Structure Formation

Baryonic matter linear clustering

Plasma perturbations

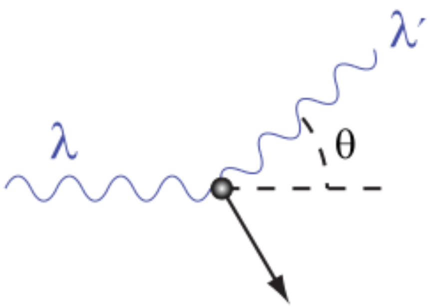
Before decoupling (z > 1100), the main components of the cosmological fluid are dark matter and the ionized **plasma of baryons+photons**.

(There are also other species that decouple earlier - like neutrinos)

In cosmology, **baryons** are free electrons and nuclear particles and all matter formed by them, i.e., atoms, molecules, stars, galaxies, clusters of galaxies, i.e., "normal matter" as opposed to dark matter.

In the early Universe, baryons are coupled to photons in a ionized plasma due to processes of interaction between radiation and matter, such as,

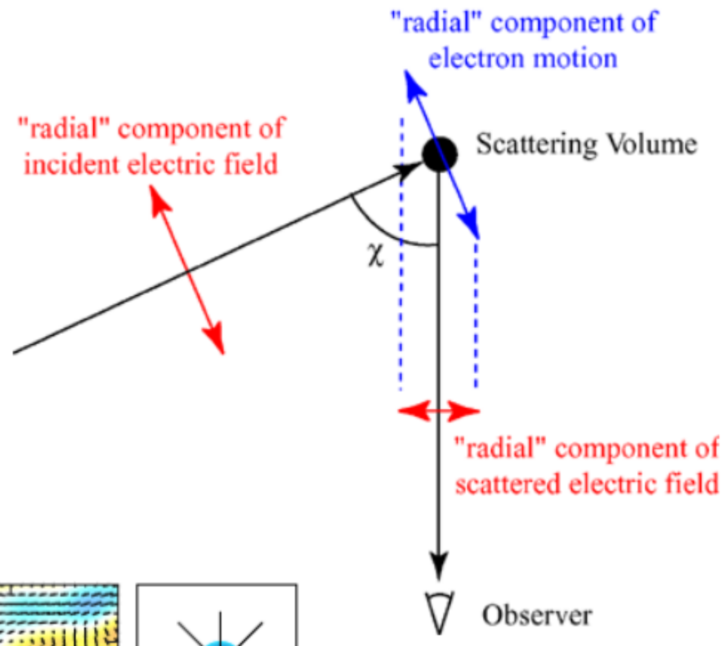
i) **Compton scattering**

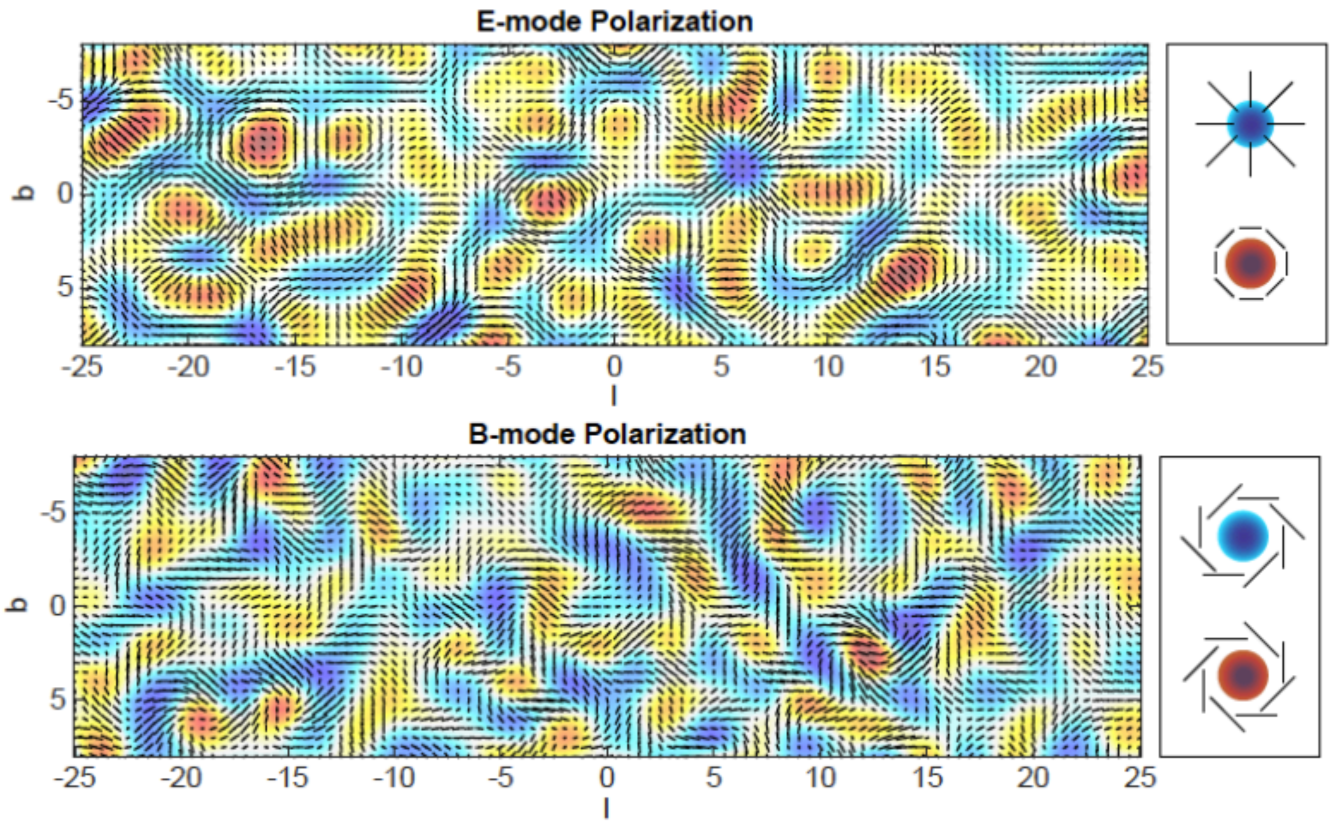


(the electron reemits a photon of lower energy)

ii) **Thomson scattering**

As the temperature decreases, the interaction starts to be dominated by Thomson scattering, (lower energy photons) where photons no longer lose energy but get polarized → this is the origin of the **CMB polarization**





We want to study the evolution of the density contrast of the "plasma perturbations",

$$\bar{\rho}_{pl}\delta_{pl} = \bar{\rho}_r\delta_r = \bar{\rho}_b\delta_b$$

in the various epochs:

- radiation dominated

- matter dominated - that is now divided in two sub-epochs:

  the **plasma epoch** ($z_{eq} > z > z_{dec}$)

  the **transparency epoch** ($z < z_{dec}$)

Unlike DM perturbations, in the case of plasma density perturbations we need to consider pressure in the fluid equations (radiation pressure, w=1/3).

Pressure affects the fluid equations in 2 different ways:

- **The mean pressure contributes to the mean energy density**
  this is more relevant in the radiation epoch

- **The pressure also have perturbations**
  this is more relevant in the early matter epoch (the plasma epoch)

Radiation-dominated epoch

In this epoch, **radiation pressure contributes to the mean energy density**

Let us see how this modifies the Newtonian fluid equations.

One of the conditions for the validity of the Newtonian approximation is not verified anymore: pressure is no longer negligible and must be accounted for as a source of gravity.

In Newtonian equations, pressure does not appear as a source of gravity. The correct calculation for energy conservation for relativistic particles in GR must be made with the Boltzmann equation.

The Euclidean approximation

However it is still possible to use an approximation, the **Euclidean approximation** → no space-time curvature, valid for sub-Hubble scales (like the Newtonian approximation) but where Special Relativity applies and we may consider mass-energy equivalence.

So this approximation corresponds to a “special relativistic Newtonian gravitational theory”, which is different from General Relativity.

Applying $\nabla_\mu T^\mu_\nu = 0.$ to a perfect fluid in a perturbed Minkowski metric (i.e. with a potential Φ), we get the special relativity continuity and Euler equations, which include density and pressure:

Continuity equation (it is now a conservation of total energy and not only mass conservation)

$$\frac{d\rho}{dt} + \nabla_r.(\rho\,\mathbf{u}) = 0 \quad \rightarrow \quad \frac{d(\rho + p)}{dt} - \frac{\partial p}{\partial t} + \nabla_r.(\rho + p)\,\mathbf{u} = 0$$

The change of energy density at one point is due not only to the flow of the fluid, but also to the pressure time-variation.

Like we did before, we need to develop the total time derivative (to account for the expansion, i.e., to introduce comoving coordinates) and insert the total velocity

$$u = \dot{r} = \dot{a}x + v$$

The comoving equation is then, (remember ρ+p = 4/3 ρ)

$$\frac{4}{3}\frac{\partial\rho}{\partial t} - \frac{\dot{a}}{a}\vec{x}.\vec{\nabla}_x(\frac{4}{3}\rho) = \frac{\partial p}{\partial t} - \frac{1}{a}\left[(\dot{a}\vec{x}+\vec{v}).\vec{\nabla}_x\frac{4}{3}\rho + \frac{4}{3}\rho(3\dot{a}+\vec{\nabla}_x.\vec{v})\right]$$

which simplifies to

$$\frac{\partial\rho}{\partial t} + 4\frac{\dot{a}}{a}\rho + \frac{4}{3a}\vec{\nabla}.(\rho\vec{v}) = 0$$

Inserting the density perturbation: $\rho = \bar{\rho} + \delta\bar{\rho}$

the equation becomes, as usually, the sum between the zero-order continuity equation (in the presence of pressure) plus first and second-order terms.

The first-order terms give the **linearized perturbed comoving continuity equation in the presence of pressure**:

$$\boxed{\frac{\partial\delta}{\partial t} + \frac{4}{3}\frac{1}{a}\nabla.\mathrm{v} = 0}$$

Euler equation

Now the source of 'force' is not only the potential but also the gradient of pressure, i.e., pressure perturbation.

$$\left(\frac{\partial u}{\partial t}\right)_r + (u.\nabla_r)u = -\nabla_r\Phi \quad \rightarrow \quad \left(\frac{\partial u}{\partial t}\right)_r + (u.\nabla_r)u = -\nabla_r\Phi - \frac{\nabla_r p}{\rho + p}$$

However, in the radiation epoch, the radiation energy density is high, and so is the mean pressure (1/3 of the energy density) → **pressure perturbations are negligible** compared to the mean pressure and the Euler force is dominated by the potential term → The Euclidean Euler equation remains identical to the Newtonian case.

So, the comoving linearized perturbed Euler equation remains the same as for dark matter:

$$\boxed{\frac{\partial v}{\partial t} + \frac{\dot{a}}{a}v = -\frac{1}{a}\nabla\Phi}$$

Poisson equation

Note that the "special relativistic Newtonian theory" is not a fully developed real working theory, it is mostly a set of arguments to modify the Newtonian approximation.

So in principle we do not know what its Poisson-like equation is.
(The reason is that Poisson equation is a gravity equation, while continuity and Euler are energy conservation equations).

We can start by considering the zero-order Euler equation in comoving coordinates, which involves the "mean gravitational potential":

$$\ddot{a}x = -\frac{1}{a}\nabla_x\bar{\Phi}$$

On the other hand, in the presence of pressure the second Friedmann equation is:

$$\frac{\ddot{a}}{a} + \frac{4}{3}\pi G(\bar{\rho} + 3p) = 0$$

Combining the 2 equations (with p = ρ/3), we get a Poisson-like equation:

$$\nabla^2\Phi = 4\pi\, G\, a^2\bar{\rho}\,\delta \quad \rightarrow \quad \boxed{\nabla^2\Phi = 8\pi\, G\, a^2\bar{\rho}\,\delta}$$

So, the 3 fluid equations for plasma perturbations (without pressure perturbations) are only slightly different than those of dark matter perturbations.

Combining the 3 equations as we did for dark matter, i.e., taking the time derivative of the continuity equation and combining with the divergence of the Euler equation (to cancel out the peculiar velocity terms),

we obtain **the evolution equation for the density contrast of the plasma**:

$$\ddot{\delta} + \frac{2\dot{a}}{a}\dot{\delta} = \frac{32\pi}{3} G\bar{\rho}\delta$$

instead of

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi\, G\,\bar{\rho}\,\delta = 0$$

To solve it we need to consider H(t) and ρ(t) in the radiation epoch.

From the Friedmann equation, dominated by radiation, we know that a (t) ~ $t^{1/2}$

and so H(t) ~ $(2t)^{-1}$

From the zeroth-order continuity equation, the mean density evolution is

$$\frac{\bar{\rho}_{pl}}{\rho_c(a=1)} = \Omega_r a^{-4}$$

Remember that the plasma is a coupled fluid of photons and baryons, and at this point, ρ_r or ρ_b are indistinguishable.

