Universo Primitivo
2024-2025 (1º Semestre)

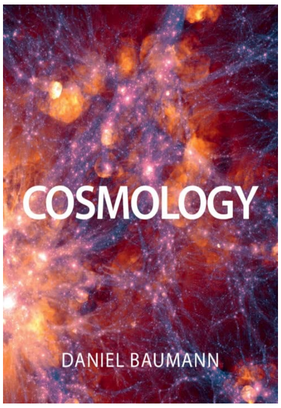
Mestrado em Física - Astronomia

Chapter 5

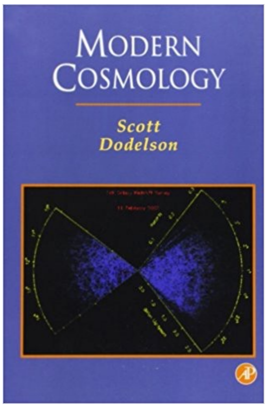
5 Dark Matter

- Observational evidences;
- Types of Dark Matter;
- WIMP decoupling: The Ricatti equation
- Dark matter relics: WIMPs freeze out;
- The WIMP ‘’miracle’’

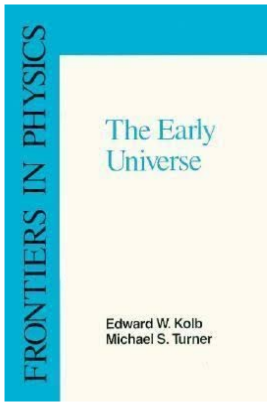
References



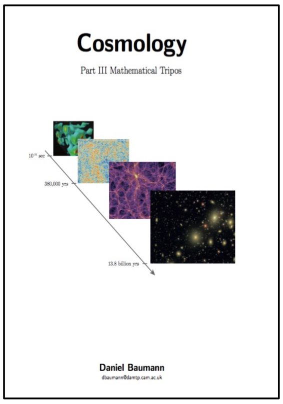
Ch. 3.2



Ch. 3.4



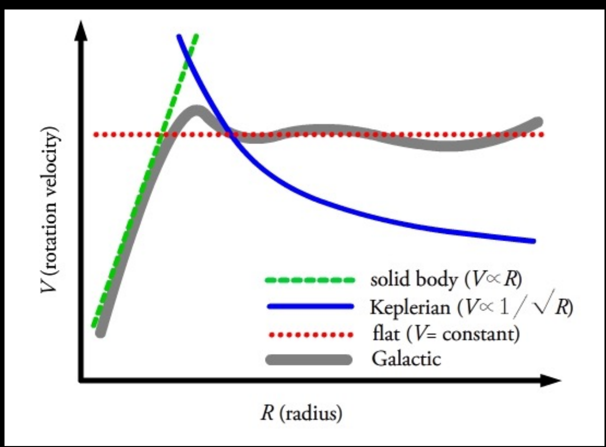
Ch. 5.2



Ch. 3.3.2

\+ Black board lectures

Dark Mater: Observational evidences



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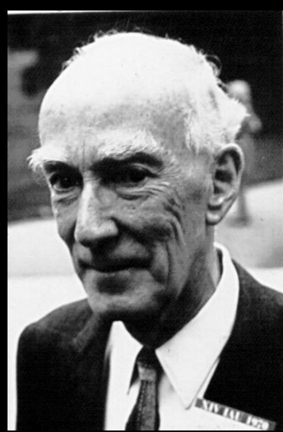
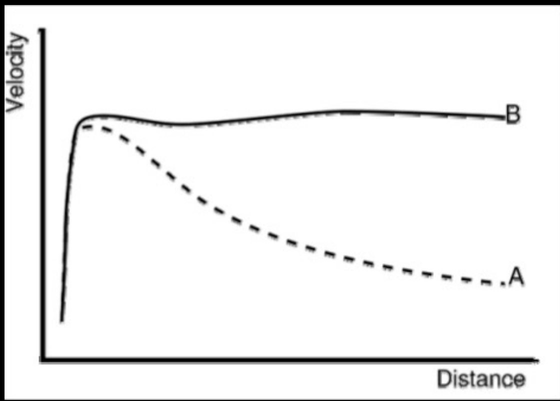


Jan Oort

1927: Jan Oort studies the rotation of stars in our galaxy and infers that their rotation is not consistent with Keplerian motion.

$$v_{circ} = \sqrt{\frac{GM(R)}{r}}$$

Dark Mater: Observational evidences



B: Observations

A: theoretical expectations

1980: Vera Rubin and others also find that stars rotate too fast in the outskirts of spiral galaxies to remain bound assuming that gravity is produced only by visible matter.

Dark Mater: Observational evidences

Fritz Zwicky

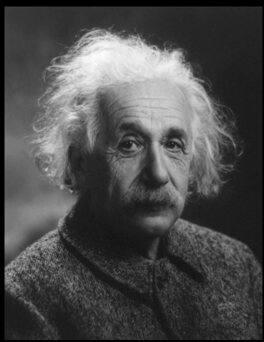
1936: Fritz Zwicky applied the Virial theorem to the velocities of galaxies in the Coma cluster and finds very high $\Upsilon = M/L$ for them to remain bound ($\Upsilon_{coma}/\Upsilon_{sun} \sim 500 \gg 2 - 10$ for galaxies).

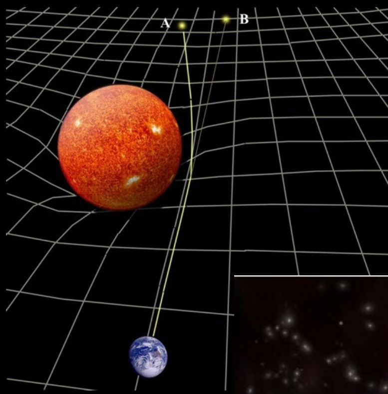
- Viral theorem (for gravitationally relaxed systems): $2\bar{E}_k + \bar{E}_p = 0$
- Mass from the virial theorem: $M_V = \langle v^2 \rangle \langle R \rangle / G$
- Visible (luminous) Mass: $M_L = N_g \, R_{ML} \, L_g$

(R_{ML} – typical mass to light ratio of galaxies;
N_g, L_g number and luminosity of individual galaxies)

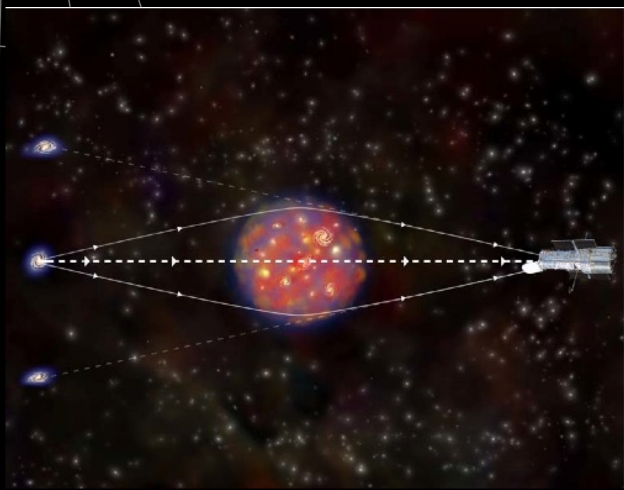
Dark Mater: Observational evidences

lensing effects: weak and strong lensing





$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

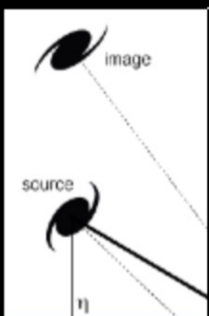


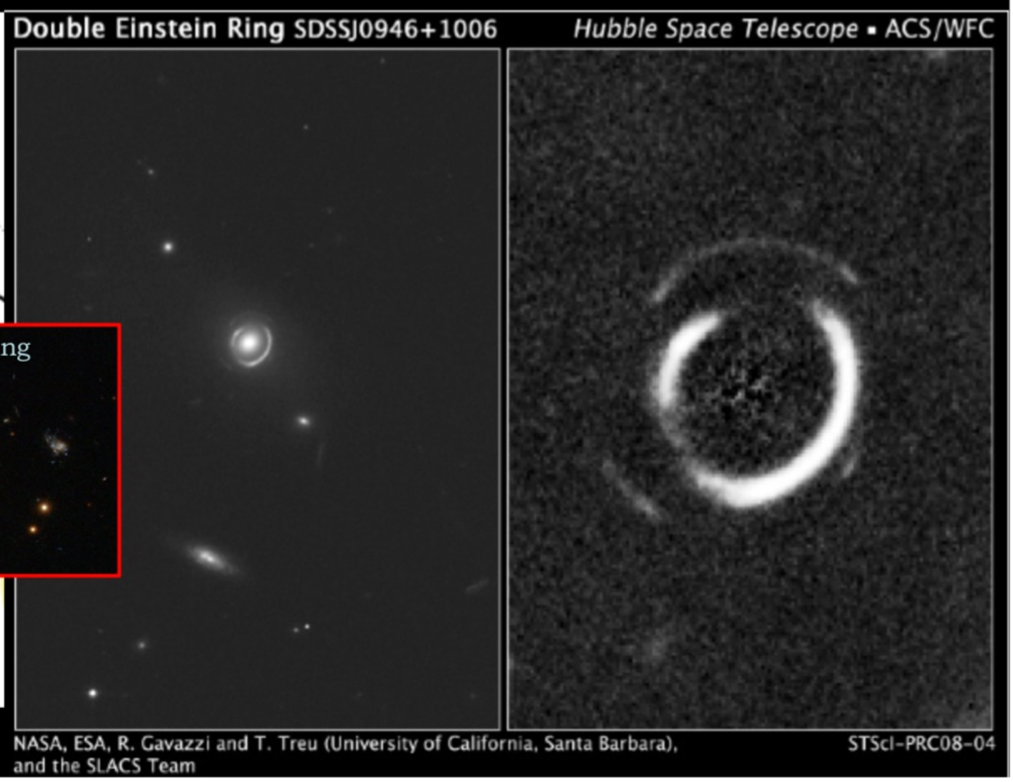
From:

