

# **Trabalho Prático 2**

Revisão Matemática

# Vetores

Vamos considerar os vetores:

$$\vec{a} = (a_1, a_2, a_3)$$

$$\vec{b} = (b_1, b_2, b_3)$$

Temos que o modulo de  $\vec{a}$  :

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$|\vec{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$$

Por sua vez, o produto interno de vetores  $\vec{a} \cdot \vec{b}$ :

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = \sum_{i=1}^3 a_i b_i$$

# Vetores

Dois **vetores são ortogonais** se estes forem perpendiculares. Como:

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

Temos que:

$$\cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2} = 90^\circ$$

Logo:

$$\vec{a} \cdot \vec{b} = 0$$

# Matrizes

Vamos considerar as matrizes:

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

A multiplicação destas matrizes será:

$$A \cdot B = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} & \dots \\ \dots & \dots \end{pmatrix}$$

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# Matrizes

Matriz transposta  $A^\dagger$ :

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$
$$A^\dagger = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \end{pmatrix}$$

The diagram illustrates the transpose operation for a 3x2 matrix A. On the left, matrix A is shown as a 3x2 grid of elements:  $a_{11}, a_{12}, a_{21}, a_{22}, a_{31}, a_{32}$ . Red curved arrows point from each element in A to its corresponding position in the transpose matrix  $A^\dagger$  on the right. The transpose matrix  $A^\dagger$  is a 2x3 grid:  $a_{11}, a_{21}, a_{31}, a_{12}, a_{22}, a_{32}$ . Green curved arrows point from each element in  $A^\dagger$  back to its original position in A, showing that the transpose operation is reversible.

# Matrizes

Diagonalização de matrizes Hermiteanas (isto é uma matriz quadrada complexa, que é igual à sua transposta conjugada):

$$\mathbf{O} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \longrightarrow \quad \Omega = \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_N \end{pmatrix}$$

## Método da transformação unitária

Precisamos de encontrar uma matriz  $\mathbf{U}$  que diagonaliza a matriz simétrica  $\mathbf{O}$ , sendo que:

$$\mathbf{U}^\dagger \mathbf{U} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Omega = \mathbf{U}^\dagger \mathbf{O} \mathbf{U} = \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_N \end{pmatrix}$$

Onde  $\omega_i$  são os valores próprios de  $\mathbf{O}$ .