Aula TP3 29-03-2023

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1) $|\psi\rangle = Ne^{-\alpha r}$

a)

$$\langle\psi|\psi\rangle = 1 \Leftrightarrow \int \psi^* \psi \, d\tau = 1 \Rightarrow \int N e^{-\alpha r} N e^{-\alpha r} d\tau = 1 \Rightarrow$$

$$\Leftrightarrow \underbrace{\int_0^{2\pi} d\varphi}_{[\varphi]_0^{2\pi} = 2\pi} \underbrace{\int_0^{\pi} \operatorname{sen}\theta \, d\theta}_{[-\cos\theta]_0^{\pi} = -\cos\pi + \cos 0 = 1+1=2} \int_0^{\infty} \underbrace{N e^{-\alpha r} N}_{N^2} e^{-\alpha r} r^2 dr = 1 \Leftrightarrow$$

$$\int_0^{\infty} x^n e^{-\alpha' x} dx = \frac{n!}{\alpha^{n+1}}$$

$$\Leftrightarrow 4\pi N^2 \int_0^{\infty} \underbrace{r^2 e^{-2\alpha r}}_{n=2 \quad \alpha' = 2\alpha} dr = 1 \Leftrightarrow$$

$$\Leftrightarrow 4\pi N^2 \frac{2!}{(2\alpha)^3} = 1 \Leftrightarrow \frac{4\pi N^2 \cancel{2}}{\cancel{2^3}\alpha^3} = 1 \Leftrightarrow N^2 = \frac{\alpha^3}{\pi} \Leftrightarrow N = \sqrt{\frac{\alpha^3}{\pi}}$$

$$\text{Logo } |\psi\rangle = \sqrt{\frac{\alpha^3}{\pi}} e^{-\alpha r}$$

b)

$$T|\psi\rangle = -\frac{1}{2}\nabla^2|\psi\rangle = -\frac{1}{2} r^{-2} \frac{d}{dr}\left[r^2 \frac{d}{dr} N e^{-\alpha r}\right] =$$

$$= -\frac{1}{2} r^{-2} \frac{d}{dr}\left[r^2 (-\alpha) N e^{-\alpha r}\right] =$$

$$\frac{d}{dx} e^{n} = (n)' e^{n}$$

$$(x_1 x_2)' = x_1 (x_2)' + (x_1)'(x_2)$$

$$= \frac{N}{2} r^{-2} \frac{d}{dr}\left[r^2 \alpha e^{-\alpha r}\right] =$$

$$= \frac{N}{2} r^{-2}\left[\alpha r^2 (-\alpha) e^{-\alpha r} + 2r\alpha e^{-\alpha r}\right] =$$

$$= \frac{N}{2}\left[\frac{\alpha \cancel{r^2}(-\alpha) e^{-\alpha r}}{\cancel{r^2}} + \frac{2\cancel{r}\alpha e^{-\alpha r}}{r^{\cancel{2}}}\right] = \frac{N}{2}\left[\frac{2\alpha e^{-\alpha r}}{r} - \alpha^2 e^{-\alpha r}\right]$$

$$\langle T\rangle = \langle\psi| -\tfrac{1}{2}\nabla^2|\psi\rangle = \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \underbrace{\int_0^{\pi} \operatorname{sen}\theta \, d\theta}_{2} \int_0^{\infty} N e^{-\alpha r} \cdot \frac{N}{2}\left[\frac{2\alpha e^{-\alpha r}}{r} - \alpha^2 e^{-\alpha r}\right] r^2 dr$$

$$\underbrace{\quad}_{2\pi} \quad \underbrace{\quad}_{2}$$

$$\Leftrightarrow \frac{4\pi N^2}{2}\int_0^\infty e^{-\alpha r}\left[\frac{2\alpha e^{-\alpha r}}{r}-\alpha^2 e^{-\alpha r}\right] r^2\, dr \Leftrightarrow$$

$$= 2\pi N^2 \int_0^\infty \frac{r^{\cancel{2}}\, 2\alpha e^{-2\alpha r}}{\cancel{r}} - r^2\alpha^2 e^{-2\alpha r}\, dr =$$

$$= 2\pi N^2\left[2\alpha \underbrace{\int_0^\infty r\, e^{-2\alpha r}\, dr}_{n=1 \quad \alpha'=2\alpha} - \alpha^2 \underbrace{\int_0^\infty r^2 e^{-2\alpha r}\, dr}_{n=2 \quad \alpha'=2\alpha}\right] =$$

$$\int_0^\infty x^n e^{-\alpha x}\, dx = \frac{n!}{\alpha^{n+1}}$$

$$= 2\pi N^2\left[\frac{2\alpha}{(2\alpha)^2} - \frac{\alpha^2 2}{(2\alpha)^3}\right] = \frac{4\pi N^2 \cancel{2\alpha}}{\cancel{2^2}\alpha^{\cancel{2}}} - \frac{\cancel{\alpha^2}\, 4\pi N^2}{2^3\alpha^{\cancel{3}}} =$$

$$= \frac{\pi N^2}{\alpha} - \frac{\pi N^2}{2\alpha} = \frac{\pi N^2}{2\alpha} = \qquad \text{como } N^2 = \frac{\alpha^3}{\pi}$$

$$= \frac{\cancel{\pi}\alpha^{\cancel{3}\,2}}{2\cancel{\alpha}\cancel{\pi}} = \frac{\alpha^2}{2} = \langle T \rangle$$

c) $$\langle V \rangle = \langle \psi | -\tfrac{1}{r} | \psi \rangle = \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \underbrace{\int_0^\pi \operatorname{sen}\theta\, d\theta}_{2} \int_0^\infty N e^{-\alpha r}\left(-\frac{1}{r}\right) N e^{-\alpha r}\, r^2\, dr =$$

$$= -4\pi N^2 \underbrace{\int_0^\infty r\, e^{-2\alpha r}\, dr}_{n=1 \quad \alpha = 2\alpha} =$$

$$\int_0^\infty x^n e^{-\alpha' x} = \frac{n!}{\alpha^{n+1}}$$

$$= -4\pi N^2 \frac{1}{(2\alpha)^2} = -\frac{4\pi N^2}{\cancel{2^2}\alpha^2} = \frac{-\pi N^2}{\alpha^2} = \qquad N^2 = \frac{\alpha^3}{\pi}$$

$$= \frac{-\cancel{\pi}\alpha^3}{\cancel{\pi}\cancel{\alpha^2}} = -\alpha = \langle V \rangle$$

d) $$\langle \epsilon \rangle = \langle T \rangle + \langle V \rangle = \frac{\alpha^2}{2} - \alpha$$

e) $$\frac{\partial \epsilon}{\partial \alpha} = 0 \Leftrightarrow \frac{d}{d\alpha}\left[\underbrace{\frac{\alpha^2}{2}}_{\alpha} \underbrace{-\alpha}_{-1}\right] = 0$$

$$\left(\frac{\alpha^2}{2}\right)' = \left(\frac{1}{2}\right)'\alpha^2 + \frac{1}{2}(\alpha^2)' =$$

$$= \frac{2\alpha}{2} = \alpha$$

2

$$\alpha - 1 = 0 \Rightarrow \underline{\underline{\alpha = 1}}$$

$$\langle \epsilon \rangle = \frac{1^2}{2} - 1 = -\frac{1}{2} = -0{,}5\,Ha$$