Para a resistência

We begin with the resistor. If the current through a resistor R is $i = I_m \cos(\omega t + \phi)$, the voltage across it is given by Ohm's law as

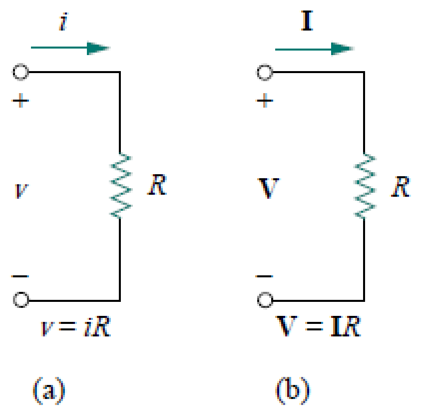
$$v = iR = RI_m \cos(\omega t + \phi) \qquad (9.29)$$

The phasor form of this voltage is

$$\mathbf{V} = RI_m \underline{/\phi} \qquad (9.30)$$

But the phasor representation of the current is $\mathbf{I} = I_m \underline{/\phi}$. Hence,

$$\mathbf{V} = R\mathbf{I} \qquad (9.31)$$



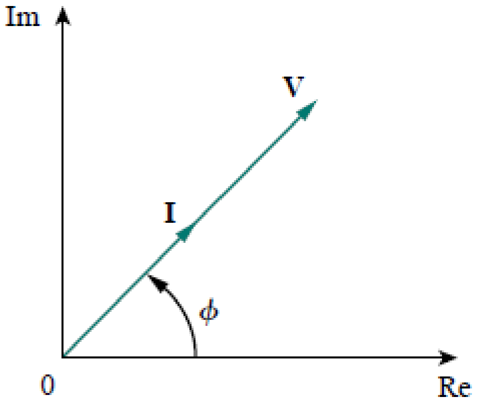


Figure 9.10 Phasor diagram for the resistor.

Para o indutor

For the inductor L, assume the current through it is $i = I_m \cos(\omega t + \phi)$. The voltage across the inductor is

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi) \qquad (9.32)$$

Recall from Eq. (9.10) that $-\sin A = \cos(A + 90^\circ)$. We can write the voltage as

$$v = \omega L I_m \cos(\omega t + \phi + 90^\circ) \qquad (9.33)$$

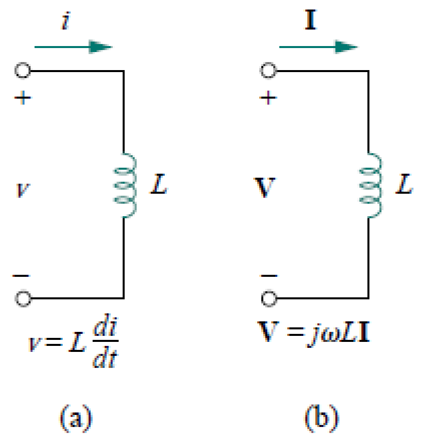
which transforms to the phasor

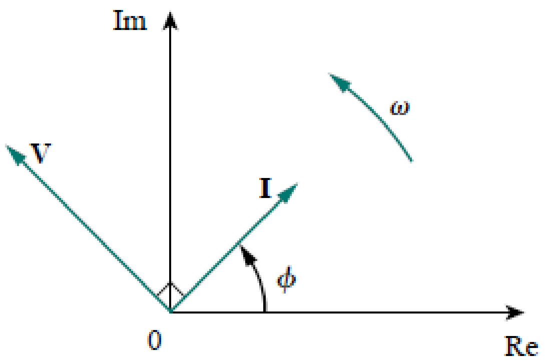
$$\mathbf{V} = \omega L I_m e^{j(\phi+90^\circ)} = \omega L I_m e^{j\phi} e^{j90^\circ} = \omega L I_m \underline{/\phi} e^{j90^\circ} \qquad (9.34)$$

But $I_m \underline{/\phi} = \mathbf{I}$, and from Eq. (9.19), $e^{j90^\circ} = j$. Thus,

$$\mathbf{V} = j\omega L\mathbf{I} \qquad (9.35)$$

showing that the voltage has a magnitude of $\omega L I_m$ and a phase of $\phi + 90^\circ$.





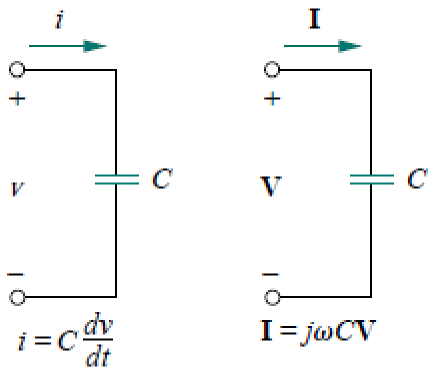
For the capacitor C, assume the voltage across it is $v = V_m \cos(\omega t + \phi)$. The current through the capacitor is

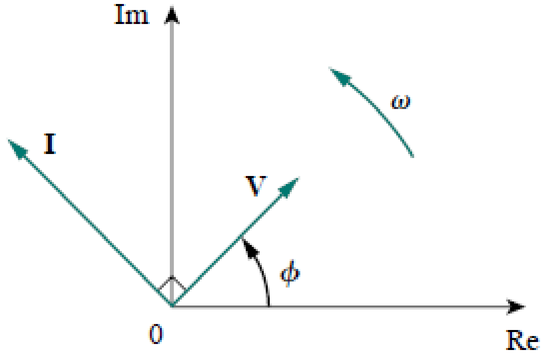
$$i = C\frac{dv}{dt} \tag{9.36}$$

By following the same steps as we took for the inductor or by applying Eq. (9.27) on Eq. (9.36), we obtain

$$\mathbf{I} = j\omega C\mathbf{V} \quad \Longrightarrow \quad \mathbf{V} = \frac{\mathbf{I}}{j\omega C} \tag{9.37}$$

showing that the current and voltage are 90° out of phase. To be specific,





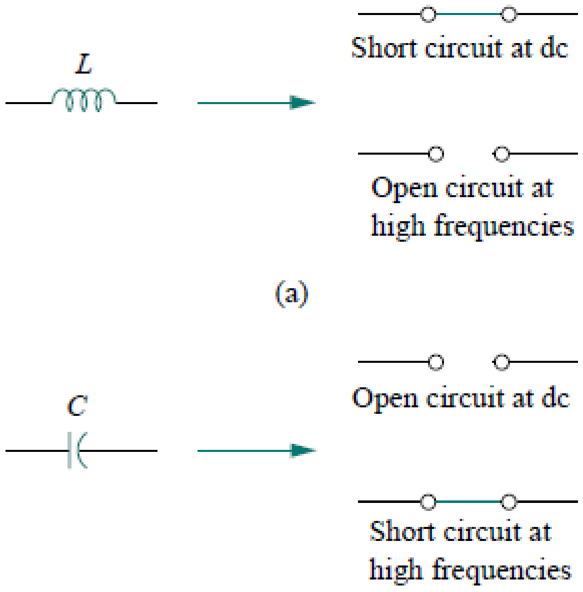
Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

Noção de impedância

$$\mathbf{V} = R\mathbf{I}, \qquad \mathbf{V} = j\omega L\mathbf{I}, \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

$$\frac{\mathbf{V}}{\mathbf{I}} = R, \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

$$\boxed{\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{Z}\mathbf{I}}$$



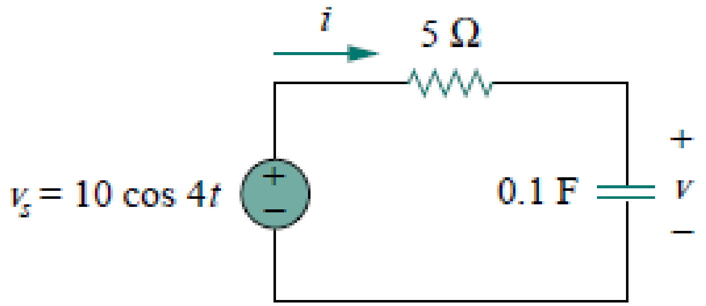
$$\mathbf{Z} = R + jX$$

A parte real de Z é a resistência

A parte imaginária de Z é a reatância

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R}$$

$$R = |\mathbf{Z}|\cos\theta, \qquad X = |\mathbf{Z}|\sin\theta$$



From the voltage source $10\cos 4t$, $\omega = 4$,

$$\mathbf{V}_s = 10\angle 0^\circ \text{ V}$$

The impedance is

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

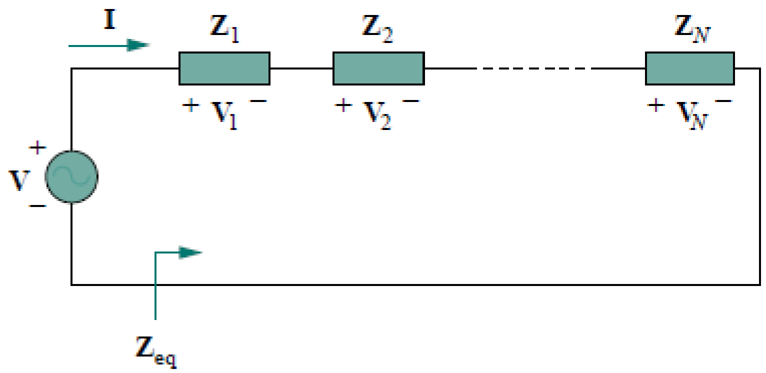
Hence the current

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\angle 0^\circ}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2}$$
$$= 1.6 + j0.8 = 1.789\angle 26.57^\circ \text{ A}$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1.789\angle 26.57^\circ}{j4 \times 0.1}$$
$$= \frac{1.789\angle 26.57^\circ}{0.4\angle 90^\circ} = 4.47\angle -63.43^\circ \text{ V}$$

