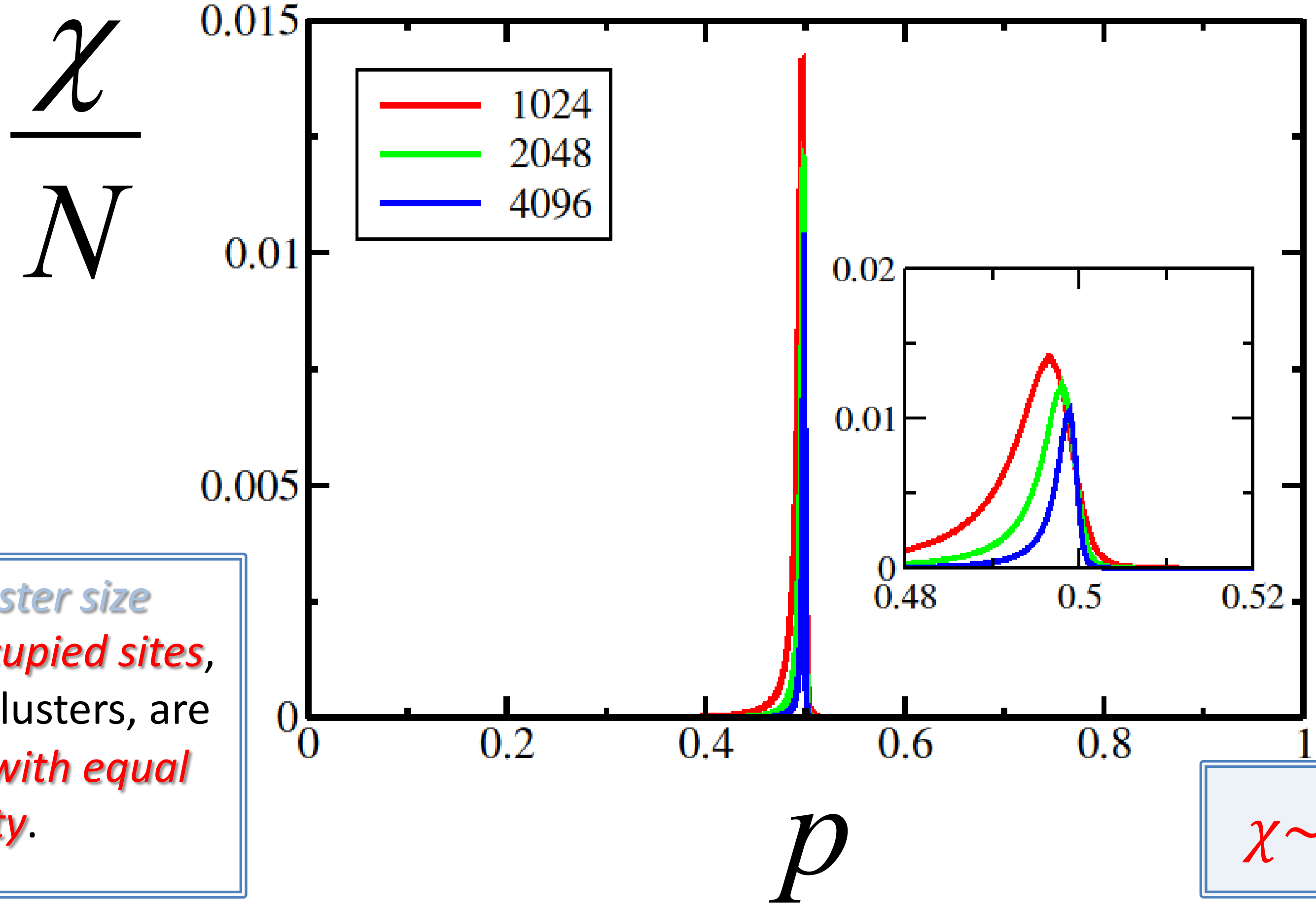


$P_\infty \sim (p - p_c)^\beta$

Percolation model

fluctuations (mean cluster size)

