PHASE TRANSITIONS

Problems 2

1. Show that the Landau free energy

$$g_s(t,h) = -m_0 h + aT_c t m_0^2 + b m_0^4 \tag{1}$$

with the corresponding equation of state,

$$-h + 2aT_c t m_0 + 4b m_0^3 = 0 \tag{2}$$

obeys a simple scaling form

$$g_s(t,h) = \lambda g_s(\lambda^x t, \lambda^y h) \tag{3}$$

where, $x = -1/2$ and $y = -3/4$. As λ is arbitrary, we can write

$$g_s(t,h) = t^2 g_s\left(1, \frac{h}{t^{3/2}}\right) = t^2 F\left(\frac{h}{t^{3/2}}\right) \tag{4}$$

The phenomenological scaling hypothesis is less strong requiring that

$$g_s(t,h) = t^{2-\alpha} F\left(\frac{h}{t^{\beta\delta}}\right) \tag{5}$$

with α and $\Delta = \beta\delta$, to be determined experimentally.

2. Consider a one dimensional chain, with Ising spins, $s = 1$, which may take values, $s_k = 1,\ 0,\ -1$, and nearest neighbour interactions, $K s_k s_{k+1}$. Show that the renormalized hamiltonian, $\bar{\mathcal{H}}'$, obtained after simple decimation with $b = 2$, contains other couplings. By deriving the recursion relations for the coupling constants, show that it is sufficient to include the terms $h_2 s_k^2$ and $K_4 s_k^2 s_{k+1}^2$, in the space of hamiltonians.