PHASE TRANSITIONS

Problems1

1. Show that for an idela paramagnet ($\mathcal{H} = -H \sum_i^N s_i$), the magnetisation per spin is given by

$$m = \tanh \beta H \tag{1}$$

and the magnetic susceptibility is

$$\chi(T, H) = \frac{\beta}{\cosh^2 \beta H} \tag{2}$$

where we used the magnetic moment per spin and the lattice constant as the system units, i.e., $m_0 = 1$ and $a = 1$.

   (a) Derive the Curie law for $\chi(T, 0)$.

   (b) Derive the Curie-Weiss law. (Suggestion: substitute H by $H_{eff} = H + \lambda m$). Calculate the solutions for $H = 0$.

   (c) Obtain the series in powers of m of the equation of state, for small m (i.e., $T \sim T_c$). Show that $\beta = 1/2$ and calculate γ and γ'.

2. Use the transfer matrix technique to calculate the free energy of the Blume-Emery-Griffiths model, in $1d$, with hamiltonian

$$\mathcal{H} = -J \sum_i s_i s_{i+1} - K \sum_i s_i^2 s_{i+1}^2 + D \sum_i s_i^2 \tag{3}$$

where, $s_i = 1,\ 0,\ -1$, for all sites i. Take $K = 0$, $J > 0$ and $D > 0$.