$$\chi = \frac{1}{N}\sum_{i\neq \max} s_i^2$$

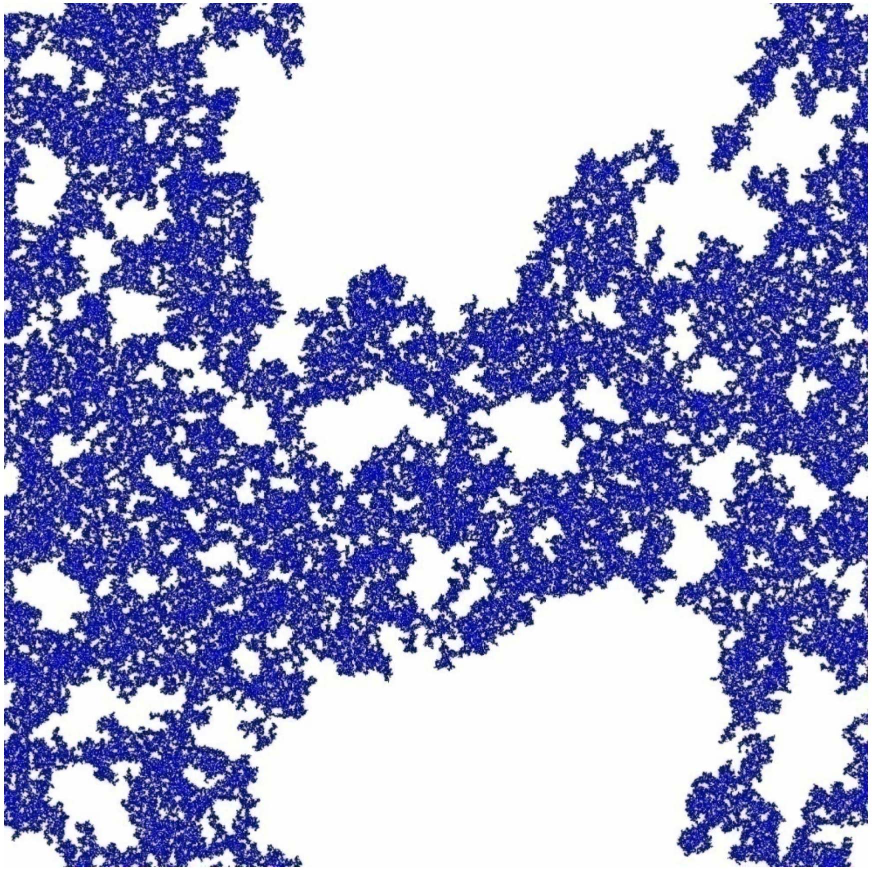


Mean cluster size when ***occupied sites***, and not clusters, are ***selected with equal probability***.