Converting **I** and **V** in Eqs. (9.9.1) and (9.9.2) to the time domain,

$$i(t) = 1.789\cos(4t + 26.57^\circ) \text{ A}$$
$$v(t) = 4.47\cos(4t - 63.43^\circ) \text{ V}$$

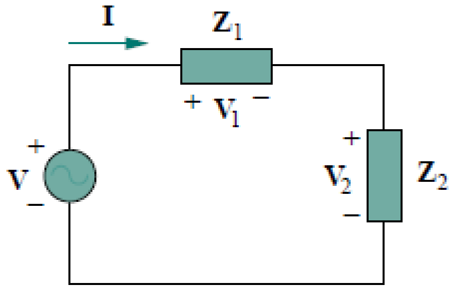


$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_N = \mathbf{I}(\mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N)$$

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$

$$\boxed{\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N}$$

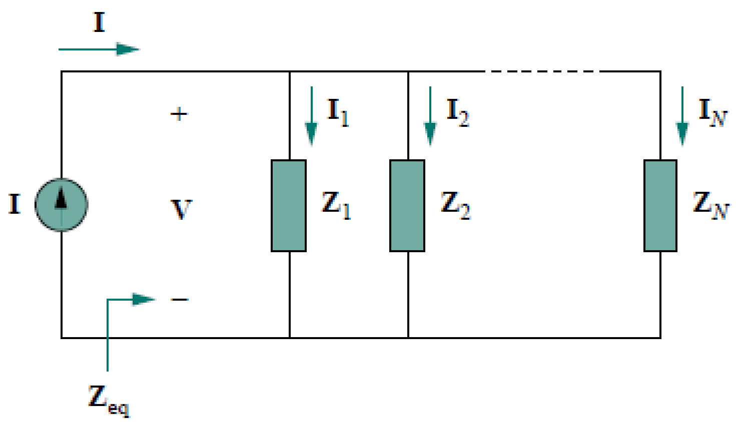
- as expressões deduzidas para as <u>associações de resistência em série e paralelo</u> podem ser generalizadas para as impedâncias.
- a Lei dos Nós e das Malhas mantêm a sua validade;
- as Leis dos Nós e das Malhas verificam-se vectorialmente!

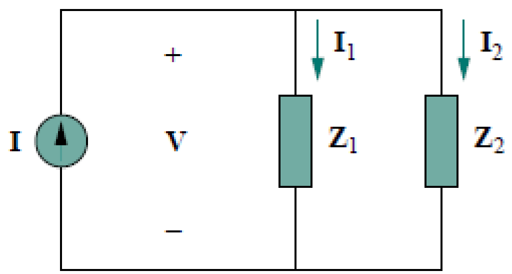


$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

Since $\mathbf{V}_1 = \mathbf{Z}_1\mathbf{I}$ and $\mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}$, then

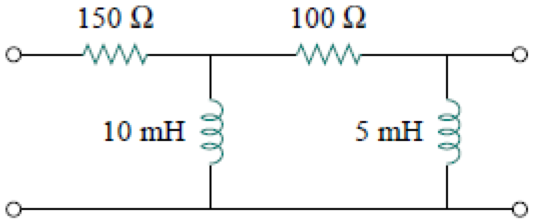
$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}, \qquad \mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$$



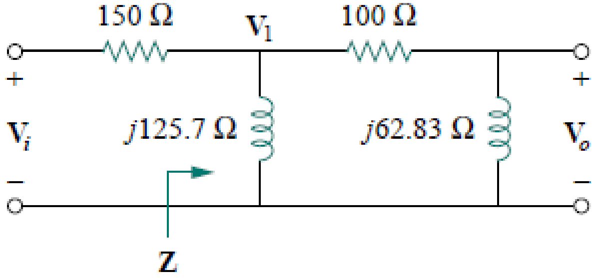


$$\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}, \qquad \mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}$$

Calcular a diferença de fase entre entrada e saída para 2 kHz



(a)



$$10\text{ mH} \implies X_L = \omega L = 2\pi \times 2 \times 10^3 \times 10 \times 10^{-3} = 40\pi = 125.7\ \Omega$$

$$5\text{ mH} \implies X_L = \omega L = 2\pi \times 2 \times 10^3 \times 5 \times 10^{-3} = 20\pi = 62.83\ \Omega$$

