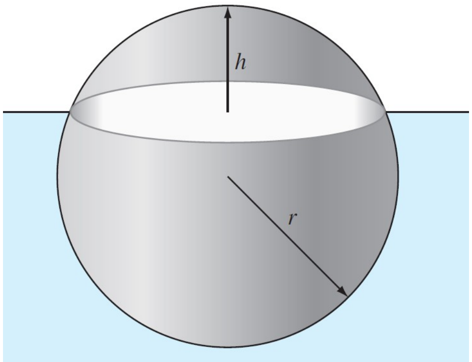
Exercise 3: Numerical integration and Ordinary Diferential Equations

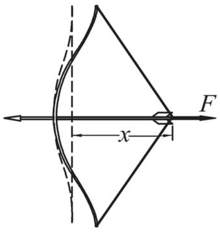
The report must be delivered within 15 days after the corresponding practical lessons.

1. **Sphere imerse in water:** consider a sphere with 2m radius being imersed in water. Knowing that the volume outside the water is given by $V=(\pi/3)h^2(3r-h)$, compute the work performed by the impulsion while submerging half the sphere (hint: place all constants out of the integrand).



   a. Implement the trapezoid and Simpson methods and use them to compute the work performed (hint: test the implementations first with a simpler integral, like x^3+x^2).
   b. Plot the difference between the numerically computed integral and its exact value as a function of the number of subdivisions, for both methods (hint: use the Integrate[] function of Mathematica to compute the exact value of the integral). Discuss the obtained results.

2. **Bow and arrow:** The force needed to draw a bow in the figure below as a function of *x* is given by the following table. Using the trapezoidal rule, compute the exit speed of an arrow weighting 75 g, when the bowstring is pulled 0.5 m (hint: use the work-energy principle).



x (m)	0.00	0.05	0.10	0.15	0.20	0.25
F (N)	0	37	71	104	134	161
x (m)	0.30	0.35	0.40	0.45	0.50	
F (N)	185	207	225	239	250	

3. **Radioactive decay:** consider the radioactive decay of Polonium-201, as described by the equation:

$$\dot{N}(t)=-kN(t),$$

   where N(t) is the density of radioactive nucleus at instant *t* and *k* is the decaying rate (with k=2.3 hour^{-1}). Taking as initial value N(0)=1:

   a. Implement the Euler method for computing ODEs. Solve the equation and plot N(t), for *t* in hours, with step sizes h={0.5, 0.7, 1}. Draw also the analytic solution (hint: although the solution is trivial, you may solve it the Dsolve[] function in Mathematica). Discuss the results.
   b. Implement the 4th order Runge-Kutta method and use it to solve the same equation. Plot the difference from the analytic solution at t=5 hours, for the Euler and RK4 methods as a function of step size, with steps $h=\{1,1/2,1/4,1/8,1/16,1/32,1/64\}$ (use a log-log scale). Discuss the order of convergence of both methods.

4. **Oblique throw:** consider an oblique throw of a projectile, where the equations of motion are given by (*x* is horizontal distance, *y* is vertical distance):

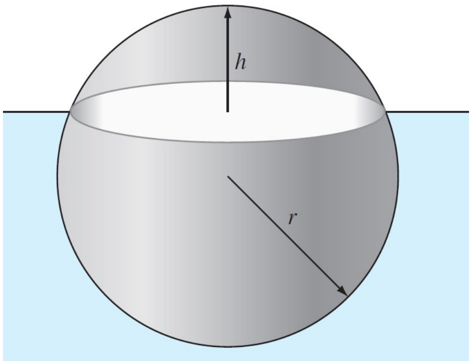
$$\dot{v}_x(t)=0$$
$$\dot{v}_y(t)=-g$$
$$\dot{x}(t)=v_x(t)$$
$$\dot{y}(t)=v_y(t)$$

   with an initial speed of 20m/s and an angle of $\pi/4$.

   a. Implement the Euler and 4th order Runge-Kutta for this set of equations (hint: implement vertical motion first and then add horizontal motion). Plot the projectile trajectory, *y* as a function of *x*, for both methods, with a step size of h=0.5. Draw also the analytical solution. Discuss the obtained results.

<u>Exercise 3 (optional)</u>: Numerical integration and Ordinary Diferential Equations

This part is optional and doesn't need to go into report.

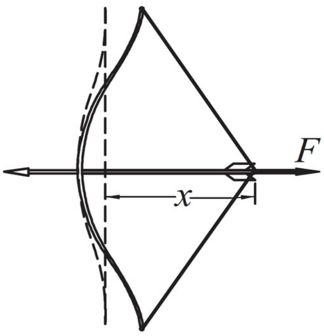


1. **Sphere imerse in water:** consider a sphere with 2m radius being imersed in water. Knowing that the volume outside the water is given by $V=(\pi/3)h^2(3r-h)$, compute the work performed by the impulsion while submerging half the sphere (hint: place all constants out of the integrand).
    a. Implement and compute the integral with Robmerg method.
    b. With Romberg method, plot difference from exact integral as a function of the number of iterations (use a log-lin scale). Discuss the results and discuss value of the integral for a single iteration.

2. Compute the integral of $1/x^2$, with all three implemented methods, and plot the difference from exact integral as a function of the number of subdivisions (use a log-log scale, draw all three curves in a single plot).

3. Calcule o integral da função $1/x^2$, para os 3 métodos, e trace o gráfico do desvio ao valor real em função do tamanho da divisão (os 3 métodos no mesmo gráfico) em escala log-log.

4. **Bow and arrow:** The force needed to draw a bow in the figure below as a function of *x* is given by the following table.



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    a. Compute the exit speed with of an arrow weitghing 75 g, when the bowstring is pulled 0.5 m (hint: use the work-energy principle).
    b. Compute the same speed in Mathematica, with the Interpolation[] function for order 1 and with the Nintegrate[] function. Compare with previous results. Plot the interpolated function.
    c. Compute the same speed with Simpsons rule.
    d. Calcule o mesmo integral usado a regra de Simpson.

5. Imagine an epidemic of zombies, knowing that: a zombie infects an human at rate *c*, an human kills a zombie at rate *a*, and a zombie kills an human at a rate *b*. The number of humans (H) and the number of zombies (Z) are given by:

$$\dot{H}(t)=-b\,H(t)Z(t)-c\,H(t)Z(t)$$
$$\dot{Z}(t)=c\,H(t)Z(t)-a\,H(t)Z(t)$$

Starting with a few zombies and a given population of humans, experiment with the previous methods to find a non-null set of parameters (*a*,*b*,*c*) that leads to apocalipse (annihilation of all humans) and a set (*a*,*b*,*c*) that leads to victory of human kind (elimination of all zombies).

6. Implement Euler and RK4 methods and use them to compute the trajectory of projectile subject to aerodinamic drag, where the acceleration has an additional term of γv^2 in the opposite direction of velocity. (hint: use function NDSolve[] in Mathmatica for comparison.)