$$\chi \sim (p_c - p)^{-\gamma}$$

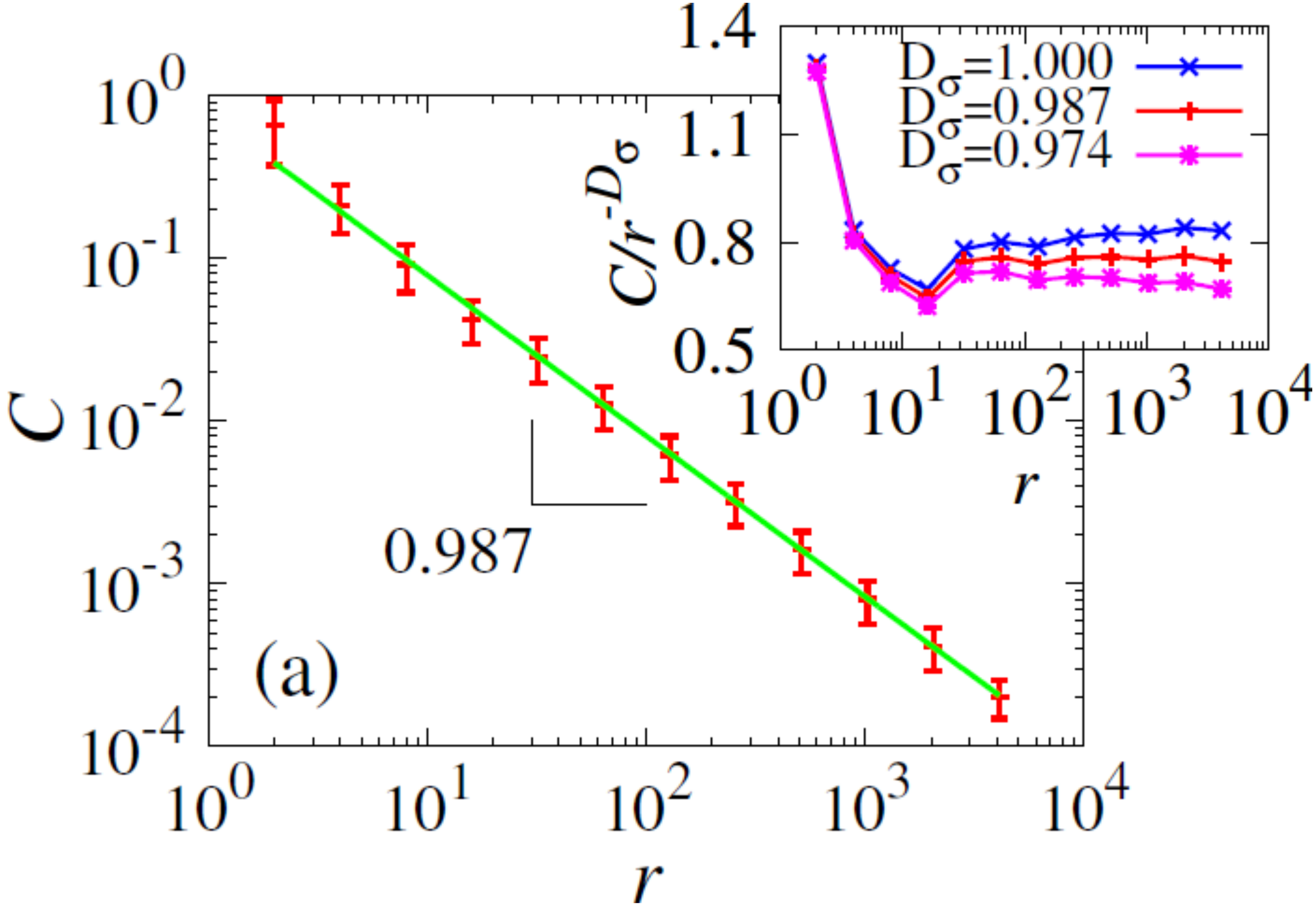
Percolation threshold

largest cluster: fractal dimension

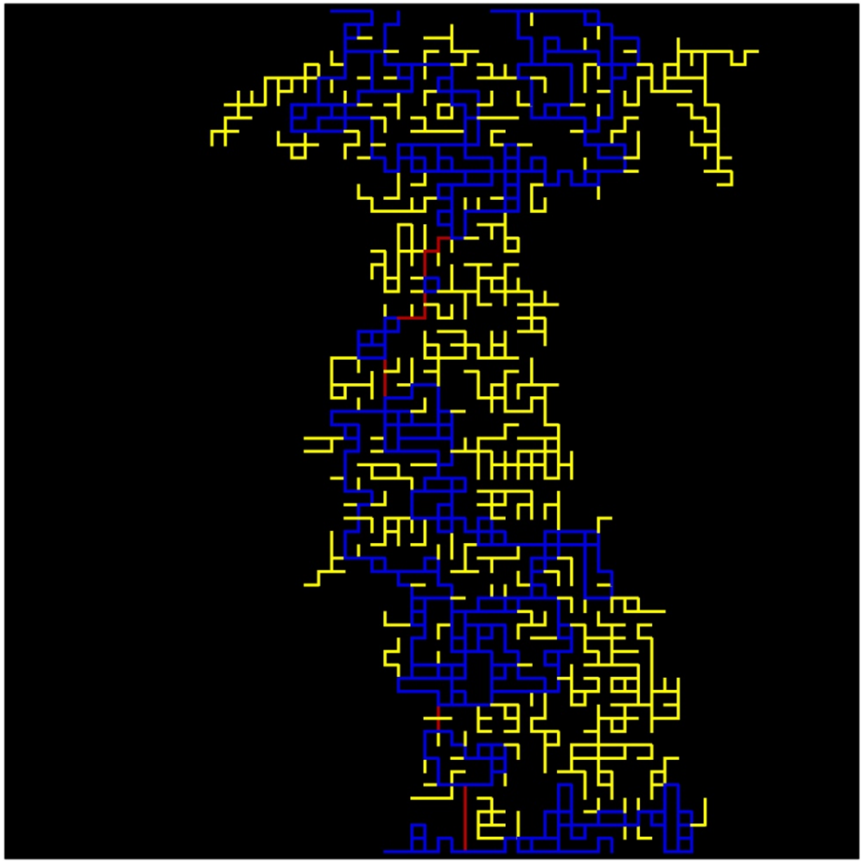


$$d_f = \frac{91}{48} \approx 1.896$$

Conductivity



Conductivity
in the largest cluster



Red bonds

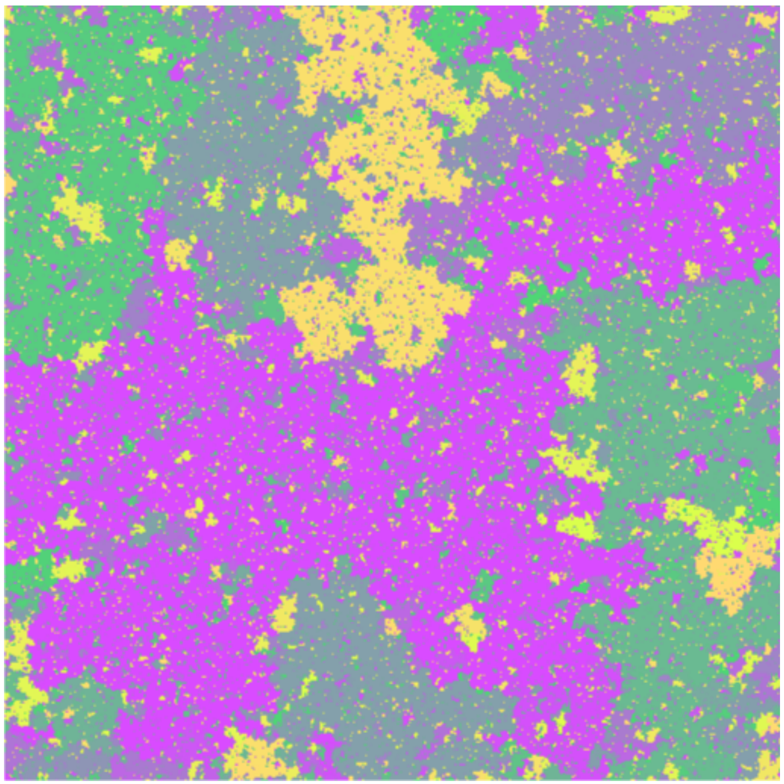