Nevertheless, we cannot write $\rho_{pl} = \Omega_b\ a^{-4}$, because after decoupling ρ_b does not continue to dilute with a^{-4}, and so the current value Ω_b cannot be used to parameterize the a^{-4} behavior.

Inserting the Hubble function and mean density time-dependences results in an **equidimensional equation**:

$$\ddot{\delta} + \frac{1}{t}\dot{\delta} - \frac{1}{t^2}\delta = 0$$

The power-law solution has index: $n^2 - n + n - 1 = 0$

and so the growing solution is

$$\delta \propto a^2$$

There is a **growth of the plasma perturbations:** the plasma (or baryonic) perturbations grow fast in the short radiation epoch ($z >> z_{eq} \sim 3500$).

Being coupled to the photons, baryonic matter is able to cluster faster than dark matter (during the brief period of the radiation epoch), because it is subject to the radiation pressure that increases the gravitational potential.

However, note that this result is valid deep in the radiation era in the approximation of zero pressure perturbations. The approximation breaks down as the radiation pressure dilutes, much before z_{eq},

Matter-dominated epoch: the plasma epoch

In the matter epoch, the dominating fluid is CDM, and the mean radiation pressure that has been diluting with a^{-4} gives now a negligible contribution to the total mean energy density.

However pressure still exist in the plasma, and perturbations around a small mean pressure are relatively more important and have an impact in the plasma behavior → we need to consider **pressure perturbations**, δp.

In the same way that the mean pressure may be related to the mean density (in the case of a **barotropic fluid**) defining an **equation-of-state** $w = p/\rho$,

the ratio of the (dimensional) pressure and density perturbations also defines an important property of the fluid: its **speed of sound c_s**:

$$c_s = \left(\frac{\delta_p \, \bar{p}}{\delta \, \bar{\rho}} \right)^{1/2}$$

Moreover, pressure and density perturbations are **thermodynamically related**:

as the universe expands and the density decreases, pressure would also decrease if the temperature was constant (pV = nRT), i.e., if the evolution was **isothermal.**

This is the case in astrophysics when the process has time to thermalize (the heat transfer is fast compared to the sound speed).

However, in cosmology, the temperature of the cosmological fluid decreases with the expansion and so the evolution is not isothermal.

A special case of a non-isothermal evolution is the **adiabatic evolution** (also called isentropic), where the temperature changes in a way that heat transfer compensates the entropy change → entropy is conserved.

In general, pressure perturbations can be separated in 2 parts: adiabatic and non-adiabatic:

$$\delta_p = \delta_{\rm pad} + \delta_{\rm pnad}$$

The adiabatic evolution has the following property:

$$\bar{p}(\rho) = p(\bar{\rho})$$

Taylor expanding p(ρ) we can write:

$$p(\rho) = p(\bar{\rho}) + \frac{\partial p}{\partial \rho}_{|\bar{\rho}} (\rho - \bar{\rho})$$

Inserting the definitions

$$\rho = \bar{\rho} + \delta\bar{\rho} \qquad\qquad p(\rho) = \bar{p} + \delta_p \bar{p}$$

in the Taylor expansion, it follows

$$\bar{p}(\rho) + \delta_p \bar{p} = p(\bar{\rho}) + \frac{\partial p}{\partial \rho}_{|\bar{\rho}} \delta\bar{\rho}$$

Inserting the adiabatic property, we find that

$$\left(\frac{\partial p}{\partial \rho}\right)_s^{1/2} = \left(\frac{\delta_p \bar{p}}{\delta \bar{\rho}}\right)^{1/2}$$

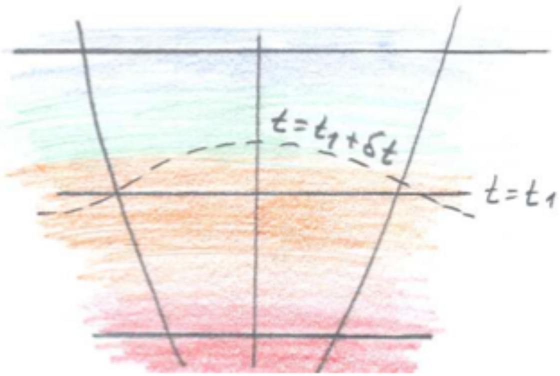
This means that the **adiabatic speed of sound** is given by

$$c_s = \left(\frac{\partial p}{\partial \rho}\right)_s^{1/2}$$

i.e., the speed of sound in an adiabatic fluid is given by a derivative of the background quantities, not requiring the information on the perturbations.

The **adiabatic property** can also be expressed in another way:

the values of ρ and p in a region x at a time t are identical to the mean quantities at a different time t+δt → some parts of the universe are "ahead" and others are "behind" in the evolution.



This implies that we can write:

$$\delta(t)\bar{\rho}(t) = \rho(t) - \bar{\rho}(t) = \bar{\rho}(t+dt) - \bar{\rho}(t) = \frac{d\bar{\rho}(t)}{dt}dt$$

i.e., **the density contrast (times the mean density) is equal to the derivative of the mean density.**

In an adiabatic evolution, similar relations can be written for all density and pressure components of the cosmological fluid.

So in a given delta time we have, for example:

$$\delta t = \frac{\delta \rho_m}{\dot{\bar{\rho}}_m} = \frac{\delta \rho_r}{\dot{\bar{\rho}}_r}$$ and also $$\frac{\delta p}{\delta \rho} = \frac{\dot{\bar{p}}}{\dot{\bar{\rho}}}$$ (the same result we found with the Taylor expansion)

This property allows us to find **the relative initial condition amplitudes between the components**.

Indeed, from the **continuity equation** for the background, we have:

$d\langle\rho_m\rangle / dt = -3 H \langle\rho_m\rangle$ and $d\langle\rho_r\rangle / dt = -4 H \langle\rho_r\rangle$

Inserting in the "time equation", we get: $\delta\rho_m\, 4 \langle\rho_r\rangle = \delta\rho_r\, 3 \langle\rho_m\rangle \rightarrow \quad \delta_r = \frac{4}{3}\delta_m$

meaning that clustering of radiation (and baryonic matter) is slightly larger than that of dark matter.

So the adiabatic condition allows us to relate perturbations with mean quantities, and relate the perturbations of various species.

The **thermodynamic type of the perturbations** is another property of the universe.

Inflation predicts that the primordial fluctuations are adiabatic → this provides an initial condition for the amplitude of the various species of perturbations.

Other types of perturbations could be possible, but are ruled out by observations:

For example in isocurvature perturbations the evolution is such that the total $\delta\rho$ is conserved → **the growth of one species implies the decrease of another** → not compatible with observations.

After this digression on pressure perturbations and speed of sound, let us return to the evolution of plasma perturbations in the early matter-dominated epoch.

For sub-Hubble scales we can use the Newtonian approximation (since there is no pressure contributing to the mean energy density). The equations are:

→ Continuity and Poisson equations: remain the same as in DM

→ Euler equation: is modified because the inhomogeneous pressure field introduces an extra force in the equation of motion of the fluid:

$$\left(\frac{\partial u}{\partial t}\right)_r + (u.\nabla_r)u = -\nabla_r \Phi - \frac{\nabla_r p}{\rho + p}$$

Inserting the comoving cordinates and keeping only the linear perturbed terms, the Euler equation is like the dark matter one with an additional term:

$$\frac{\partial v}{\partial t} + \frac{\dot{a}}{a} v = -\frac{1}{a} \nabla \Phi - \frac{1}{a} \frac{\nabla \bar{p} \delta_p}{\bar{\rho}}$$

The additional term depends on a ratio that we now see is the **speed of sound** in the plasma multiplied by the density contrast

$$c_s^2 = \frac{\delta_p \, \bar{p}}{\delta \, \bar{\rho}}$$

Inserting the speed sound in the Euler equation, we can write:

$$\frac{\partial v}{\partial t} + \frac{\dot{a}}{a} v = -\frac{1}{a} \nabla (\Phi + c_s^2 \delta)$$

The Euler equation contains now a trade-off between 2 effects: attractive gravity (the potential, which is determined by the density contrast through Poisson equation) and repulsive pressure (the sound velocity), that works against the clustering.

In addition there is the usual Hubble drag term due to the expanding background.

Note that physically, this Euler equation is equivalent to the Jeans equation that describes the instability of a cloud of particles (gas) of density n as a mechanism for **star formation**:

$$\frac{\partial(n\langle v_j\rangle)}{\partial t} + n\frac{\partial\Phi}{\partial x_j} + \sum_i \frac{\partial(n\langle v_i v_j\rangle)}{\partial x_i} = 0$$

In the case of a cloud of particles, the repulsive term is usually given by the dispersion velocity of the collection of particles, while in the case of a fluid the repulsive term is given in terms of pressure gradients (the fluid sound speed). The other terms of the original Jeans equation are the acceleration of the particles and the gravitational attraction, just like in Euler equation.

Now, going to Fourier space, the spatial derivative is replaced by -ik, and this **introduces an explicit scale-dependence in the Euler equation**.

Combining the 3 comoving, linear and first order equations (continuity, Euler, Poisson) in the usual way, and written in Fourier space, we get the new equation for the evolution of δ_{pl} in the plasma epoch:

$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\,\dot{\delta}_k - \left(4\pi G\bar{\rho} - \frac{c_s^2\,k^2}{a^2}\right)\,\delta_k = 0$$

This equation is also known as the **Jeans equation.**

Its solutions for δ will be scale-dependent (due to the new k dependence)**:**

the equation defines a **threshold scale**, the **Jeans scale**, that separates **two regimes (gravity vs pressure dominated):**

$$k_J = \sqrt{4\pi G\bar{\rho}}\,\frac{a}{c_s}$$ (the comoving Jeans scale)

It is also usual to define its reciprocal → the **Jeans length**:

$$\lambda_J(a) = \frac{2\pi}{k_J(a)}\,a = \frac{c_s(a)\sqrt{\pi}}{\sqrt{G\bar{\rho}(a)}}$$ (the proper [non-comoving] Jeans length)

This means the Jeans equation has two types of solutions:

i) on **super-Jeans scales** ($k < k_J$) (i.e., the larger scales - but still sub-Hubble)

gravity dominates → the third term of the equation is negative.

This guarantees there is a **growing solution**:

- In the limiting (unrealistic) case of no expansion (no Hubble drag) → the growth would be exponential.

- In the limit of very large sub-Hubble scales $k << k_J$
(or also in the limit of $c_s = 0$ - which only happens after z_{dec} -)
→ the pressure term is zero and the equation is identical to the dark matter one, where we saw the solution is $\delta(a) \sim a$, or more precisely,

$$\delta(a) \sim a^f$$

(remember the growth parameter f accounts for different dark energy models)

ii) on **sub-Jeans scales** ($k > k_J$) (i.e., the smaller scales)

pressure dominates → the third term of the equation is positive.

This guarantees there is an **oscillating solution**. This means the clustering increases and decreases, oscillating in time, there is no net growth of the clustering → it is equivalent to a **frozen period** of no growth.

- In the limiting (unrealistic) case of no expansion (no Hubble drag) → the growth would be sinusoidal:

$$\delta = \sin\left(\sqrt{\frac{k^2\, c_s^2}{a^2} - 4\pi\, G\bar{\rho}}\, t\right)$$

- With background expansion, the Hubble drag term acts as friction → the solution is a **damped oscillation:**

$$\delta = \frac{A}{k^2\, c_s^2}\left[1 - \frac{\sin(kc_s\eta)}{kc_S\eta}\right]$$

(analytical solution written in terms of conformal time η)

Notice that:

- the frequency in the solution is proportional to k → in **smaller scales the growth oscillates with larger frequencies.**

- the amplitude in the solution is inversely proportional to k → **larger scales reach higher clustering amplitudes** in their oscillating movement**.**

- the fact that the amplitudes of the perturbations oscillate in time in their spatial locations, creates a **propagation of the plasma overdensities in space, that propagates with the plasma sound velocity and creates spatial correlations in the plasma density field.**

- the scales oscillate in phase → at any given time, the clustering of any scale is slightly larger (or smaller) than the clustering of its neighboring scale → **the power spectrum of the plasma density will have an oscillatory shape.**

We have encountered a second period where clustering is frozen:

For DM a freezing period occurs while scales are smaller than the Hubble radius during the (shorter) radiation epoch.

For plasma (baryons) the freezing (with damped oscillations) period occurs while scales are smaller than the Jeans scale during the (longer) matter-plasma epoch.