Consider the circuit in Fig. 9.35(b). The impedance **Z** is the parallel combination of $j125.7\ \Omega$ and $100 + j62.83\ \Omega$. Hence,

$$\mathbf{Z} = j125.7 \parallel (100 + j62.83) = \frac{j125.7(100 + j62.83)}{100 + j188.5} = 69.56\angle 60.1^\circ\ \Omega \qquad (9.14.1)$$

Using voltage division,

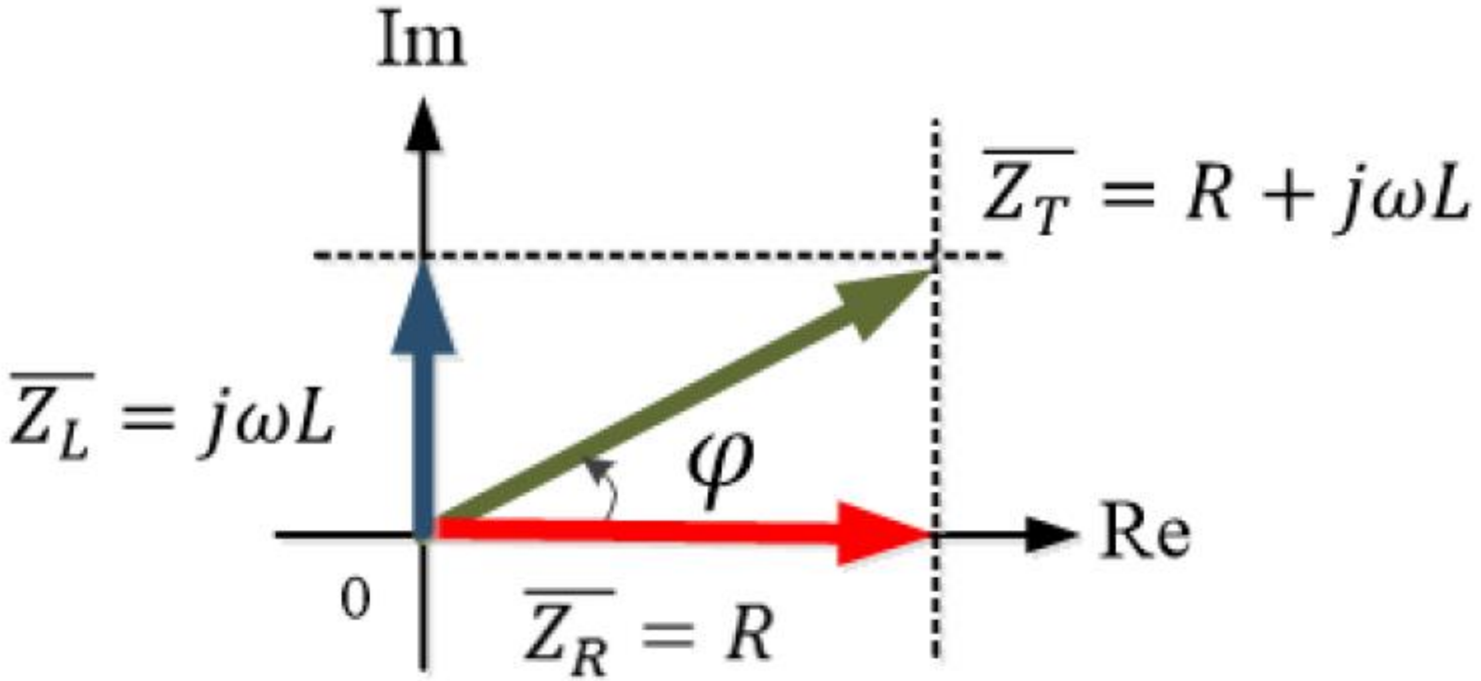
$$\mathbf{V}_1 = \frac{\mathbf{Z}}{\mathbf{Z} + 150}\mathbf{V}_i = \frac{69.56\angle 60.1^\circ}{184.7 + j60.3}\mathbf{V}_i = 0.3582\angle 42.02^\circ\ \mathbf{V}_i \qquad (9.14.2)$$

and

$$\mathbf{V}_o = \frac{j62.832}{100 + j62.832}\mathbf{V}_1 = 0.532\angle 57.86^\circ\ \mathbf{V}_1 \qquad (9.14.3)$$

Combining Eqs. (9.14.2) and (9.14.3),

$$\mathbf{V}_o = (0.532\angle 57.86^\circ)(0.3582\angle 42.02^\circ)\ \mathbf{V}_i = 0.1906\angle 100^\circ\ \mathbf{V}_i$$


Im
$\overline{Z_T} = R + j\omega L$
$\overline{Z_L} = j\omega L$
φ
Re
0
$\overline{Z_R} = R$