Blue bonds

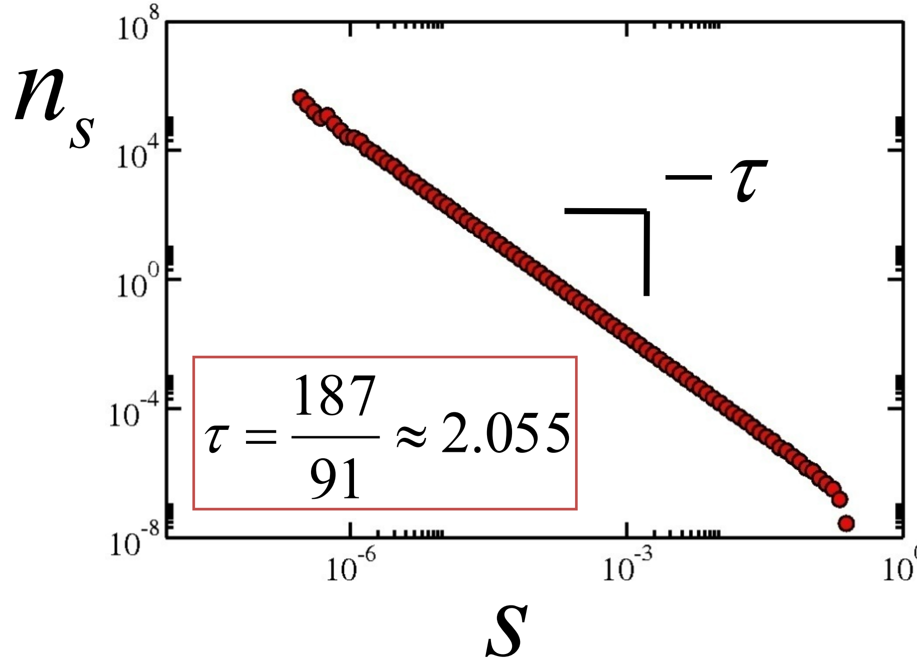
Yellow bonds

Percolation threshold

cluster-size distribution

$$n_s \sim s^{-\tau}$$

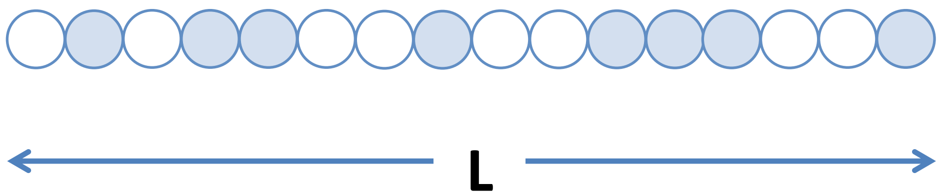




Exact solution in one dimension

cluster number density

p	occupied
1-p	empty



Cluster number frequency:

$$N(s,p;L) = L(1-p)^2 p^s$$

Probability that a site belongs to a cluster of size s :

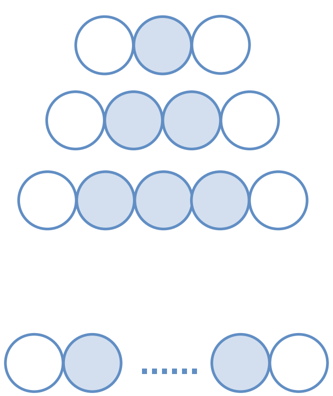
$s = 1$: $(1-p)p(1-p) = p(1-p)^2$

$s = 2$: $2(1-p)p^2(1-p) = 2p^2(1-p)^2$

$s = 3$: $3(1-p)p^3(1-p) = 3p^3(1-p)^2$

...

$s(1-p)p^s(1-p) = sp^s(1-p)^2$



Cluster number density:

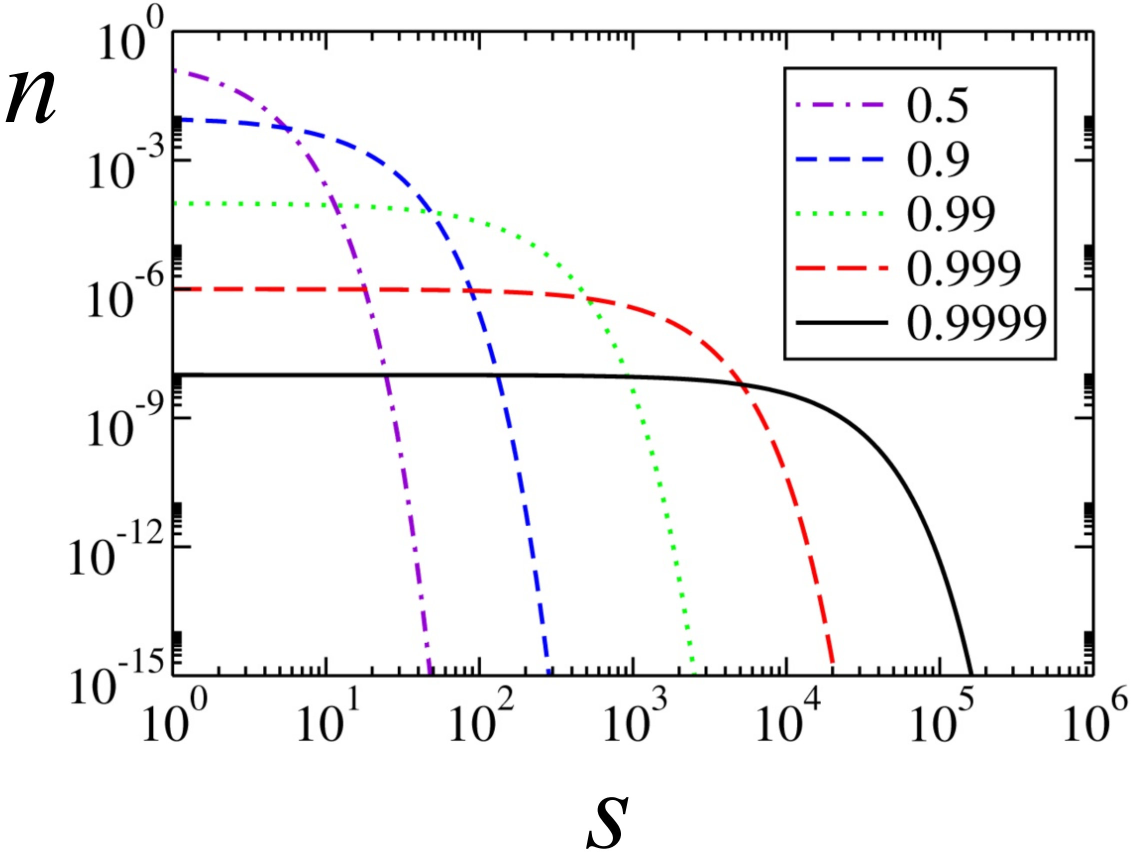
$$n(s,p) = \frac{N(s,p;L)}{L}$$

$$n(s,p) = (1-p)^2 p^s$$

Exact solution in one dimension

cluster number density

$$n(s,p) = (1-p)^2 p^s$$



$$\begin{aligned} n(s,p) &= (1-p)^2 p^s \\ &= (1-p)^2 \exp(\ln p^s) \\ &= (1-p)^2 \exp(s \ln p) \\ &= (1-p)^2 \exp(-s/s_\xi) \end{aligned}$$

$$s_\xi = -\frac{1}{\ln p}$$

$$s_\xi \sim (1-p)^{-1}$$

K. Christensen and N. R. Moloney. *Complexity and Criticality.* Imperial College Press (2005)

Exact solution in one dimension

cluster number density and fluctuations

Probability that a site belongs to a cluster of size s : $sn(s,p) = s(1-p)^2 p^s$

$p < p_c$

$$\sum_s sn(s,p) = \sum_s s(1-p)^2 p^s = p$$

$p > p_c$

$$P_\infty + \sum_{s=1}^{\infty} sn(s,p) = p$$

$p < p_c$

$$\chi(p) = \frac{\sum_s s^2 n(s,p)}{\sum_s sn(s,p)} = \frac{1+p}{1-p}$$

$$\chi(p) \sim (1-p)^{-1}$$

$p < p_c$