Evolution of the Jeans' scale

Like the Hubble radius, also the Jeans scale evolves with time. Its evolution depends on the evolutions of the mean density, scale factor, and **sound speed**.

$$k_J = \sqrt{4\pi G\bar{\rho}}\,\frac{a}{c_s}$$

In standard cosmologies, the expansion is adiabatic (even though other cases exist in alternative models), where the speed of sound is given by

$$c_s^2 = \frac{\partial p}{\partial \rho}_{|\bar{\rho}}$$

So, since w=1/3 → p = 1/3 ρ_r c^2, and $\rho = \rho_b + \rho_r$, the sound speed in the plasma is

$$\frac{\partial P}{\partial \rho}|_S = \frac{c^2}{3}\frac{\partial \rho_\gamma}{\partial(\rho_\gamma + \rho_B)}|_S = \frac{c^2}{3}\left(1 + \frac{\partial \rho_B}{\partial \rho_\gamma}|_S\right)^{-1}$$

Using the (homogeneous) continuity equation for a fluid with 2 components, we can get an expression for the derivative of ρ_b with respect to ρ_r :

$$\frac{\partial \rho_B}{\partial \rho_\gamma}|_S = \frac{3}{4}\frac{\rho_B}{\rho_\gamma}$$

and the speed of sound is then

$$c_s(z) = \frac{c}{\sqrt{3}} \left(1 + \frac{3\Omega_b(z)}{4\Omega_r(z)}\right)^{-1/2}$$

It is usual to introduce the **baryonic fraction** $f_b = \Omega_b / \Omega_m$ ~0.17

Noting that $\Omega_b(a)/\Omega_r(a) = a\, \Omega_b/\Omega_r = a\, f_b\, \Omega_m/\Omega_r = f_b\ a/a_{eq}$, the speed of sound becomes,

$$c_s(a) = \frac{c}{\sqrt{3}} \left(1 + \frac{3\, f_b}{4} \frac{a}{a_{\rm eq}}\right)^{-1/2}$$

From this expression, we can see that **the sound speed decreases as the Universe expands** (this comes from the fact that the radiation density decreases faster than the matter density):

- **$z > z_{eq}$** → $\rho_r >> \rho_m$ → $c_s = c/\text{sqrt}(3)$ → c_s is roughly constant at its maximum value.

- **$z = z_{eq}$** → $c_s = c/\text{sqrt}(3.3)$

- **$z = z_{dec}$** → $c_s \sim c/3$ (using $a_{dec} \sim 3\, a_{eq}$)

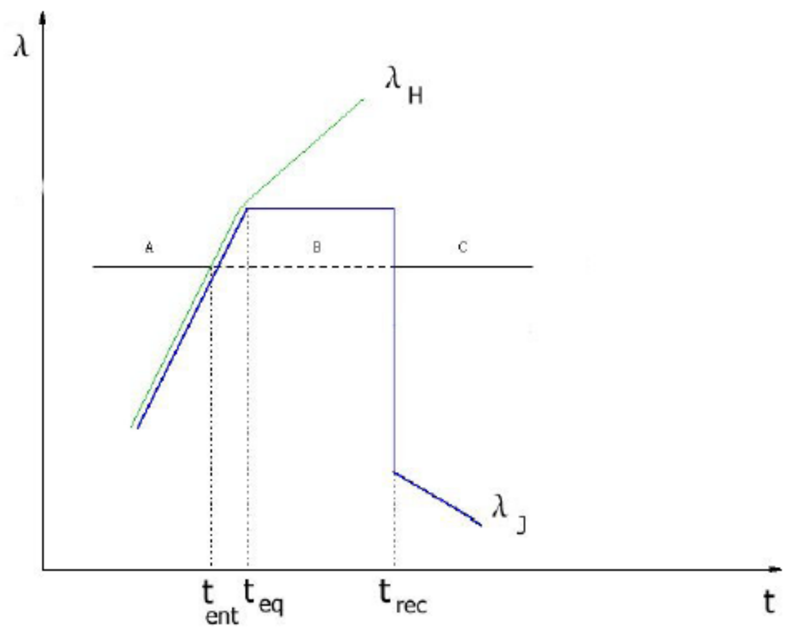
- **$z < z_{dec}$** → $c_s = 0$ (pressure drops to zero)

Now, the **comoving Jeans length** is $r_J \sim c_s / (a \, \mathrm{sqrt}(\rho))$

- **radiation epoch** → $c_s = c/\mathrm{sqrt}(3)$, $\rho \sim a^{-4}$ → $r_J \sim a$ → r_J increases

- **plasma epoch** → $c_s \sim c/3$, $\rho \sim a^{-3}$ → $r_J \sim c_s a^{1/2}$, c_s decreases a factor of 1.6, $a^{1/2}$ increases a factor of 3 → r_J constant

- **transparency epoch** → $c_s = 0$ → r_J has a sudden drop to zero at $z = z_{dec}$

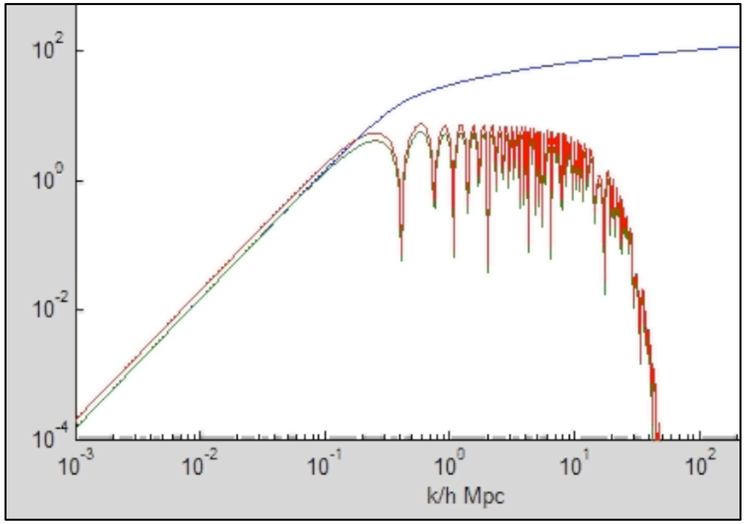


(evolution of Jeans and the Hubble lengths)

A → during this shorter period **baryon perturbations grow, while dark matter perturbations** $r > r_H$ also grow but DM with **$r < r_H$ does not grow** (this is responsible for the DM transfer function) → δ_b may be larger than δ_{dm} during this short period.

B → During this longer period all **DM perturbations grow, while most of the baryonic ones do not grow** → this is responsible for a **strong suppression of growth in baryonic matter (due the coupling with radiation → existence of pressure perturbations → speed of sound → Jeans equation).**

The resulting (dimensionless) **baryonic matter power spectrum** after decoupling from the radiation looks like (green line):



It has a **lower amplitude** than the dark matter power spectrum (blue line) and strong oscillations on **intermediate scales**.

On **small scales** there are no overdensities due to yet another **free-streaming** effect. This time due to the fact that the last scattering surface has a finite thickness, i.e., **decoupling is not instantaneous → this is known as the Silk damping or diffusion damping.**

Silk damping

During this delta time, the plasma is "**partially coupled**" → photons start to stream out of the overdensities and drag baryons with them → **destroying the overdensity**.

On large perturbations, the photon crossing time is larger than the "thickness of the last-scattering surface" (i.e. the duration of the decoupling process) → they stream out of the perturbation already fully decoupled → no baryon dragging → **the overdensity remains in place.**

The resulting effect is an exponential damping of the power spectrum amplitude by a factor:

$$e^{-\ell^2/\ell_D^2}$$ (the effect is larger for large l - small scales)

here given in terms of angular scales l : $l(z) = k\, f_K(z) = k\, d_{A,com}(z)$

l_D is known as the diffusion angular scale $\quad \ell_D \sim k_D d_A^c(t_{\rm dec})$

which is computed as $$k_D^{-1} \sim \frac{1}{\rm few}\frac{a_0}{a}\sqrt{\frac{\lambda_\gamma(t_{\rm dec})}{H_{\rm dec}}}$$

from the free-streaming length Λ_γ is the size that the photons with v=c can travel during the thickness of the last scattering surface

The result is $l_D \sim 1500$ (concordance model), which corresponds to

k_D (z = 1100) ~ 100 h M_{pc}^{-1}

The plasma perturbations (i.e. the existence of **baryonic acoustic oscillations**) that we just described are a crucial ingredient in the computation of **three cosmological quantities**:

- (Total) Matter power spectrum

- BAO peak in the matter correlation function

- Anisotropies of the CMB

Total matter power spectrum: the BAO wiggles

Matter-dominated epoch: the transparency epoch

Once the plasma is dissolved, baryons become pressureless matter like dark matter (as far as gravity is concerned, there is of course gas pressure once structures are formed, but that is not relevant for cosmological structure formation, only to the process of galaxy formation).

Its subsequent evolution is the same we found for DM, i.e, $\delta_b (a) \sim a$.

However, the baryonic density field has a lower clustering amplitude since the baryon growth was suppressed for a long time (Jeans equation) → **δ_b is smaller than δ_{cdm} at z_{dec}**

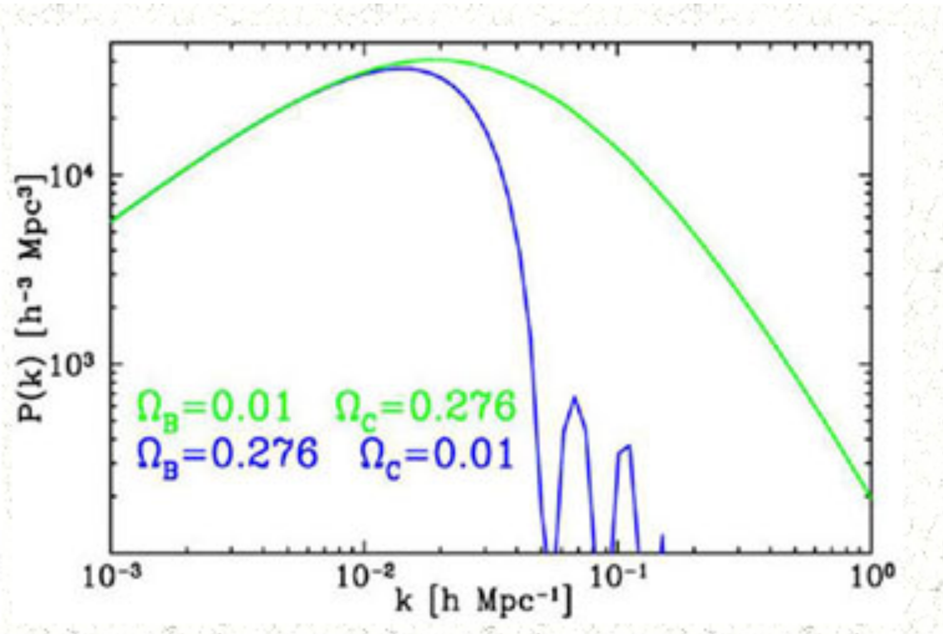
The value of δ_b at z_{dec} can be found by measuring the CMB anisotropies (as we will see later).

These are $\delta T \sim 10^{-5}$ → the baryonic inhomogeneities are $\delta_b (z_{dec}) = 10^{-5}$

This means that from 1+z ~1100 until 1+z = 1 (today), δ_b can grow a factor of 1000, reaching δ_b (z=0) ~10^{-2}

This does not agree with the observations → **there would exist no baryonic structures in the Universe**
(for example no astrophysical galaxies formed, only dark matter galactic halos).

Moreover, if baryons would be the dominating matter species, the matter power spectrum would look very different than the one observed. For example:



Notice that any species that introduces a sound speed in the cosmological fluid would create oscillations in the overdensities.

This also happens in some dark energy models. The speed of sound needs to be kept to a very small value for those models to be viable.

These two arguments:

amplitude and shape of the matter power spectrum

led to the conclusion that structure formation needs to be driven by a fluid that does not couple with photons, and is not subject to oscillations for such a long time → **This was the reason to introduce dark matter in the cosmological model.**

There is also another reason to introduce dark matter; to explain observed rotation curves of galaxies.

These two arguments are different and deal with completely different scales. In principle, dark matter does not need to be the same in both cases.

How does the presence of dark matter help in increasing baryonic clustering and in limiting the oscillations?

After recombination, the matter perturbations evolve in the matter dominated background ($a \sim t^{2/3}$), with the system of equations:

$$\ddot{\delta}_{dm} + \frac{2\dot{a}}{a}\dot{\delta}_{dm} = 4\pi G(\rho_{dm}\delta_{dm} + \rho_b\delta_b)$$

$$\ddot{\delta}_b + \frac{2\dot{a}}{a}\dot{\delta}_b = 4\pi G(\rho_{dm}\delta_{dm} + \rho_b\delta_b)$$

i) Since $\rho_b\ \delta_{b,} << \rho_{dm}\ \delta_{dm}$ let us neglect it in the third term of the equations:

dark matter: with $\rho_{dm}\ \delta_{dm} >> \rho_b\ \delta_b$, the equation for dark matter reduces to the usual one, with solution $\delta_{dm} \sim a$

baryonic matter: inserting the solution $\delta_{dm} = C\ a$ (where C is a constant) and neglecting $\rho_b\ \delta_b$ in the third term, the equation becomes,

$$\ddot{\delta}_b + 2\frac{\dot{a}}{a}\dot{\delta}_b - \frac{3}{2}\frac{H_0^2\Omega_m}{a^3}C\,a = 0$$

Changing the variable from t to “a” and with $a \sim t^{2/3}$, the equation becomes

$$\frac{d}{dt} = \dot{a}\frac{d}{da} = H_0 a^{-1/2}\frac{d}{da}$$

$$\delta_b'' + \frac{3}{2a}\delta_b' = \frac{3C}{2a}$$

(where the derivatives are taken with respect to the scale factor)

and it follows that δ_b = C a is a solution, i.e. the baryons not only start to cluster with the same rate as dark matter, but the solution also has the same amplitude C.

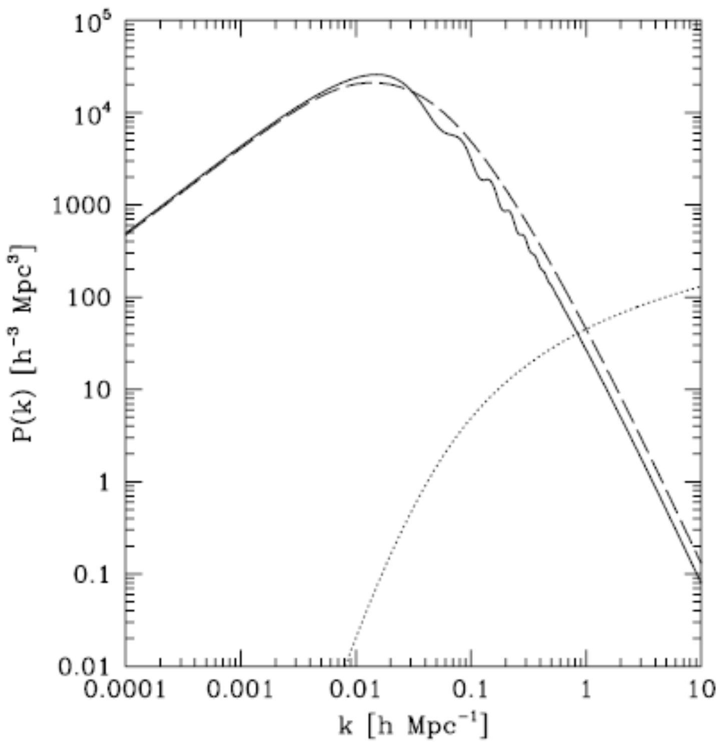
→ **This means that baryonic density contrast reaches the same amplitude as DM** soon after z_{dec}, i.e, **“baryons fall into the potential wells of dark matter”,** getting a great increase in amplitude → **it is the presence of dark matter that enables the formation of baryonic structures.**

Afterwards, δ_b continues to grow inside the DM potential wells (the dark matter halos) with $\delta_b \sim a$. Their gravitational properties are now exactly the same as the ones of dark matter (dust) since they are no longer coupled to photons.

When $\delta_b \sim 1$, the baryonic perturbations are dense enough for the fluid to be in the form of gas → it starts to have important non-gravitational interactions. Pressure starts to be important again (not relativistic radiation pressure, but 'astrophysical pressure') → the process of galaxy formation starts, resulting in the formation of baryonic structures in the dark matter halos

ii) If we keep the term δ_b in the coupled equations (which contains the oscillating solution) a more precise solution is obtained:

the **total matter power spectrum** is a combination of the growth solution $\delta \sim a$ with the small amplitude oscillations of δ_b → **the small baryon oscillations become imprinted in the total matter power spectrum:**



(total matter power spectrum: solid
dark mater power spectrum: dashed)

The "relic" **baryon acoustic oscillations (BAO)** from the plasma epoch are still noticeable in the matter power spectrum at z=0, on intermediate scales.

On small scales there are no oscillations due to Silk damping.

The wiggles become less prominent in the matter power spectrum as time goes by (since the amplitude of the power spectrum increased by a factor $(1000)^2$ from the time it got the oscillations until today).

Interestingly, we can still use the same **fitting transfer function** use for the **dark matter power spectrum** as a fit to the **total matter power spectrum**,

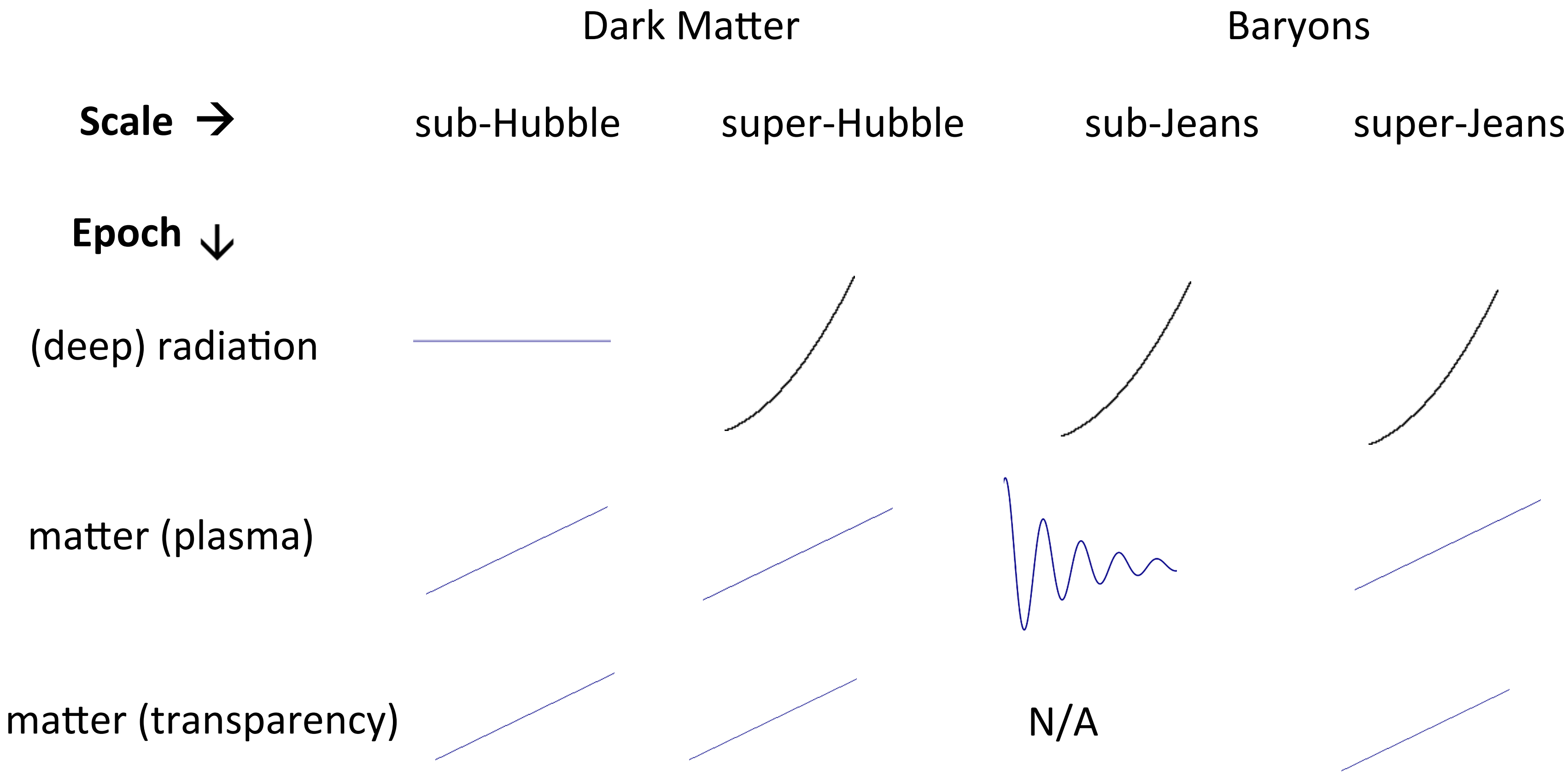
$$T(k) = \frac{\ln 1 + 2.34q}{2.34q} \left[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\right]^{-1/4}$$

Indeed the general shape or the total matter power spectrum is the same, the only difference are the overlaid oscillations.

The only change required in the transfer function is to include a dependence on Ω_b in the scale, that now is not k but q given by:

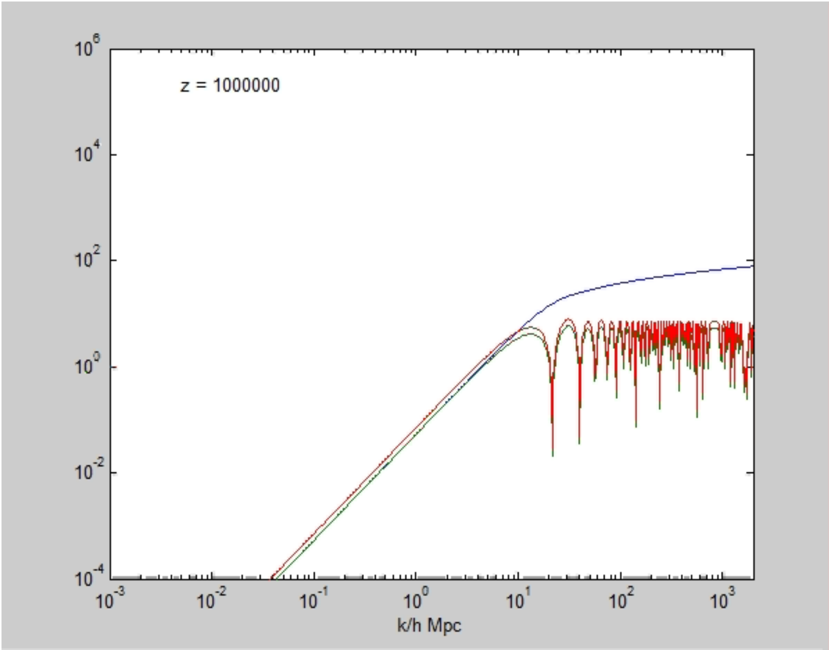
$$q = \frac{k}{\Omega h^2} e^{2\Omega_b}$$

Summary:
evolution of linear dark and baryonic matter perturbations



$z > z_{eq}$ (radiation epoch)

The animation seems to move to the left: Dark matter perturbations **grow** until caught by the expanding Hubble radius and "**freeze**".
Plasma perturbations start to **oscillate** (the zero δ_p approximation is already not valid).



z_eq > z > z_dec: (plasma matter epoch)

All dark matter scales **grow** $\delta_{DM}(a) \sim a$, while the plasma density contrast continues to **oscillate**.

z < z_dec: (transparency matter epoch)

Photons and baryons decouple at z_{dec} , photon perturbations wash out, while baryons fall into the DM potential wells.
Dark matter perturbations continue to grow, but the rate decreases as dark energy becomes important.

BAO peak

As we saw, the scale-dependent amplitude and the time frequency of the oscillating clustering, creates a power spectrum with oscillations that shows a characteristic wavelength.

This implies that the Fourier transform of the power spectrum (the correlation function) will contain one single peak, at spatial separation given by that wavelength.

What is the physical meaning of that peak ?

Let us go back to the plasma epoch. During that epoch, perturbations of different scales oscillate in phase at each spatial location. **The oscillation of a density field at a spatial location produces a density wave that propagates through the plasma with the plasma sound speed.**

This means that **secondary overdensities** travel through the plasma from every location where there is an overdensity. At each instant, the secondary overdensity is at a distance D(t) from the primary overdensity.

At decoupling, the plasma dissolves, the speed of sound goes to zero, and the wave stops. **The traveling overdensity then stops at the maximum distance that the wave could travel from $t=0$ to $t=t_{dec}$.**

During the **transparency epoch**, the overdensity remains frozen on that scale, as the density contrast continues to evolve linearly with 'a' for all scales.

That overdensity is still imprinted today in the statistical cosmological functions on real space → it appears as a **single peak** in the matter correlation function → the **BAO peak.**

The location of this peak in the correlation function is the maximum distance the sound wave travelled up to decoupling time (z=1100) → it is the (comoving) sound horizon at decoupling.

This is another **characteristic scale of the universe**.

Its value is given by the **sound comoving distance** from a=0 to a_{dec}

Using the sound speed, instead of the light speed, this distance is given by

$$r_s(z_*) = \int_0^{a_*} \frac{c_s(a')}{a'^2 H(a')} da'$$

where c_s (a) is a decreasing function from c/sqrt(3) to c/3.

The integration limit **z_*** is a more precise version of z_{dec} = 1100, where the redshift is computed using a fitting function (function of the cosmological parameters), that accounts for the details of the recombination process:

$$z_* = 1048\left[1 + 0.00124(\Omega_b h^2)^{-0.738}\right]\left[1 + g_1(\Omega_m h^2)^{g_2}\right]$$

with $$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}} \qquad g_2 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}}$$

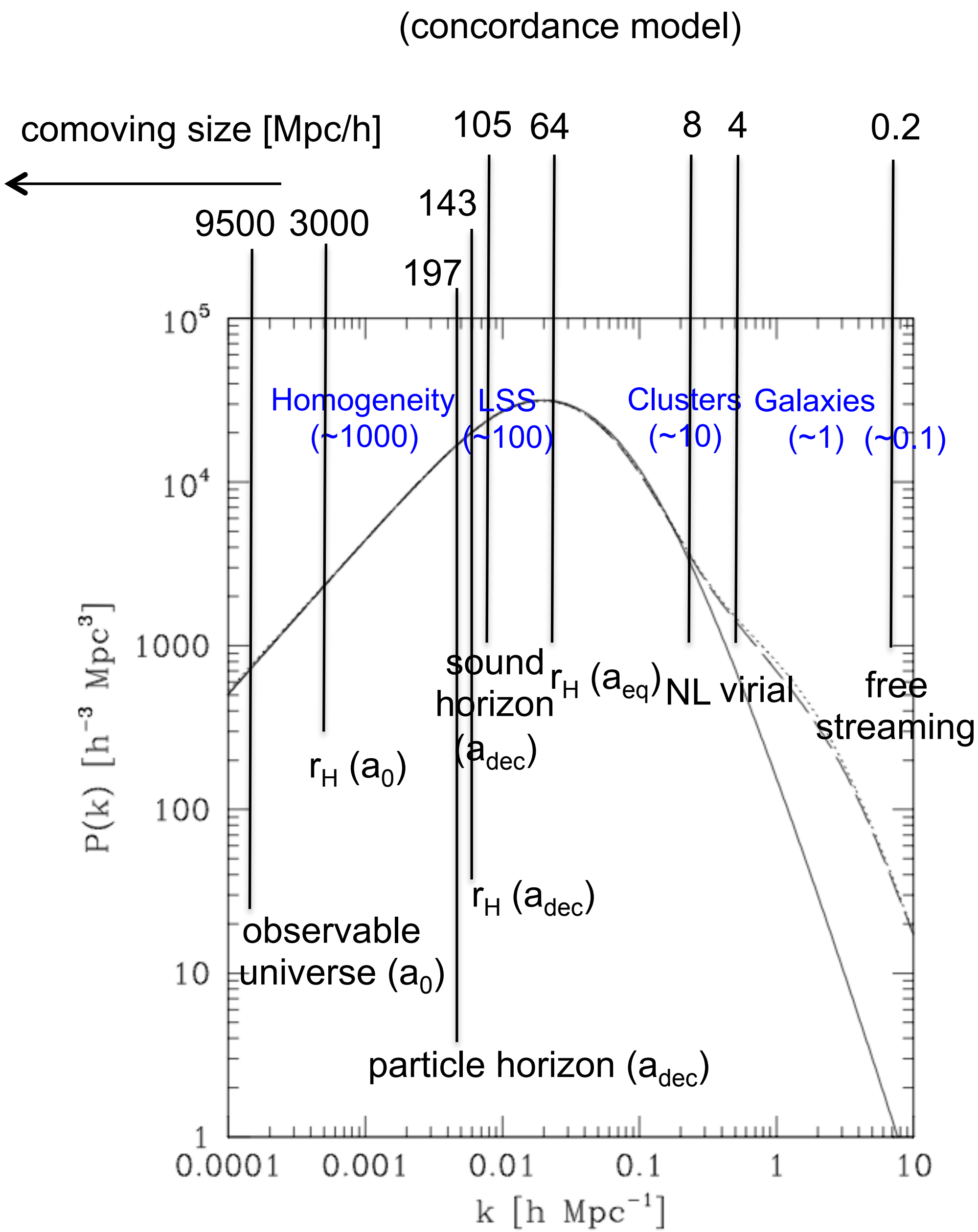
The resulting value for the **comoving sound horizon** is then:

r_s (z_*) = 105 Mpc/h = 150 Mpc (concordance model)

(the proper size of the sound horizon at z_{dec} is a factor of 1000 smaller ~ 0.1 Mpc/h)

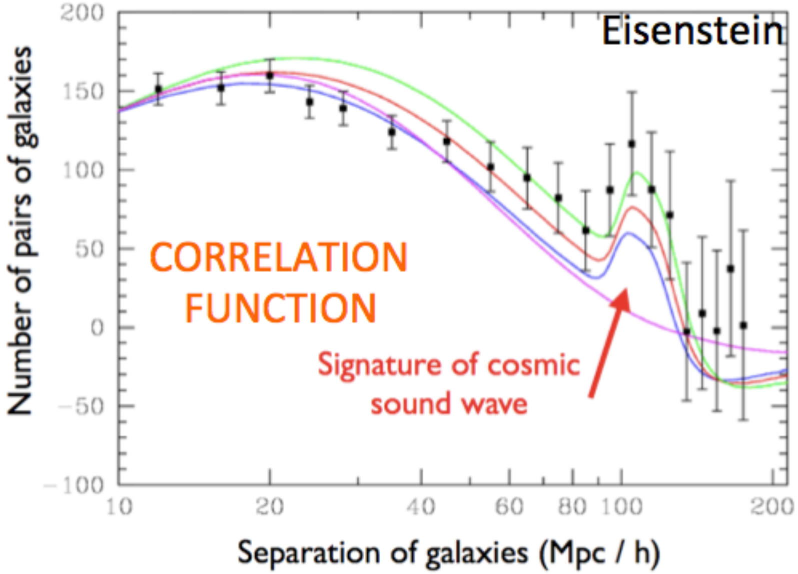
Notice that the sound horizon is smaller than the particle horizon, since the speed of sound is smaller than the speed of light:

r_c (z_*) = 197 Mpc/h = 281 Mpc (concordance model)

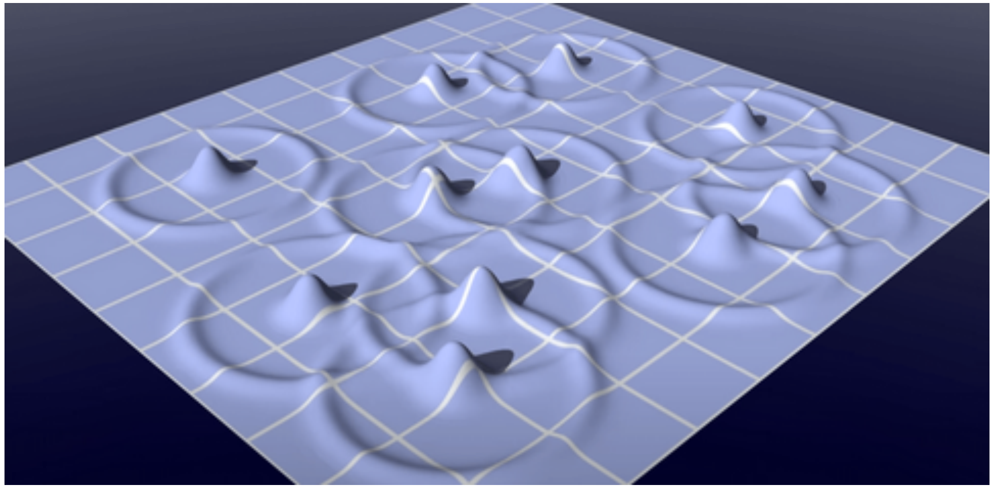


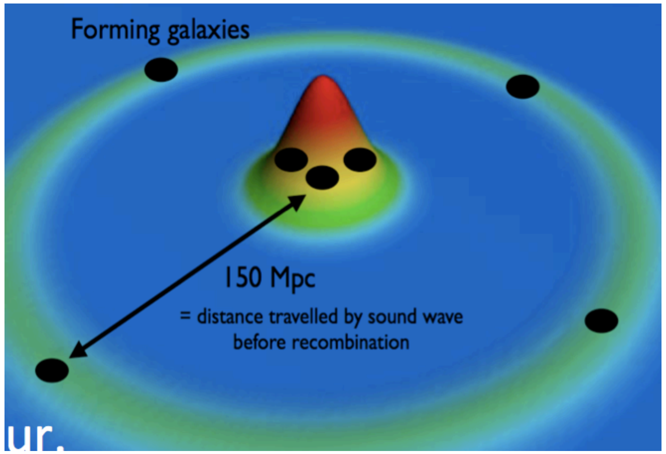
(concordance model)
comoving size [Mpc/h]
9500
3000
197
143
105
64
8
4
0.2
Homogeneity
(~1000)
LSS
(~100)
Clusters
(~10)
Galaxies
(~1)
(~0.1)
sound horizon (a_{dec})
r_H (a_{eq})
NL
virial
free streaming
r_H (a_0)
r_H (a_{dec})
observable universe (a_0)
particle horizon (a_{dec})
10^5
10^4
1000
100
10
1
0.0001
0.001
0.01
0.1
1
10
P(k) [h^{-3} Mpc^3]
k [h Mpc^{-1}]

So a secondary peak in the correlation function appears at this separation:
(this peak was detected for the first time in 2005)

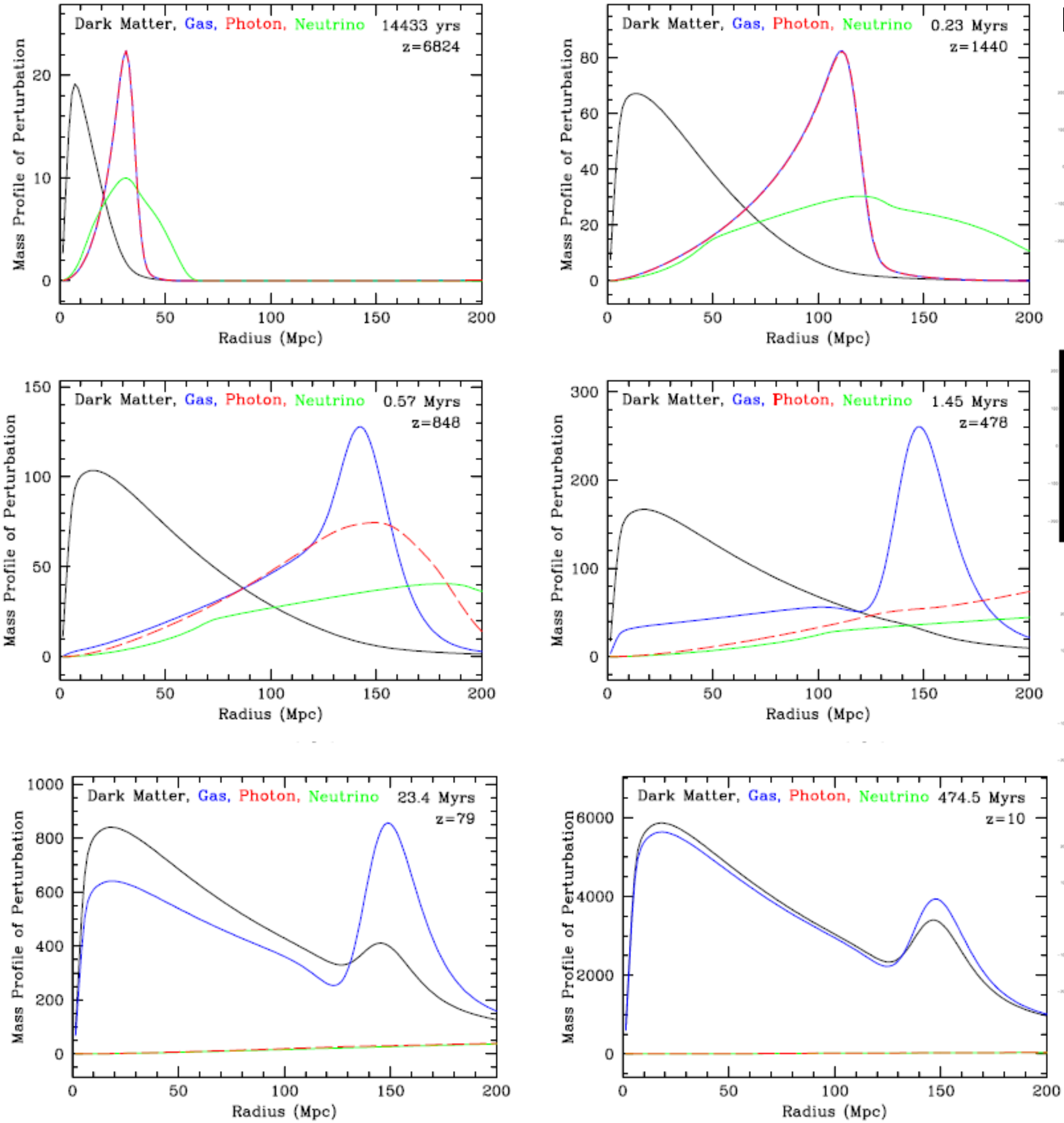


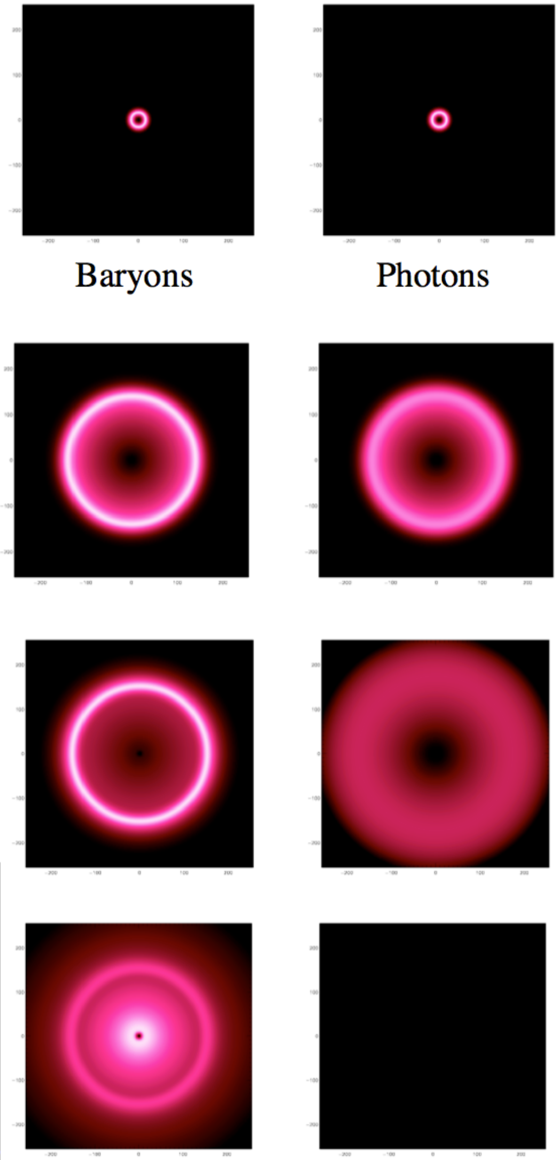
In other words, **on average there is a secondary overdensity at a separation of 150 Mpc from every primary overdensity:**





Formation of the BAO peak





CMB anisotropies

At the end of the **decoupling process**, baryons and radiation fully decouple, i.e.

the plasma is dissolved, the photons no longer scatter from the baryons

and are free to propagate → forming a radiation background that propagates through all the Universe - known as the **cosmic microwave background radiation** - **CMB**.

Notice that during the plasma epoch, there are **photon perturbations** (which are identical to the baryon perturbations → there are overdensities of photons → they can be clustered, forming structures of light! (immediately absorbed and reemitted or scattered).

Those should look like baryonic structures with light (since both are coupled) but the light is not emitted by the baryons (which are not dense enough to form stars), but it is rather made of primordial photons.

Of course, this speculative picture cannot be observed, since the photons cannot travel far in the plasma.

After decoupling, the photons move and all ‘structures of light’ are destroyed. They become smoothly distributed → **the current photon spatial distribution is homogeneous**. Today there are no photon perturbations.

However, the homogeneous distribution of primordial photons is not isotropic. It is anisotropic because the energy distribution (the temperature) of the primordial photons released at z_{dec} is determined by the conditions at their emission from the last scattering surface which is inhomogeneous → it contains information of the plasma inhomogeneities at z_{dec}.

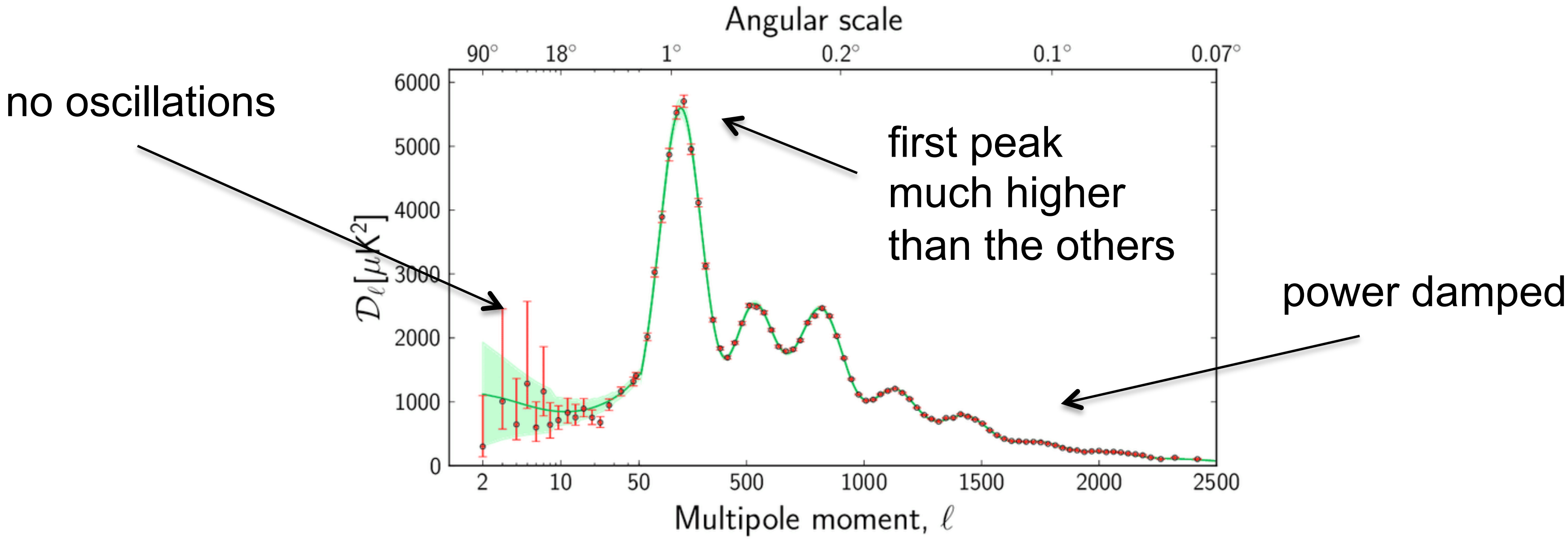
So today we detect a homogeneous distribution of photons, but each direction of detection comes from a specific point of the last scattering surface at z_{dec} and its frequency (temperature) is mostly determined by the inhomogeneities on that point.

The anisotropies are very small, their standard deviation is **10^{-5}** around the mean value of **2.7K** (which is in the **microwave band**, hence the name). Given the redshift produced by the expansion, the frequency at emission was **~10 μm** (in the infrared).

These were the only photons existing in the universe until the process of reionization started (at z_{re} ~ 20) when the first collapsed baryonic structures formed in the dark matter halos and started emitting new photons.

The CMB power spectrum: primary temperature anisotropies

The CMB power spectrum is the angular power spectrum (angular because the field is defined on a single redshift) of the temperature contrast of the field of photons temperature.



There are several contributions to the temperature contrast:

$$\left(\frac{\delta T}{T}\right)_{\text{obs}} = \frac{1}{4}\delta_\gamma^N - v^N \cdot \hat{n} + \Phi(t_{\text{dec}}, x_{\text{ls}}) + 2\int \dot{\Phi}\, dt$$

Note: the formal derivation requires the Einstein eqs. for the radiation relativistic component (no Newtonian approx.,no perfect fluid), and the Boltzmann eq. for energy conservation.

Density contrast

$$\left(\frac{\delta T}{T}\right)_{\rm obs} = \frac{1}{4}\delta_\gamma^N - v^N \cdot \hat{n} + \Phi(t_{\rm dec}, x_{\rm ls}) + 2\int \dot{\Phi} dt$$

The term δ_γ is the most important contribution.

Density contrast in sub-Jeans scales : acoustic oscillations

This is what we saw already, the clustering produced by the Jeans equation:

$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\,\dot{\delta}_k - \left(4\pi G\bar{\rho} - \frac{c_s^2\,k^2}{a^2}\right)\,\delta_k = 0$$

There is a slight difference in amplitude to be considered, since the density of radiation is not exactly equal to the density of the plasma, but they are related by:

$$\delta\_\text{plasma} = \frac{1+R}{1+\frac{4}{3}R}\delta_\gamma \qquad \text{with} \qquad R \equiv \frac{3}{4}\frac{\bar{\rho}_b}{\bar{\rho}_\gamma}$$

which explains the factor 1/4

Furthermore, the Jeans equation needs to be slightly changed since the photons are relativistic → the gravity term is no longer $4\pi G\rho$ but $4\pi G(1+w)\rho$, since pressure cannot be neglected for the radiation part of the plasma.

Of course, all this is an approximation, in reality the temperature fluctuations must be computed using the full formalism of the Einstein-Boltzmann equations.

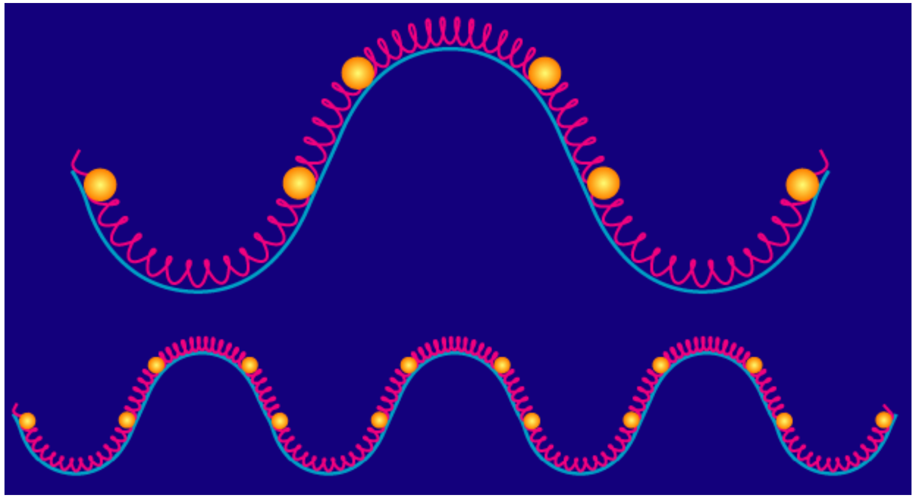
But anyway, the behavior found from Jeans equation is valid. It is the oscillatory pattern we saw for baryons.

We can interpret Jeans equation as follows (referring to the sketch on next page):

- The **gravity term** represents the potential (dominated by **dark matter potential wells**).

- The **pressure term** represents the movement of the coupled **baryon-photon plasma** (spheres with strings).

This forms the following picture:



springs - photons

spheres - baryons

potential wells - dark matter

The plasma clustering grows up to a maximum δ (compression) and then starts to decrease.

Larger-scale perturbations reach larger amplitude of clustering and have lower time-frequency.

At z_{dec} each photon is released from its position, which has a certain density (i.e. from a position in a potential well).

All photons coming from perturbations of equal scale have the same energy (temperature or frequency), because the oscillations are in phase → the oscillation pattern survives in the released CMB power spectrum.

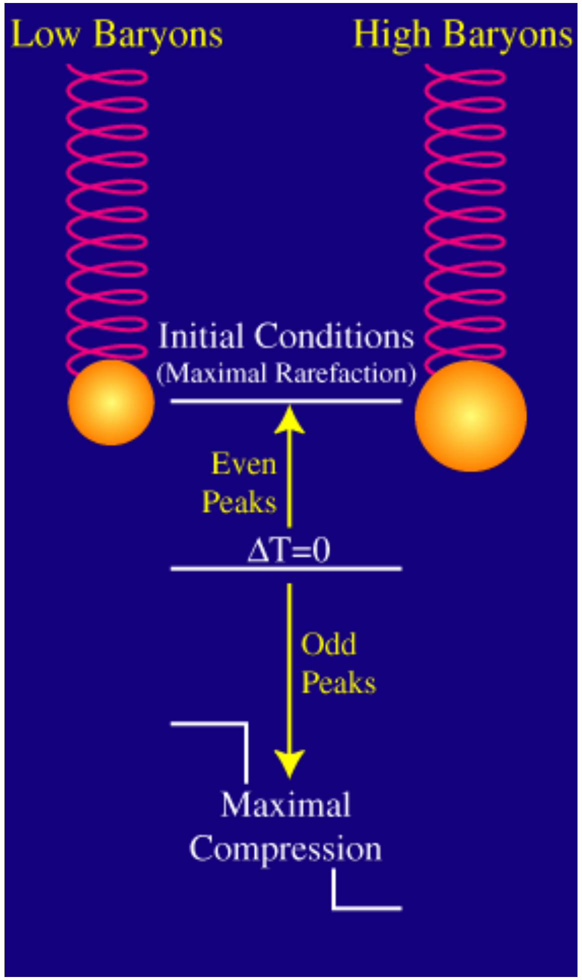
So the anisotropy of the radiation detected today shows the inhomogeneities of the baryon density at $z = z_{dec}$

Now, note that odd peaks in the power spectrum represent scales of maximum compression and even peaks represent scales of maximum rarefaction (because the power spectrum is the variance, i.e., it is a square, positive quantity δ^2).

In models with **larger values of Ω_b**, the baryons also contribute to the potential → the maximum compression has higher amplitude → **larger amplitude of odd peaks compared to even peaks**

(the 2nd peak of the CMB power spectrum can be smaller than the 3rd peak for some values of the cosmological parameters).