Dark Mater: Observational evidences

lensing effects: strong lensing

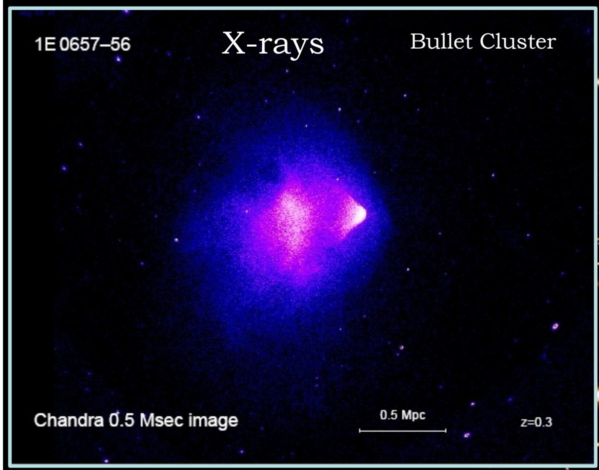
Einstein Rings



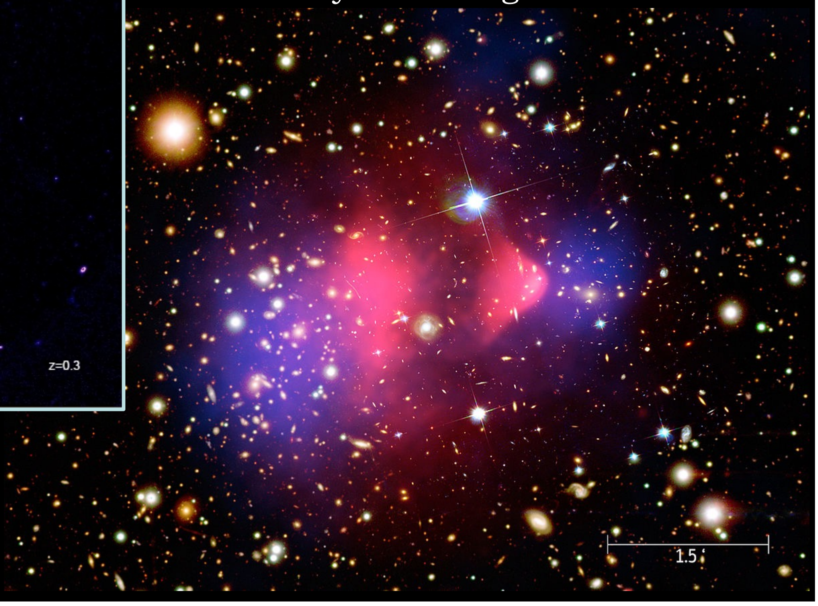


Dark Mater: Observational evidences

2003: X-ray (produced by extremely hot gas – in red) vs weak lensing observations (probing the total mass distribution in blue) of the Bullet Cluster put in evidence that galaxy clusters must contain “dark matter”



X-rays + lensing + visible



Refs:
astro-ph/0309303
astro-ph/0312273

Types of “Dark Matter”

Some definitions:

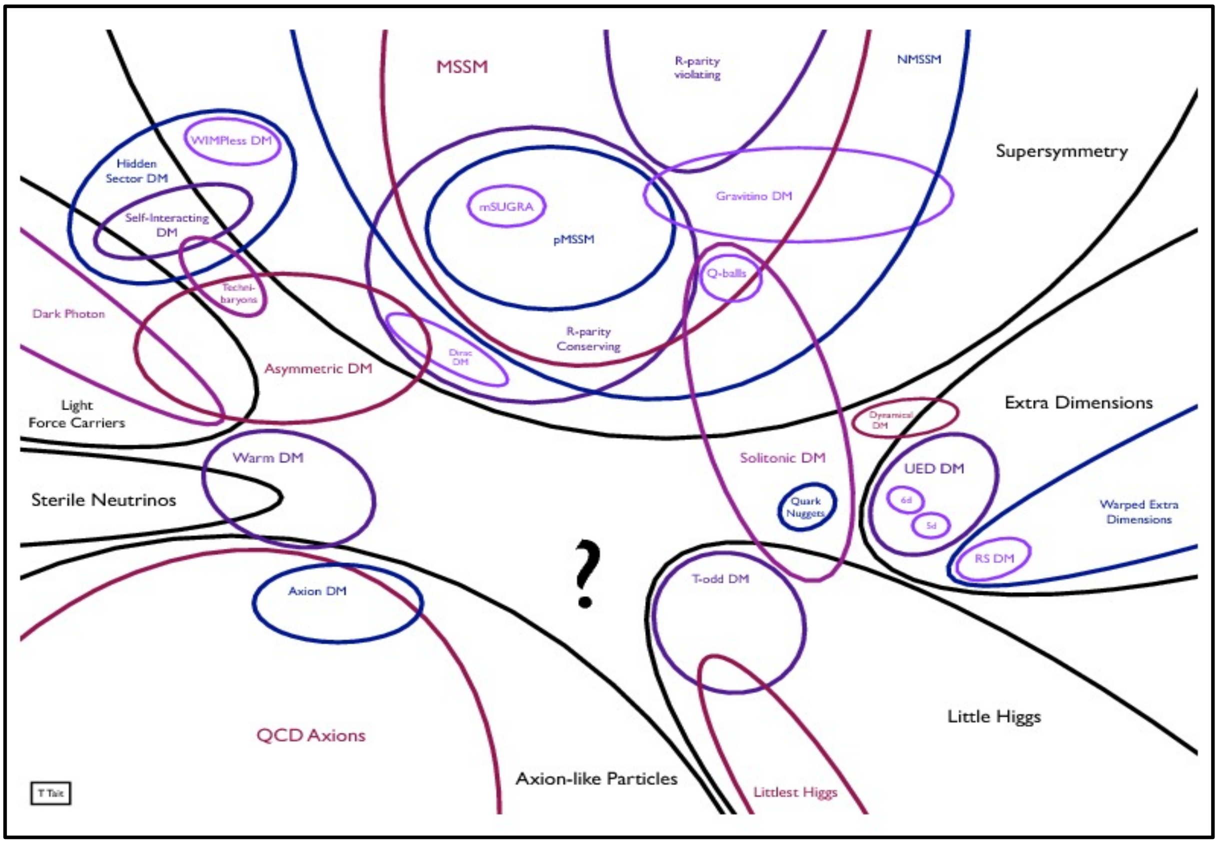
Dark matter **is not non-luminous** (invisible) ordinary matter!

Non-luminous baryonic “dark matter”:
- White dwarfs
- Neutron stars
- Black holes
- MACHOS (Massive Compact Halo Objects)
- …

“Non-baryonic” dark matter:
- **Hot (HDM)**: massless or very small mass relativistic particles, (e.g. neutrinos)
- **Warm (WDM)**: hypothetical with intermediate properties, (e.g. keVins and GeVins - keV and Gev inert fermions)
- **Cold (CDM)**: massive, non-relativistic particles, (e.g. Axions, light supersymmetric particles - gravitino, neutralino, WIMPS - Weak Interactive Massive Particles)

Types of “Dark Matter”

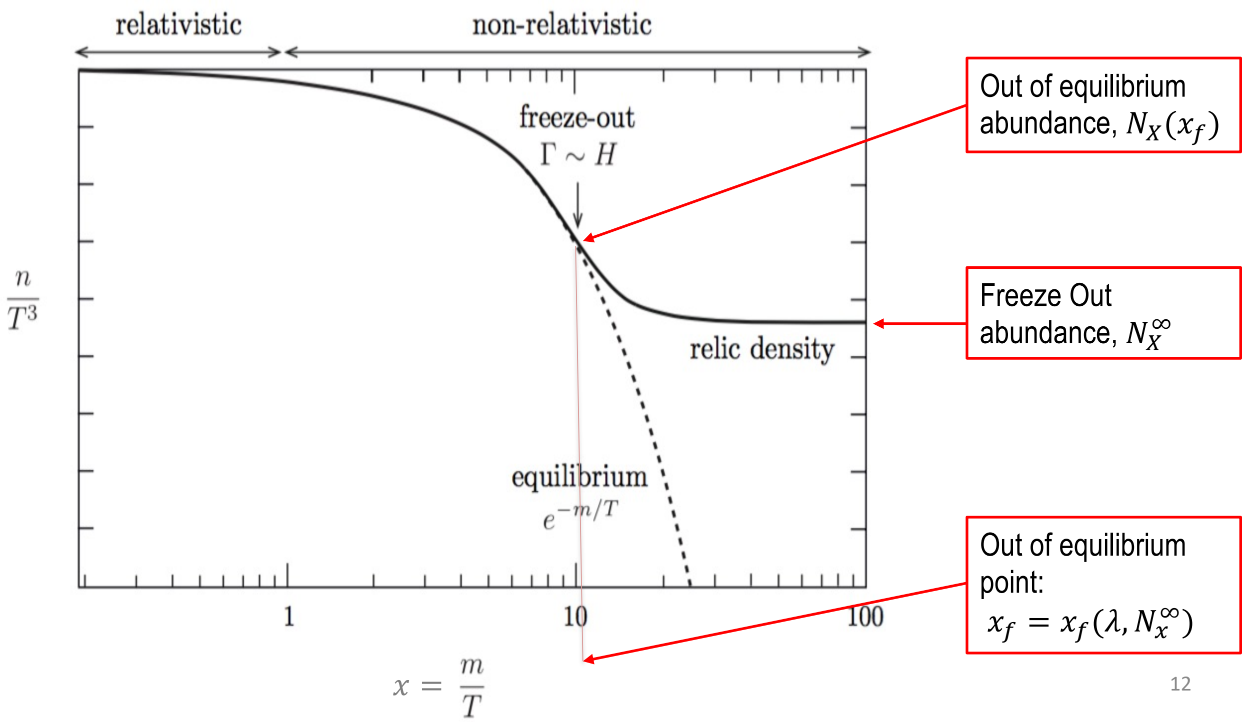
Model candidates:



Dark Matter relics: WIMP Freeze out

Weakly Interactive Massive Particles:

Black board lecture



WIMP decoupling

Weakly Interactive Massive Particles (WIMP)

A **WIMP** particle species is a hypothetical type of massive particles, of mass M_X, that **under certain conditions provides a viable mechanism to explain the observed energy density parameter of Dark Matter**, Ω_{DM}. WIMPs are assumed to interact via the Weak Force. They are not part of the Standard Model of Particle Physics (SMPP) but can be sought as part of SMPP extensions such as Super-Symmetric Models (SM).

A simple (viable) WIMP model assumes that the WIMP particle species X remains in equilibrium with the primordial fluid until it decouples via an annihilation process of the form:

$$X + \bar{X} \rightleftarrows l + \bar{l}$$

where $l, \bar{l}$ are massless and tightly coupled particles (e.g. neutrinos) to the fluid. It is also often assumed that there's no initial particle-antiparticle asymmetry for X. Under these conditions one has:

- $n_l = n_l^{\mathrm{eq}}$
- $n_{\bar{l}} = n_{\bar{l}}^{\mathrm{eq}}$
- $n_X = n_{\bar{X}}$; $n_X^{\mathrm{eq}} = n_{\bar{X}}^{\mathrm{eq}}$

The Boltzmann equation then reads

$$\frac{1}{a^3}\frac{d(n_X a^3)}{dt} = -\langle\sigma v\rangle\left[n_X n_{\bar{X}} - \left(\frac{n_X n_{\bar{X}}}{n_l n_{\bar{l}}}\right)_{\mathrm{eq}} n_l n_{\bar{l}}\right] =$$

$$= -\langle\sigma v\rangle\left[n_X n_{\bar{X}} - \left(\frac{n_X n_{\bar{X}}}{n_l n_{\bar{l}}}\right)_{\mathrm{eq}} n_l^{\mathrm{eq}} n_{\bar{l}}^{\mathrm{eq}}\right]$$

WIMP decoupling

Weakly Interactive Massive Particles (WIMP)

Using the X particle-antiparticle symmetry ($n_X = n_{\bar{X}}$ and $n_X^{\mathrm{eq}} = n_{\bar{X}}^{\mathrm{eq}}$) assumption, gives:

$$\frac{1}{a^3}\frac{d(n_X a^3)}{dt} = -\langle\sigma v\rangle\left[n_X n_X - n_X^{\mathrm{eq}} n_X^{\mathrm{eq}}\right] = -\langle\sigma v\rangle\left[n_X^2 - \left(n_X^{\mathrm{eq}}\right)^2\right]$$

One can now convert number densities into number of particles by using $N_X = n_X s$ and $N_X^{\mathrm{eq}} = n_X^{\mathrm{eq}} s$ (the $N_X = n_X s$ definition is kept regardless the state of equilibrium of the particle species):

$$\frac{1}{a^3}\frac{d(N_X s a^3)}{dt} = -\langle\sigma v\rangle\, s^2\left[N_X^2 - \left(N_X^{\mathrm{eq}}\right)^2\right] \Leftrightarrow$$

$$\Leftrightarrow \frac{s a^3}{a^3}\frac{dN_X}{dt} = -\langle\sigma v\rangle\, s^2\left[N_X^2 - \left(N_X^{\mathrm{eq}}\right)^2\right] \Leftrightarrow$$

$$\Leftrightarrow \frac{dN_X}{dt} = -\langle\sigma v\rangle\, s\left[N_X^2 - \left(N_X^{\mathrm{eq}}\right)^2\right]$$

Note that sa^3 is constant due to entropy conservation. The objective is to integrate this equation at any epoch, particularly around and beyond the X mass threshold, i.e. for $T \gtrsim M_X$. For this it is useful to change the integration variable, t, to the variable $x = M_X/T$ which is $x = 1$ for $T = M_X$ and gives $x \to 0$ as $T \to \infty$ (equilibrium) and $x \to \infty$ as $T \to 0$ (out of equilibrium). So, one has

$$\frac{dN_X}{dt} = \frac{dN_X}{dx}\frac{dx}{dt} = \frac{dN_X}{dx}\frac{d\,(M_X T^{-1})}{dt} = \frac{dN_X}{dx}\left(-\frac{M_X}{T^2}\frac{dT}{dt}\right) = -\frac{dN_X}{dx}\frac{x}{T}\frac{dT}{dt}$$

WIMP decoupling

Weakly Interactive Massive Particles (WIMP)

To proceed one needs to compute the time derivative of temperature, which can be achieved by **solving numerically the temperature - scale factor relation**, $T = A_i\, g_{*S}^{-1/3}(T)\, a(t)^{-1}$. Since g_{*S} is constant away from the mass threshold $x = 1$ ($T = M_X$) one can still **proceed analytically with an approximation** for $x \gg 1$ (and $x \ll 1$). So:

$$\frac{dN_X}{dt} = -\frac{dN_X}{dx}\frac{x}{T}\frac{dT}{dt} = -\frac{dN_X}{dx}\frac{x}{T}\frac{d}{dt}\left(A_i g_{*S}^{-1/3}\, a^{-1}\right) \simeq \frac{dN_X}{dx}\frac{x}{T}A_i g_{*S}^{-1/3}\, a^{-2}\frac{da}{dt} =$$

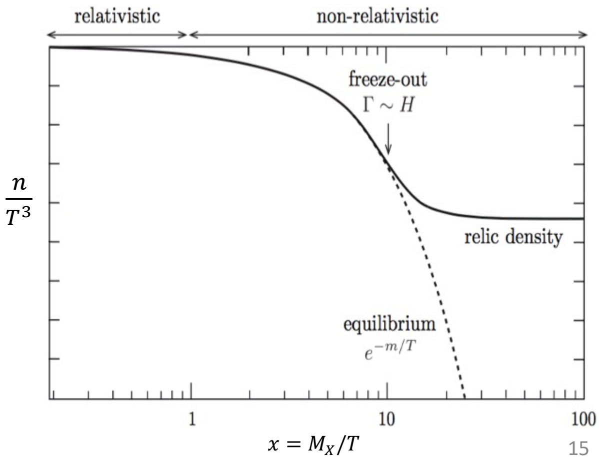
$$= \frac{dN_X}{dx}\, x\, \frac{A_i g_{*S}^{-1/3}\, a^{-2}}{A_i g_{*S}^{-1/3}\, a^{-1}}\frac{da}{dt} = \frac{dN_X}{dx}\, x \frac{1}{a}\frac{da}{dt} = \frac{dN_X}{dx}\, x\, H$$

Going back to the Boltzmann equation one has:

$$\frac{dN_X}{dx}\, x\, H \simeq -\langle\sigma v\rangle\, s\left[N_X^2 - \left(N_X^{eq}\right)^2\right] \Leftrightarrow$$

$$\boxed{\frac{dN_X}{dx} \simeq -\langle\sigma v\rangle\,\frac{s}{H}\frac{1}{x}\left[N_X^2 - \left(N_X^{eq}\right)^2\right]}$$

The latter is known as the **Ricatti equation** that can be easily integrated providing analytical and accurate solutions of the **relic** abundance of X, i.e. $\boldsymbol{N_X^\infty \equiv N_X(x \to +\infty)}$.



Dark Matter relics: WIMP Freeze out

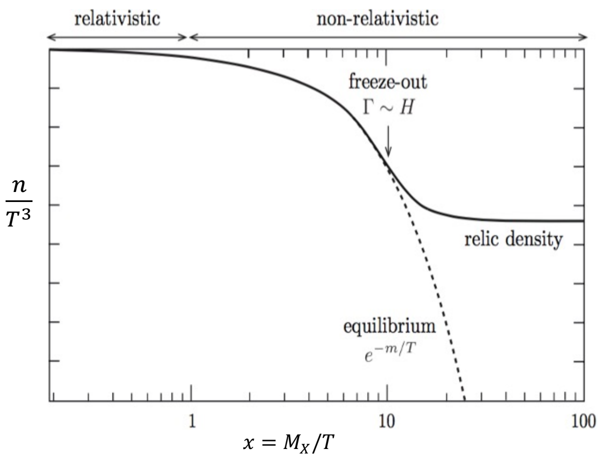
Ricatti Equation

The integration of the **Ricatti equation**

$$\boxed{\frac{dN_X}{dx} = -\langle\sigma v\rangle\,\frac{s}{H}\frac{1}{x}\left[N_X^2 - \left(N_X^{eq}\right)^2\right]}$$

requires the evaluation of s and H as a function $x = M_X$/T. Starting with the former one has:

$$s = \frac{2\pi^2}{45}\, g_{*S} T^3 = \frac{2\pi^2}{45}\, g_{*S}\left(\frac{M_X}{x}\right)^3$$



Given that the decoupling of X is mediated by the weak force, which is effective at $T \gtrsim 1$ MeV, it is reasonable to assume that the **decoupling happens during radiation domination**, so:

$$H = \frac{\pi}{3M_{pl}^2}\left(\frac{g_*}{10}\right)^{1/2} T^2 = \frac{\pi}{3M_{pl}^2}\left(\frac{g_*(M_X/x)}{10}\right)^{1/2}\left(\frac{M_X}{x}\right)^2 \simeq H(M_X)\left(\frac{1}{x}\right)^2$$

where $H(M_X)$ is the value of the Hubble function around $T \simeq M_X$ i.e. $x \sim 1$. If the transition at the mass threshold is fast (or instantaneous) $H(M_X)$ below or above the is $T \simeq M_X$ is a constant, reading:

$$H(M_X) = \frac{\pi}{3}\left(\frac{g_*(M_X)}{10}\right)^{1/2}\left(\frac{M_X}{M_{pl}}\right)^2$$

Dark Matter relics: WIMP Freeze out

Ricatti Equation

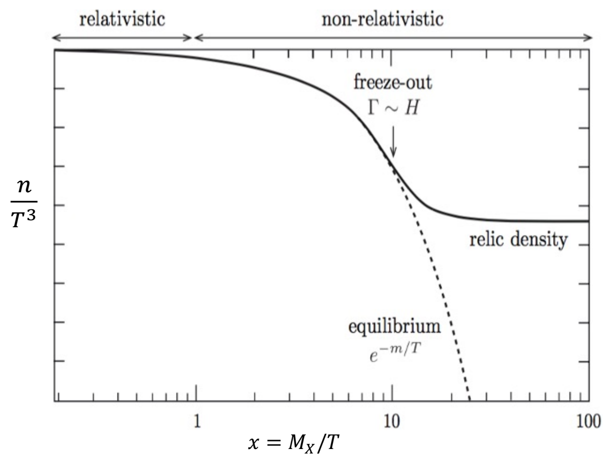
Combining the expressions of $s(x)$ and $H(x)$ in the the Ricatti equation yields:

$$\frac{dN_X}{dx} = -\langle\sigma v\rangle \frac{s}{H}\frac{1}{x}\left[N_X^2 - \left(N_X^{eq}\right)^2\right] =$$

$$= -\langle\sigma v\rangle \frac{\frac{2\pi^2}{45} g_{*S}\left(\frac{M_X}{x}\right)^3}{H(M_X)\left(\frac{1}{x}\right)^2}\frac{1}{x}\left[N_X^2 - \left(N_X^{eq}\right)^2\right] =$$

$$= -\frac{2\pi^2}{45}\langle\sigma v\rangle \frac{g_{*S}M_X^3}{H(M_X)}\frac{1}{x^2}\left[N_X^2 - \left(N_X^{eq}\right)^2\right] =$$

$$= -\frac{\lambda}{x^2}\left[N_X^2 - \left(N_X^{eq}\right)^2\right]$$



where,

$$\boxed{\lambda = \frac{2\pi^2}{45}\langle\sigma v\rangle \frac{g_{*S}M_X^3}{H(M_X)}}$$

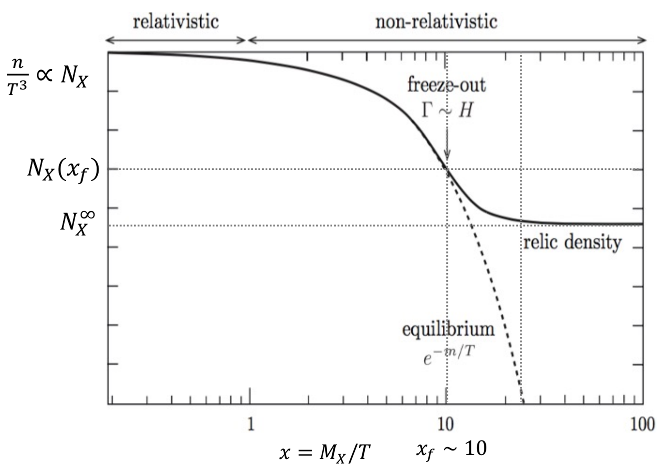
is a time independent quantity whenever $g_{*S}(T)$ is constant, i.e., way from mass thresholds and, in particular, away from the X mass threshold ($x = 1$). Note that the Ricatti equation is by itself a valid approximation of the Boltzmann equation only for $x \gg 1$ (and $x \ll 1$). Soon after WIMP decoupling, i.e. below $T = 1$ MeV, $g_{*S}(x)$ remains constant until the present, so $\lambda =$ constant is a good approximation to compute **analytically** the relic abundance of WIMP particles.

Dark Matter relics: WIMP Freeze out

Ricatti Equation

So let us know **integrate the Ricatti equation for $x \gg 1$** (where it is a valid approximation of the WIMP's Boltzmann equation and **λ is constant** because $g_{*S}(x) \simeq 3.94$ until the present:

$$\boxed{\frac{dN_X}{dx} = -\frac{\lambda}{x^2}\left[N_X^2 - \left(N_X^{eq}\right)^2\right]}$$



At late times $x \gg x_f \sim 10$, $N_X \gg N_X^{eq}$ (the relic abundance is much larger than the equilibrium abundance because the latter decreases exponentially with x). So,

$$\frac{dN_X}{dx} \simeq -\frac{\lambda}{x^2}N_X^2 \quad \Leftrightarrow \quad dN_X \simeq -\frac{\lambda}{x^2}N_X^2\, dx$$

The mass to temperature ratio x_f is usually defined by the condition $\Gamma(x_f) = H(x_f)$ yielding $x_f = 8$. Integrating, for $x \gg x_f$, on has:

$$\int_{N_X(x_f)}^{N_X^\infty} N_X^{-2}\, dN_X \simeq -\lambda \int_{x_f}^{\infty} x^{-2}\ dx \quad \Leftrightarrow \quad \left[-N_X^{-1}\right]_{N_X(x_f)}^{N_X^\infty} \simeq -\lambda\left[-x^{-1}\right]_{x_f}^{\infty} \Leftrightarrow$$

