Cosmologia Física

Topics for the Presentations

I. The structure of the Universe

1. Boltzmann equation for dark matter

The dynamics of the Universe is described by a set of constraint and conservation equations, the Einstein-Boltzmann equations. Present the Boltzmann equation in the inhomogenous Universe. Apply it to the case of dark matter, to derive the continuity and Euler equations.

References:
S. Dodelson and F. Schmidt (2021) "Modern Cosmology, 2nd edition"[book] - Sections 3.3, 3.4, 5.4

2. CMB anisotropies

The coupled radiation-baryon cosmological fluid can only be properly described in the formalism of the Boltzmann equation. Present (and derive when possible) the perturbation equations for CMB temperature anisotropies (eq. 4.170, eq. 4.179) and its solution $\Theta_\ell(k)$ (eq. 5.23).

References:
L. Amendola and S. Tsujikawa (2010) "Dark Energy"[book] - Sections 4.9, 5.3

3. Sachs-Wolfe effect

Photons travelling on the perturbed space-time suffer deflections on the direction of propagation (gravitational lensing) and change of redshift (Sachs-Wolfe effect). Present a derivation of the Sachs-Wolfe effect, i.e. of the effect of the potential on the temperature fluctuations.

References:
L. Amendola and S. Tsujikawa (2010) "Dark Energy"[book] - Sections 4.11, 4.11.1

4. Weak lensing theory

Introduce the effect of gravitational lensing. Explain the lens equation $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\theta)$, and the relation between the deflection and the gravitational potential. Define also convergence and shear from the linearized lens equation. Then, specialize on the regime of cosmological weak lensing and present the derivation of cosmic shear and cosmic convergence for light propagation in the inhomogeneous Universe.

References:
I. Tereno (2023) - Slides Gravitational Lensing (pg 1-20), Slides 21 (pg 1-22)
S. Dodelson (2017) "Gravitational lensing"[book] - Section 2.6
M. Bartelmann and P. Schneider (2001) - Chapter 6: https://arxiv.org/pdf/astro-ph/9912508.pdf

5. Redshift-space distortions

When measuring the correlation functions of galaxies, the distances between them must be known. However, the peculiar velocities of the galaxies introduce an error on the measured distances that affects the resulting correlations function. This is known as a redshift-space distortion (RSD). Present the RSD effect, describing its theory in the linear regime. Explain also how RSD is measured in galaxy surveys with the multipole estimators.

References:
I. Tereno (2023) - Slides 20
L. Amendola and S. Tsujikawa (2010) “Dark Energy”[book] - Sections 4.8, 14.3
A.Hamilton (1997) - Sections 2, 4.1, 4.2, 5.2: https://arxiv.org/pdf/astro-ph/9708102.pdf

6. CMB polarization

Present the effect of polarization of the CMB. Describe the formalism of the Stokes parameters, their power spectra and the decomposition in E and B modes. Mention also gravitational waves as a source of CMB polarization.

References:
P.Cabella and M.Kamionkowski (2004) - Section 1, 2, 3, 7: https://arxiv.org/pdf/astro-ph/0403392.pdf
A.Balbi et al (2006): https://arxiv.org/pdf/astro-ph/0606511.pdf
S. Naess et al (2014) - example of a CMB polarization survey: https://arxiv.org/pdf/1405.5524.pdf

7. Clustering dark energy

Like dark matter, dark energy could also be inhomogeneous and cluster. Present the equations of linear and non-linear (spherical collapse) evolution of possible dark energy fluctuations and their impact on observations.

References:
R. Batista (2022): https://arxiv.org/pdf/2204.12341.pdf

8. Neutrinos

Present the neutrino properties that are relevant in cosmology (mass, effective number, neutrino hierarchy) and describe their effects on the main cosmological probes. Discuss also (briefly) the complementarity between cosmological and laboratory neutrino searches.

References
M. Gerbino and M. Lattanzi (2018) - Sections 2, 3, 4, 8, 9, 10: https://arxiv.org/pdf/1712.07109.pdf

II. ΛCDM problems

There is a long list of different types of challenges to the standard ΛCDM model that emerged in the past years as the accuracy of cosmological observations improved. You can find a recent review on: L. Perivolaropolous and F. Skara (2022) https://arxiv.org/pdf/2105.05208.pdf. The topics of this section deal with some of these problems.

9. The Hubble tension

Describe the H_0 tension, a fundamental open problem in the ΛCDM model, presenting its early and late time measurements. Mention also its relation with the sound horizon scale and discuss the shortcomings of trying to modify the sound horizon scale as a way to solve the tension (in a general way, without addressing specific thoretical early time models).

References
E. Di Valentino et al (2021) - Sections 2 and 3: https://arxiv.org/pdf/2103.01183.pdf
K. