Cosmologia Física

Homework 4

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Exercise 1: Newtonian perturbed fluid equations

(In the following, "boldface"denotes a vector).

1.1) Consider the continuity equation for the dark matter fluid in an expanding background in comoving coordinates:

$$\frac{\partial \rho}{\partial t} - \frac{\dot{a}}{a}\mathbf{x}.\nabla\rho + \frac{1}{a}\nabla.(\rho\mathbf{u}) = 0\,.$$

a) Inserting the density and velocity perturbations through, $\rho = \bar{\rho}(1+\delta)$ and $\mathbf{u} = \dot{a}\mathbf{x} + \mathbf{v}$, derive the perturbed linearized comoving continuity equation (i.e., the first equation given in 1.3 below).

1.2) Consider the Euler equation for the dark matter fluid in an expanding background in comoving coordinates:

$$\frac{\partial \mathbf{u}}{\partial t} - \frac{\dot{a}}{a}(\mathbf{x}.\nabla)\mathbf{u} + (\mathbf{u}.\frac{1}{a}\nabla)\mathbf{u} = -\frac{1}{a}\nabla\Phi\,.$$

a) Inserting the density and velocity perturbations through, $\rho = \bar{\rho}(1+\delta)$ and $\mathbf{u} = \dot{a}\mathbf{x} + \mathbf{v}$, derive the perturbed linearized comoving Euler equation (i.e., the second equation given in 2.3 below).

1.3) Consider the system of perturbed linearized comoving Newtonian equations of fluid mechanics (continuity, Euler and Poisson):

$$\dot{\delta} + \frac{1}{a}\nabla.\mathbf{v} = 0\,, \qquad \frac{\partial \mathbf{v}}{\partial t} + \frac{\dot{a}}{a}\mathbf{v} = -\frac{1}{a}\nabla\Phi\,, \qquad \nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta\,.$$

a) Combine the linearized equations to derive the equation of motion of δ,

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi\, G\,\bar{\rho}\,\delta = 0\,.$$

Exercise 2: Dark matter perturbations

2.1) Consider a flat Universe with some amount of non-relativistic pressureless massive neutrinos and no baryonic matter. In this Universe, the mean matter density Ω_m has two contributions: cold dark matter $\Omega_{\rm cdm}$ and neutrinos Ω_ν, i.e., $\Omega_{\rm m} = \Omega_{\rm cdm} + \Omega_\nu$. We assume that only the cold dark matter component clusters and forms structure.

a) Compute the evolution of the cold dark matter density contrast for this universe in the matter-dominated epoch.

b) Assume the neutrino fraction is 10% of the total matter density. In this case, using the result found in a), what is the growth rate (n) of the dark matter overdensities: $\delta_{\rm cdm} \propto a^n$?

2.2) Consider a flat Universe with dark matter and dark energy (where Ω_ν is now 0) where the dark matter growth rate is exactly the same found in 2.1 (i.e. $n = 0.94$).

a) Consider that dark energy is a cosmological constant. What is the value of Ω_Λ needed to produce that same growth rate?

b) Consider now a dynamical dark energy (i.e. w=w(z)). Assume also that this model is indistinguishable from the concordance ΛCDM at z=0. Show that, in order to produce that same growth rate, w(z) changed by roughly an order of magnitude since $z = 1$.

2.3) Consider a flat Universe with dark matter and a cosmological constant, such that the transition from matter to dark energy epochs occurs only today, at $a = 1$. In the matter epoch, the matter density contrast grows as $\delta \propto a$. Assume (unrealistically) that $\delta(t)$ keeps that rate all the way until the transition (i.e., dark energy does not make the rate to decrease during the matter epoch). This means that we can write $\delta(a) = a\,\delta_0$, where δ_0 is the clustering amplitude today. After the transition, the universe is dominated by the cosmological constant. Assume that the mean dark matter density can be neglected immediately after the transition: $\Omega_m(a > 1) = 0$.

a) Solve the equation of motion for the dark matter density constrast δ in the dark energy dominated epoch $a > 1$.

b) In a) you must have found that the solution for $\delta(t)$ is the sum of a decaying solution plus a constant term (an integration constant). Show that the constant term in the solution can be written in the form $n\delta_0$. Find out the value of n.

c) Assume that no collapsed (non-linear) structures have yet formed in this universe at $a = 1$, i.e., $\delta_0 < 1$. What is the minimum value that δ_0 must have for non-linear structure to be able to form in this universe in the future?

2.4) We know that the linearized Newtonian fluid equations are not valid to describe the non-linear clustering (when $\delta > 1$). However, N-body simulations use the linearized Newton's law and Poisson's equation in order to compute the interaction between masses and evolve the density field until $z = 0$. The resulting density map contains very large values of overdensities and is used to compute the non-linear matter power spectrum.

a) Do you think this is a valid approach that produces a reliable non-linear power spectrum? Or on the contrary is there some inconsistency? Justify your answer.

Exercise 3: Baryonic matter perturbations

3.1) The Jeans length is an important scale for baryonic matter clustering. Consider the standard flat ΛCDM Universe with Planck 2018 parameters, i.e., total matter density $\Omega_\mathrm{m} = 0.315$, $\Omega_\mathrm{b} = 0.049$, $\Omega_\mathrm{r} = 5.44 \times 10^{-5}$ and h = 0.674.

a) Compute the comoving Jeans length at z=1100 (assume this is just before decoupling, still in the plasma epoch). Give the result in terms of fraction of today's Hubble length c/H, and in Megaparsecs.

b) Theoretically, the Jeans length at decoupling is the largest scale that can have baryonic oscillations. In the standard model is there any oscillatory feature in the CMB power spectrum on that scale? Why (or why not)?

c) Compute the Jeans mass of baryonic matter at z=1100. Give the result in Laniakea masses, i.e., relative to the mass of the very massive Laniakea supercluster that has 10^{17} solar masses.

3.2) The total (baryonic + dark) matter power spectrum can be approximately computed using the primordial power spectrum and the extended BBKS transfer function (the one shown in page 41 of the slides chapter 11), where the scale q is given by

$$q = \frac{k}{\Omega_m h^2} \, e^{2\Omega_b}.$$

a) Make a plot of the matter power spectrum as function of scale k using a log-log x,y scale, with k in the range 0.001 to 1. Use $n_s = 1$, $\Omega_m = 0.315$, $\Omega_b = 0.049$.

b) Make another plot for $n_s = 1$, $\Omega_m = 0.315$, $\Omega_b = 0.266$.

Note: You can either write a simple code in a language of your choice, or you can compute the values of the power spectrum for some scales using some software you are familiar with and make the plot.