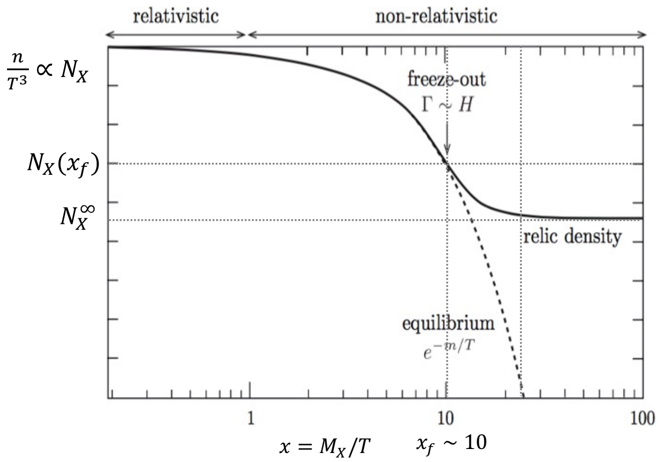
$$\Leftrightarrow \left(\frac{1}{N_X^\infty} - \frac{1}{N_X(x_f)}\right) = -\lambda\left(\frac{1}{\infty} - \frac{1}{x_f}\right) \Leftrightarrow \frac{1}{N_X^\infty} - \frac{1}{N_X(x_f)} = \frac{\lambda}{x_f}$$

Dark Matter relics: WIMP Freeze out

Ricatti Equation

The solution of the Ricatti equation for $x \gg x_f \sim 10$ can be equated from

$$\boxed{\frac{1}{N_X^\infty} - \frac{1}{N_X(x_f)} \simeq \frac{\lambda}{x_f}}$$



Since $\boldsymbol{N_X}$ is typically more than one order of magnitude smaller than $\boldsymbol{N_X(x_f)}$ (i.e. $\boldsymbol{N_X(x_f) > 10\, N_X}$) the 2nd term of the right-hand-side of the equation above is often neglected, yielding the following approximate relic abundance for the WIMP particles:

$$\frac{1}{N_X^\infty} \simeq \frac{\lambda}{x_f} \quad \Longleftrightarrow \quad \boxed{N_X^\infty \simeq \frac{x_f}{\lambda}}$$

where $x_f \sim 10$, and:

$$\lambda = \frac{2\pi^2}{45} \langle \sigma v \rangle \, \frac{g_{*S}(M_X)\, M_X^3}{H(M_X)} \simeq \text{constant}$$

depends on the weak interaction thermally averaged cross-section, the mass of the WIMPs, the Hubble function at $T \simeq M_X$ and $g_{*S}(T > M_X) \simeq 3.98$ (which is constant for $T \gtrsim 10\, M_X$).

Dark Matter relics: WIMP Freeze out

WIMP miracle

Let us now **compute the energy density parameter of WIMP** relic particles, $\boldsymbol{\Omega_{X,0}}$, and **compare** our findings **with** the observed Dark Matter energy density parameter $\boldsymbol{\Omega_{DM,0} = 0.27}$. By definition:

$$\Omega_{X,0} = \frac{\rho_{X,0}}{\rho_{crit,0}}$$

$$\rho_{crit,0} = \frac{3H_0^2}{8\pi G} = \frac{3H_0^2}{1/M_{pl}^2} = 3H_0^2 M_{pl}^2$$

where $\rho_{X,0}$ is the present WIMP energy density and the critical density at present, $\rho_{crit,0}$, was expressed in terms of the Planck mass $M_{pl}^2 = 1/(8\pi G)$ and the Hubble constant, $H_0 = h \times 100$ Km/s/Mpc.

Since the WIMP species is already decoupled and non-relativistic at late times, one has $\rho_{X,0} = M_X n_{X,0}$. Using this and the conversion between number and number density, $n_{X,0} = N_{X,0}\, s_0$, one has:

$$\Omega_{X,0} = \frac{\rho_{X,0}}{\rho_{crit,0}} = \frac{M_X n_{X,0}}{3H_0^2 M_{pl}^2} = \frac{M_X\, N_{X,0} s_0}{3H_0^2\, M_{pl}^2} \simeq \frac{s_0 M_X}{3H_0^2\, M_{pl}^2} N_X^\infty \simeq \frac{s_0 M_X}{3H_0^2\, M_{pl}^2} \frac{x_f}{\lambda}$$

where in the last equality we approximate the present number of WIMP particles by its asymptotic relic value given by the solution the Riccati Equation $N_{X,0} \simeq N_X^\infty = x_f/\lambda$. Recalling the expression of s_0 and λ, one has:

$$\Omega_{X,0} \simeq \left(\frac{2\pi}{45} g_{*S}(T_0) T_0^3\right) \frac{M_X\, x_f}{3H_0^2\, M_{pl}^2} \left(\frac{2\pi^2}{45} \langle \sigma v \rangle \frac{g_{*S}(M_X)\, M_X^3}{H(M_X)}\right)^{-1}$$

Dark Matter relics: WIMP Freeze out

WIMP miracle

So, recalling the expression of $H(M_X) = \frac{\pi}{3}\left(\frac{g_*(M_X)}{10}\right)^{1/2}\left(\frac{M_X}{M_{pl}}\right)^2$ and rearranging one obtains:

$$\Omega_{X,0} = g_{*S}(T_0)\,T_0^3\,\frac{M_X}{3H_0^2\,M_{pl}^2}\,\frac{x_f}{\langle\sigma v\rangle}\,\frac{H(M_X)}{g_{*S}(M_X)\,M_X^3}$$

$$= \frac{g_{*S}(T_0)\,T_0^3}{3H_0^2\,g_{*S}(M_X)}\,\frac{1}{M_X^2\,M_{pl}^2}\,\frac{x_f}{\langle\sigma v\rangle}\left(\frac{\pi}{3}\left(\frac{g_*(M_X)}{10}\right)^{1/2}\left(\frac{M_X}{M_{pl}}\right)^2\right)$$

$$= \frac{\pi}{9}\,\frac{g_{*S}(T_0)}{g_{*S}(M_X)}\left(\frac{g_*(M_X)}{10}\right)^{1/2}\frac{x_f}{\langle\sigma v\rangle}\,\frac{T_0^3}{M_{pl}^4 H_0^2}$$

