Cosmologia Física

Homework 3

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Exercise 1: Two-point functions

1.1) Consider the ΛCDM universe. At early times, right after inflation, the so-called primordial matter density contrast field has small values at all scales, and its power spectrum is given by a power-law: $P(k) \propto k^\alpha$.

a) Derive the corresponding correlation function, i.e. its Fourier transform,

$$\xi(r) = \frac{1}{(2\pi)^3} \int P(k)\, e^{-ik.x}\, d^3k\,,$$

showing it is given by

$$\xi(r) \propto r^{-(3+\alpha)}\,.$$

Note: The following integral will be needed in the derivation:

$$\int_0^\infty x^{n+1} \sin(x)\, dx = \Gamma[n+2]\, \sin(\pi(n+2)/2)\,.$$

b) The power spectrum of the matter density contrast field today has evolved from the primordial one. Its shape is now a continuous sequence of power-laws $k^{\alpha(k)}$, with α ranging from 1 on the largest scales to -3 on the smallest scales. Measurements of the spatial distribution of astrophysical objects at $z \approx 0$ show that the correlation function for a certain type of objects (let us call it type A) is $\xi \propto r^{-1.7}$, while for another type of objects (type B) is $\xi \propto r^{-2.1}$, where one of the types is galaxies and the other is galaxy clusters. From these measurements and the result of exercise 2.1a, are galaxy clusters the type A or type B objects? Why?

1.2) Consider an alternative universe where the matter correlation function is a Gaussian function centered in $r = 0$ and with variance σ^2.

a) Compute the power spectrum of this universe.

Note: The following integral will be needed:

$$\int_0^\infty x\, \sin(x)\, \exp(-x^2/(2c^2)) dx = (\pi/2)^{1/2} c^3\, \exp(-c^2/2).$$

b) Does this universe verify the cosmological principle? Why (or why not) ?

Exercise 2: Galaxy clustering

2.1) Consider a galaxy survey that observed a 3-dimensional spherical volume of radius R and found that all the galaxies were lying on the equatorial disk inside the sphere (and not randomly distributed throughout the volume)

a) Show that the correlation function in that volume is

$$\xi(r) = \frac{2}{3}\frac{R}{r} - 1,$$

where r is the radial coordinate, i.e., the 3D separation from the center of the sphere.
Hint: Remember that $1 + \xi(r)$ can be measured as the ratio between the number of galaxies at separation r from the origin in the clustered sample and the number of galaxies at separation r from the origin in a random sample.

Exercise 3: Shot noise and Cosmic variance

3.1) Shot noise and cosmic variance are two different sources of uncertainties in measurements of cosmological power spectra.

a) Explain (in words) what is shot noise in cosmology, and what is its effect in the measurements of power spectra. Is there a way to make its effect negligible in the power spectrum measurements on all scales?

b) Explain (in words) what is this special type of sample variance in cosmology known as cosmic variance. Is there a way to make its effect negligible in the power spectrum measurements on all scales?

c) Compute the cosmic variance of the angular power spectrum C_ℓ of a Gaussian random field, showing the result is

$$\sigma^2_{C_\ell} = \frac{2}{2\ell + 1} C_\ell^2.$$

d) What is the minimum uncertainty with which the quadrupole moment of the CMB power spectrum can be estimated in a half-sky survey?

Exercise 4: Power spectrum

4.1) Assume a flat CDM universe (no dark energy) where the matter power spectrum today is given by the following expression, where k_p is a characteristic scale chosen to be $k_p = 0.02$ h/Mpc (corresponding to a size $R_p = 2\pi/k_p$):

$$P_\delta(k, z = 0) = \frac{A(\frac{k}{k_p})}{1 + \beta(\frac{k}{k_p})^4}.$$

Remember also that the corresponding dimensionless power spectrum is defined as $\Delta^2(k) = P_\delta(k)\, k^3$. Assume that at the end of the radiation-dominated epoch ($z = z_{eq} = 3499$) the amplitude of the power spectrum was 0.00340136484(Mpc/h)^3 (it is important to use these exact values in the computations).

a) What is the value of the spectral index paramter n_s in this universe?

b) Write the expression of the transfer function in this universe.

c) If we choose the characteristic scale k_p to be the scale of the peak, then this choice fixes the parameter β. Show that $\beta = 1/3$.

d) What is the meaning of the parameter A in the expression of the power spectrum, and what are its units?

e) What is the amplitude of the power spectrum today at the peak?
Hint: Remember the growth rate of the power spectrum.

f) Find out which scale is collapsing today (find both its k value and its R size).

g) Is the scale of the peak already collapsed today? If not, when does it collapse?

h) Find out at which redshift does the scale of R=0.1 Mpc/h collapse (which is a typical scale of a galaxy). Is this result consistent with the observations of the real universe?

i) What is the feature in the transfer function used in this exercise that led to the unrealistic result found in h)? How could you improve the transfer function to obtain a more realistic description of our universe?