The ratio between the amplitudes of the 2nd and 3rd peaks is a way to measure Ω_b (a completely different method than from the big bang nucleosynthesis).



The **first peak of the CMB power spectrum** corresponds to the largest scale with oscillations: the scale that reached maximum compression at decoupling → it is given by the maximum distance a perturbation can propagate from $a=0$ to $a=a_{dec}$ → it is the (comoving) sound horizon at decoupling.

This is the same scale responsible for the BAO peak in the matter power spectrum

$$r_s(z_*) = \int_0^{a_*} \frac{da'}{a'^2} \frac{c_s}{H(a')} \quad \sim \textbf{100 Mpc/h}$$

Now, the angular scale of the 1st peak is the ratio: $l_a \equiv \pi \frac{r(z_*)}{r_s(z_*)}$

where r (z*) is the (comoving) angular diameter distance to the CMB surface:

$$r(z_*) = \frac{c}{H_0} \int_0^{z_*} \frac{dz'}{E(z')}$$

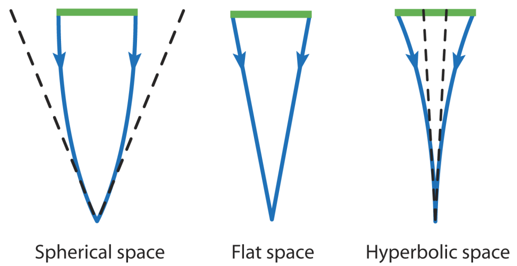
The comoving distance to z=z* is r (z*) ~ **6 Gpc/h** (concordance model)

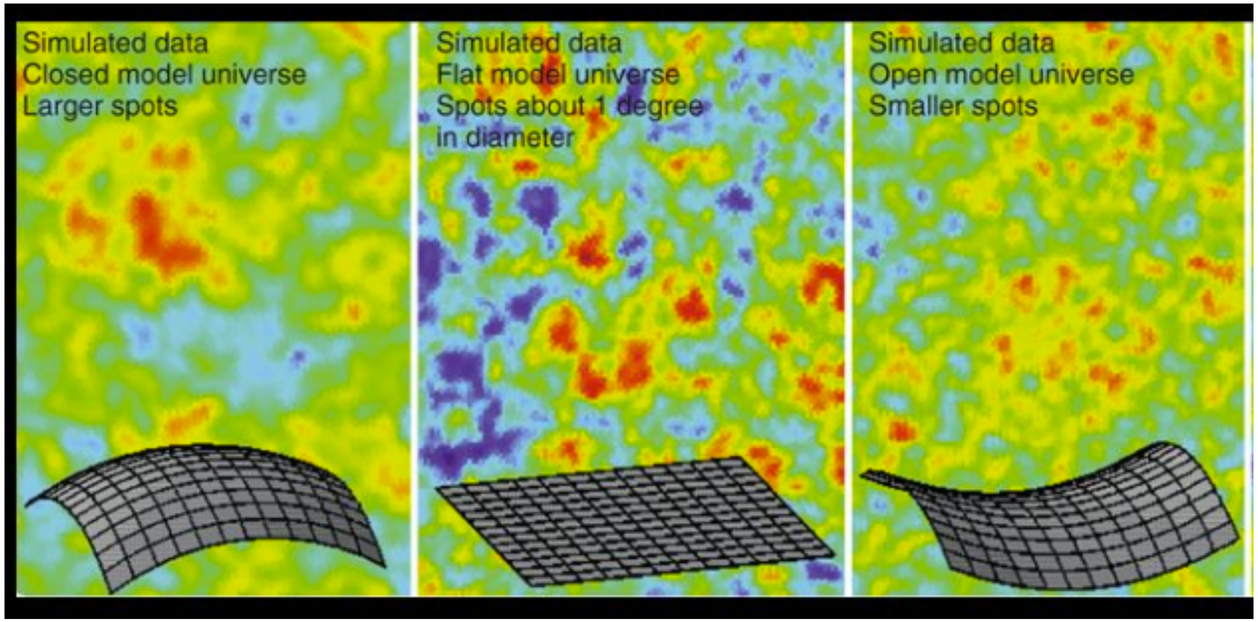
This means that the peak appears at l ~200 → θ ~1 deg

The position of the CMB first peak can be used as a geometric probe : measuring the peak angular scale constrains the ratio between the size of the sound horizon and the distance to the sound horizon → **the position of the first peak constrains the cosmological parameters** on which this ratio depends (i.e., H_0 and all Ω parameters, i.e., the background cosmological parameters or the parameters of geometry)

(Note that differently from the BAO method, the CMB 1^{st} peak can only be used in a single way, since there is only one source redshift possible → it is less powerful than BAO as a geometric probe).

In particular, angular power spectrum geometric probes (like the CMB first peak or the BAO peak) are good probes of the **curvature of the Universe**, because the l/θ relation depends explicitly on the curvature:





Peculiar velocity (Doppler effect)

$$\left(\frac{\delta T}{T}\right)_{\rm obs} = \frac{1}{4}\delta_\gamma^N - v^N \cdot \hat{n} + \Phi(t_{\rm dec}, x_{\rm ls}) + 2\int \dot{\Phi} dt$$

The plasma perturbations have a peculiar velocity, since they are moving on the potential wells.

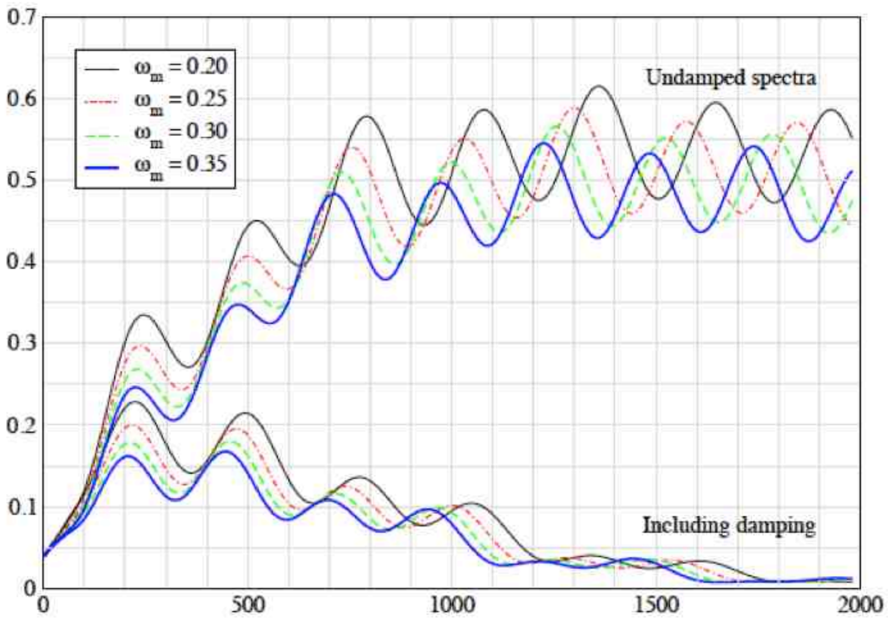
This is zero at the turnaround points (corresponding to maximum compression or rarefaction), i.e., at the positions of the CMB peaks.

However, at decoupling, the emissions made from other points of the plasma suffer a Doppler shift → there is a Doppler effect that is larger for scales in-between peaks → **smoothing of the peak structure of CMB.**

The effect depends on the peculiar velocity, which is function of the potential and thus related with the density contrast.

The **v.n** contribution in the expression for ΔT/T is thus a Doppler effect.

The (dimensionless) CMB temperature power spectrum produced only by the acoustic oscillations (including Doppler shifts) would look like the upper grey curve:



Similar to the dimensionless matter power spectrum, but with peaks of large amplitudes.

This is quite different from what we observe, because there are still other important contributions to consider.

General relativistic effects

$$\left(\frac{\delta T}{T}\right)_{\text{obs}} = \frac{1}{4}\delta_\gamma^N - v^N \cdot \hat{n} + \Phi(t_{\text{dec}}, x_{\text{ls}}) + 2\int \dot{\Phi} dt$$

i) **Density contrast on super-Hubble scales**

On large scales, Jeans equation is not valid and there is no trade-off between pressure and gravity → **gravity dominates.**

The growth of δ_γ is related to the growth of the potential (through Poisson equation).

The Einstein equations include equations for the evolution of δ, for the evolution of the potential (a metric perturbation), and a relativistic Poisson equation relating the two quantities. In the plasma epoch, a combination of those equations results in the following relation:

$$\delta = \left[1 + \frac{2}{3}\left(\frac{k}{aH}\right)^2\right]\Phi$$

This shows that on large scales, the perturbations do not relate with the potential only through the Laplacian ($k^2 \Phi$) but have an extra **scale-independent term**.

On very large scales (k small) this term is the dominant one and δ ~ Φ

ii) **Gravitational time-delay**

There is a second GR gravitational effect, which arises because photons coming from denser regions suffer a larger gravitational redshift → this translates into a time-delay → **they decouple slightly later, when the temperature of the universe is lower**.

Since in the matter epoch, $a(t) \sim t^{2/3}$ and $T \sim 1/a$, we can write:

$$\frac{\Delta T}{T} = -\frac{\Delta a}{a} = -\frac{2}{3}\frac{\Delta t}{t} = -\frac{2}{3}\Phi$$

(the potential is responsible for the time-delay).

This means that there is an effect on the emission temperature just because they are emitted later (due to the time-delay created by the potential) and not just because of larger density contrast (also created by the same potential).

The two gravitational effects have opposite impacts on the density contrast:

- through **Poisson equation**: larger potential → larger density contrast → higher CMB δT
- through **time-delay**: larger potential → larger gravitational redshift → lower CMB δT

The sum of the 2 effects is known as the **Sachs-Wolfe effect**:

$\Delta T/T\ (z_{dec}) = 1/3\ \Phi\ (z_{dec})$ → **on large scales, the amplitude of the CMB power spectrum is increased by the value of $\Phi/3$ at z_{dec}**

We need now to consider the **scale-dependence of Φ**, i.e., its power spectrum.

We saw that the primordial dimensionless potential power spectrum is scale-independent:

$$k^3\, P_\Phi^0 \propto k^{n_s - 1}$$

On large scales, the time-evolution of the potential has no transfer function, i.e., it evolves equally on all scales → at z_{dec}, P_Φ still keeps its original shape → a scale-independent dimensionless power spectrum.

This contribution to the CMB TT power spectrum is known as the Sachs-Wolfe plateau → **the dimensionless CMB power spectrum is constant on large scales**:

$l(l+1)/2\pi\ C_l$ is scale-independent on very large scales

On the scales near the first peak, both the acoustic and the gravitational effects contribute (oscillations and constant Sachs-Wolfe amplitude) → there is an added SW amplitude to the peak → **this is why the first CMB peak is much higher than the others.**

Integrated Sachs-Wolfe effect **(ISW)**

$$\left(\frac{\delta T}{T}\right)_{\rm obs} = \frac{1}{4}\delta_\gamma^N - v^N \cdot \hat{n} + \Phi(t_{\rm dec}, x_{\rm ls}) + 2\int \dot{\Phi} dt$$

After decoupling, there are extra SW contributions to the CMB temperature power spectrum, if the potential metric perturbation evolves in time.

Indeed, if the potential evolves in time during the time a photon takes to cross it, then the net gravitational frequency shift (redshift entering and blueshift exiting) is not zero → this extra contribution to ΔT/T is the term:

$$2\int \dot{\Phi} dt$$

This does not happen during the matter dominated epoch (where the solution for the evolution of Φ is Φ ~ constant), but it happens during radiation and dark energy epochs, which originates two different effects:

i) **Early-time ISW**

During the recombination period, radiation density is still high enough to produce some effect through the potential → the primary CMB (the emitted one) is still affected by this and the SW contribution needs to include this early-time ISW effect → amplitude of CMB further increases on large scales.

In models with **larger values of Ω_m**, the radiation epoch ends earlier → radiation effects such as the early ISW effect are less important at z_{dec} → no added contribution to the SW plateau → in particular, the **amplitude of the first peak is lower.**

The amplitude of the first peak directly constrains Ω_m

ii) **Late-time ISW**

In the late universe, when **dark energy** starts to be important, the potential is again no longer constant and there is an important contribution from an ISW effect.

This is called late-time ISW effect → amplitude of the secondary anisotropies of the CMB increases on large scales.

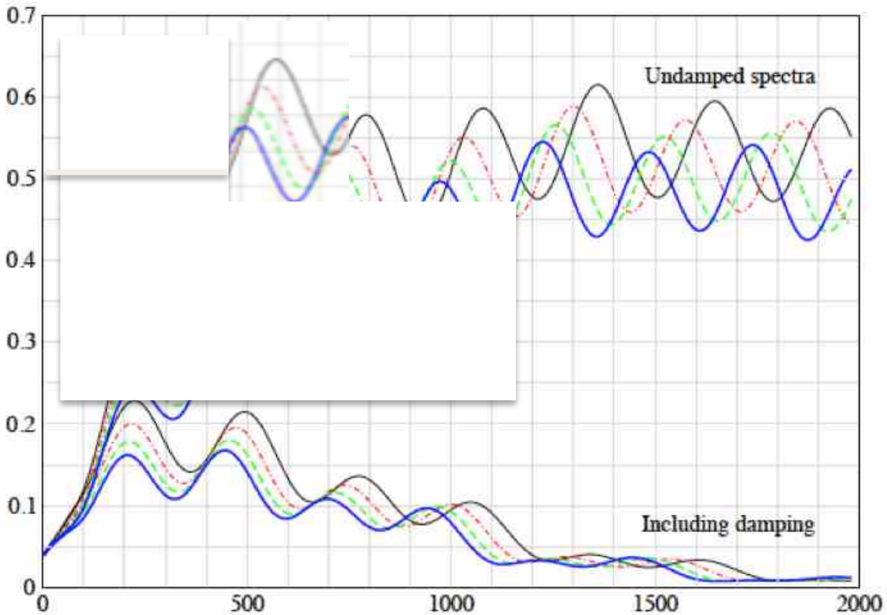
The potential perturbations of different scales can evolve in different ways. Different dark energy (or modified gravity) models may have specific signatures for the scale-dependence of this effect.

Because it is a large-scale effect at very low redshift, the angular scales are very large → it shows up on the SW plateau and not on the first peak → the observed SW plateau is no longer flat.

The amplitude and shape of the SW plateau constrains dark energy and modified gravity models.

Diffusion damping **(Silk damping)**

Putting the four effects together, the CMB should look like a flat line on large scales with an oscillatory pattern on smaller scales, with equal peak amplitudes (except for a higher first peak), as shown in the upper grey curve



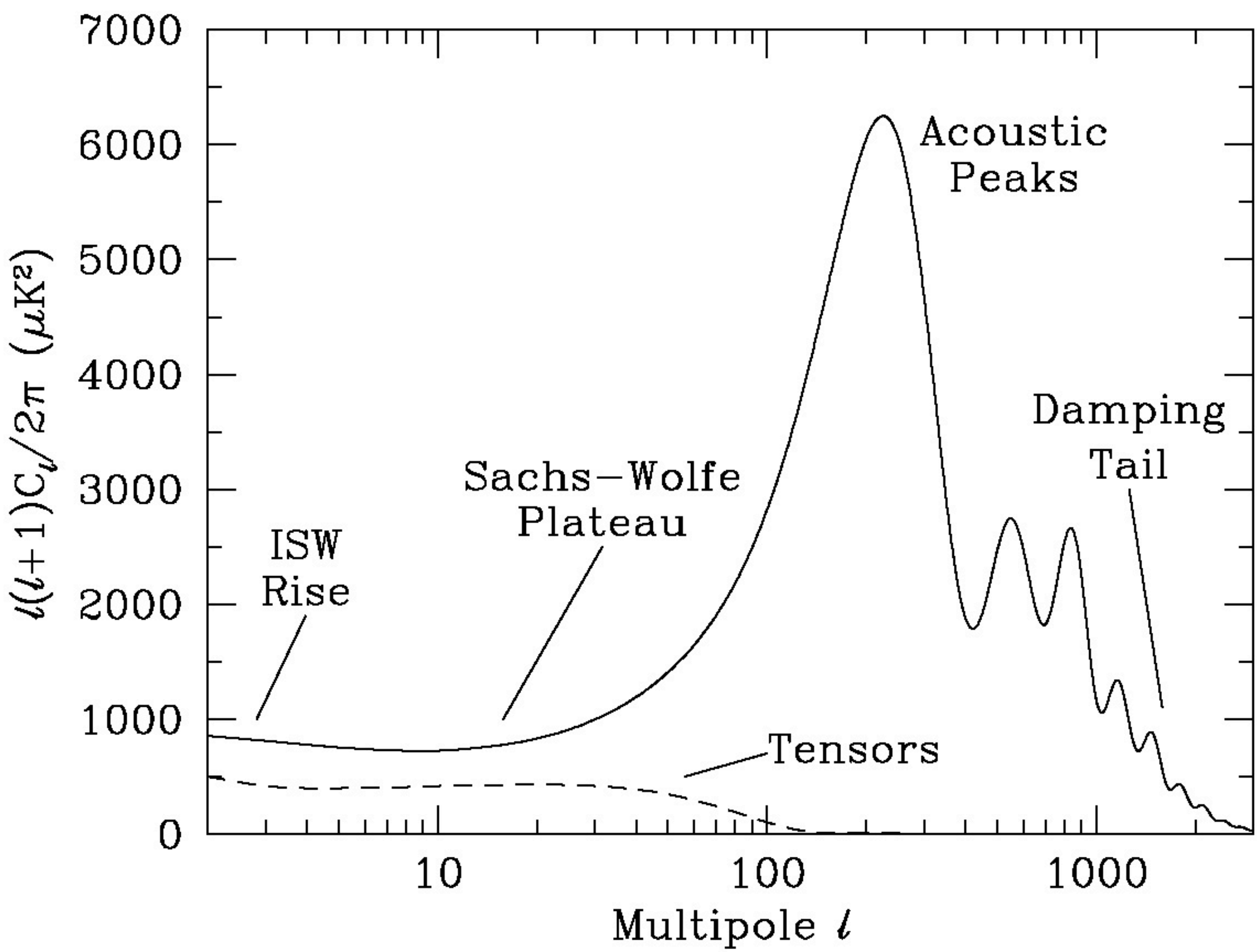
It is no longer similar to a dimensionless matter power spectrum with peaks, because on large scales it is flat (instead of increasing) → on large scales C_l is basically a C_Φ

However, the observed amplitudes of the peaks decrease with scale.

This is due to the fact that the last scattering surface has a finite thickness, i.e., **decoupling is not instantaneous.** During this delta time, the plasma is "**partially coupled**" → photons start to stream out of the overdensities and drag baryons with them → **destroying the overdensity** on scales below the damping scale.

Summary

Putting all effects together, the dimensionless CMB power spectrum looks like this:



The CMB power spectrum: secondary temperature anisotropies

In addition to these effects responsible for the CMB primary anisotropies, there are other effects that change the CMB power spectrum during its propagation, the secondary anisotropies → **the (theoretical) observationally estimated CMB is different from the one emitted at z_{dec}.**

Besides the Late-time ISW effect, other secondary effects are:

Rees-Sciama effect - Similar to ISW, but due to peculiar velocities of the potentials (instead of time variations).

Gravitational waves - They may also change the potentials and create another ISW effect.

Gravitational lensing - Deflects each photon to another direction in a coherent way. It mixes the directions → smearing out the anisotropies **(smoothing the peaks)**. In addition, it also changes the original polarization of the CMB.

Sunyaev-Zeldovich effect - Scattering of CMB photons if they pass in the hot gas of galaxy clusters

Reionization - New free electrons start to be available from first stars (galaxy formation) → they scatter CMB photons, which become more isotropically distributed → this reduces the amplitude of the anisotropies on all scales by a factor exp(-τ^2) → If the effect was too strong it would completely destroy the CMB anisotropies.

The optical depth of this scattering is **another fundamental parameter of the ΛCDM cosmological model**. It is defined as:

$$\tau = \int_0^{z_{\rm rec}} dz\, n_e\, \sigma_T \qquad\qquad n_e = \Omega_{\rm gas}\frac{3H_0^2}{8\pi G}\frac{1}{\mu m_p}(1+z)^3$$

it depends on the electron density in the stars, and the cross section for the interaction

Alternatively to τ, the effect can also be parameterized by the redshift of reionization, z_{re}

The amplitude of the CMB power spectrum constrains the redshift of the first stars.

Conclusion

We derived and described the features of the two central cosmological functions of the inhomogeneous ΛCDM universe:

Matter power spectrum → structure in the dark matter density field + effect of baryons (oscillations).

It is a function of redshift.
It is not directly observable, but the power spectra of several observable cosmological fields are derived from it (galaxy clustering, weak lensing, peculiar velocity, cluster abundance, Lyman-alpha forest, etc.)

CMB Temperature power spectrum → anisotropy in the cosmological photons temperature field → structure in the radiation/baryon plasma density field.

At a fixed redshift (decoupling).
It is directly observable.

Our information on the properties of the inhomogeneous universe comes mostly from measurements of these two power spectra.

The CMB power spectrum is the most powerful to constrain the cosmological parameters because:

- it has a rich structure (much more features than the matter power spectrum)

- it depends on many cosmological parameters in different ways

- it contains information from both the early universe and the recent universe (through the secondary anisotropies)

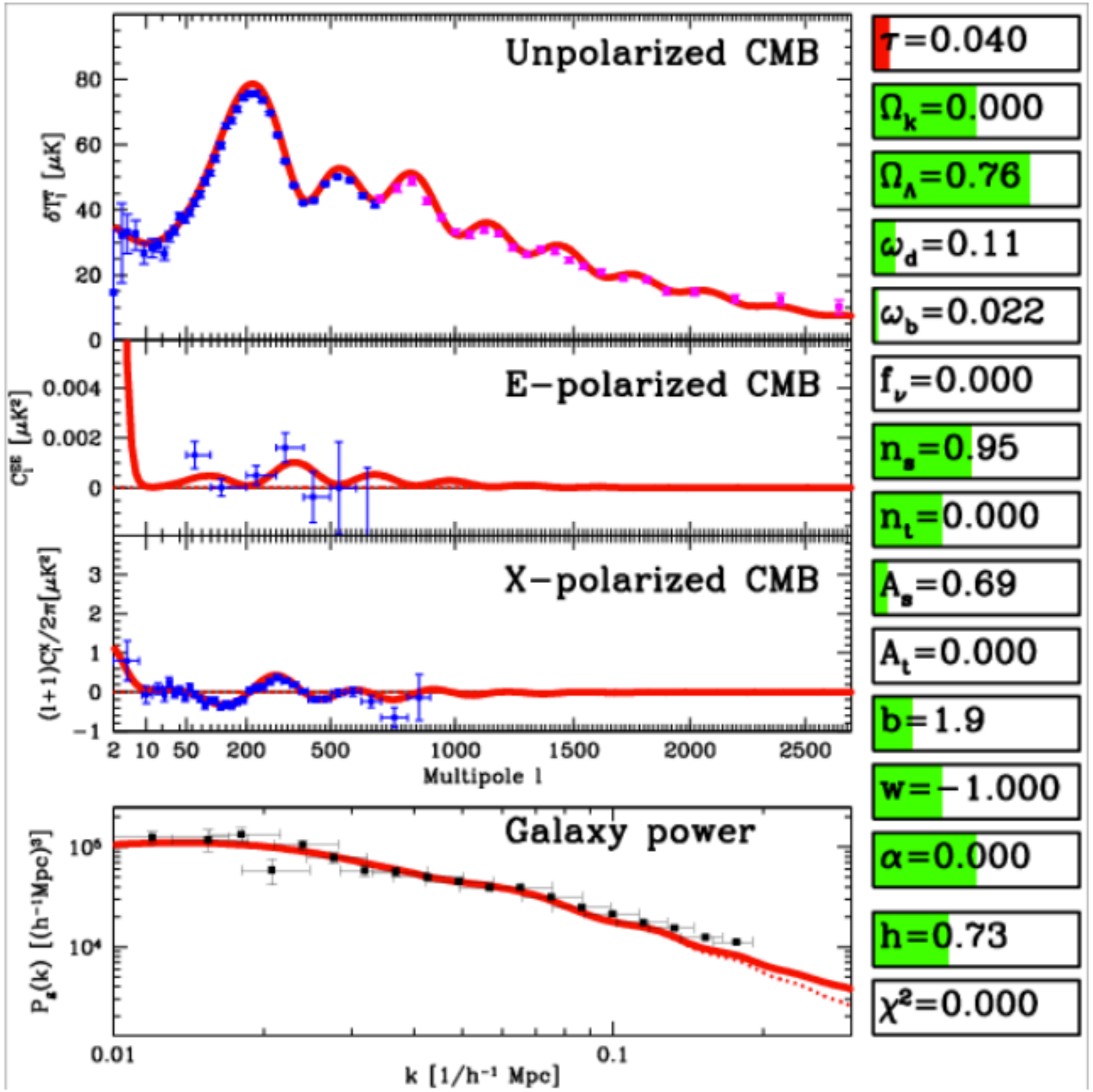
- it can be measured with high precision and on the whole sky

- it is less affected by theoretical biases than other probes because its perturbations are linear

- it is less affected by astrophysical biases than other probes (the primary anisotropies)

The various power spectra can be computed for **input values of the cosmological parameters.**

There are several **codes** that implement the set of needed equations (Newtonian, Einstein, Boltzmann, transfer function, non-linear fits): CLASS, CAMB, Hi_class, EFTCAMB.



Interactive "CMB and matter power spectra movies" made from the outputs of those codes can be found here: *https://space.mit.edu/home/tegmark/parmovies*

This is just one of the steps of the full process of **finding the parameters of the universe**