Setting $\Omega_{X,0} = \Omega_{DM}$ one can estimate the order of magnitude of $\langle\sigma v\rangle$ and $g_*(M_X) \sim g_{*S}(M_X)$, from cosmological observations. Conversely, if WIMPs **decouple via the weak force one can estimate** $\Omega_{X,0}$ from what we know (from Particle Physics) about the weak force, which is not able to keep a species in equilibrium below 1 MeV. Assuming WIMPs decouple around that epoch one has:

- $H_0 = h \times 100\ kms^{-1}Mpc^{-1}$; $T_0 = 2.726\ K$; $h = 0.7$; $\Omega_{DM,0} = 0.27$; $\Omega_{DM,0}h^2 = 0.1323$
- $g_{*S}(M_X) \simeq g_{*S}(T_0) = 3.94$; $g_*(M_X) \simeq 3.38$; $\langle\sigma v\rangle \sim 10^{-8}GeV^{-2}$
- $x_f \gtrsim 10$; $M_{pl} \simeq 2\times 10^{18}GeV$

Plugging these in the expression above (after unit conversions), one obtains:

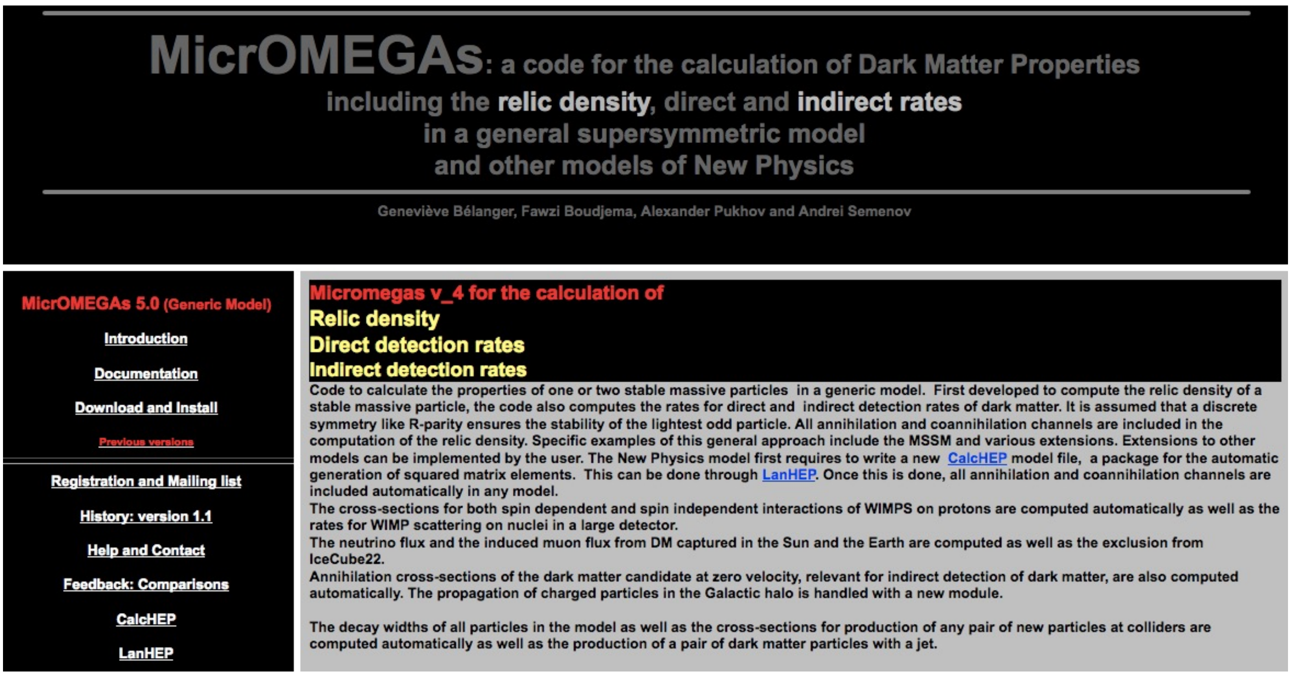
$$\Omega_{X,0}h^2 \simeq 0.1\left(\frac{x_f}{10}\right)\left(\frac{10}{g_*(M_X)}\right)^{1/2}\left(\frac{10^{-8}GeV^{-2}}{\langle\sigma v\rangle}\right) \simeq 0.15$$

The fact that this value is consistent with the observed $\boldsymbol{\Omega_{DM,0}h^2}$, **is known as the WIMP miracle!**

Dark Matter relic (Boltzmann) code packages

Example 1:

http://lapth.cnrs.fr/micromegas/



Dark Matter relic (Boltzmann) code packages

Example 2:

http://superiso.in2p3.fr/relic/

SuperIso Relic

By Alexandre Arbey, Farvah Nazila Mahmoudi & Glenn Robbins

SuperIso
- Description
- Manual

SuperIso Relic
- Description
- Manual

Download
- SuperIso
- SuperIso Relic

AlterBBN

Links

Calculation of flavour physics and dark matter observables

SuperIso Relic is a mixed C - Fortran code which computes the dark matter observables in the MSSM and NMSSM. SuperIso Relic is an extension of SuperIso and therefore gives access to many flavour observables at the same time.

The computation of the relic density requires the calculation of thousands of annihilation and coannihilation Feynman diagrams. In SuperIso Relic, all these diagrams have been analytically computed at tree level using FeynArts/FormCalc. They are then calculated numerically during execution. The widths of the Higgs bosons are also computed using FeynHiggs or Hdecay. The necessary libraries are included in the SuperIso Relic packages.

From the cosmological point of view, SuperIso Relic performs the relic density calculation in the cosmological standard model, but also offers the possibility to alter the equation of state of radiation or modify the density and entropy content of the pre-BBN Universe. BBN constraints to check the validity of the altered model are automatically computed using the AlterBBN code included in the package.

Since its version 4, SuperIso Relic also includes routines to compute observables related to dark matter direct and indirect detections, and incorporates in particular experimental constraints from FERMI-LAT, AMS-02, XENON1T, PANDA-X and PICO60.

For any comment, question or bug report please contact Alexandre Arbey, Nazila Mahmoudi or Glenn Robbins.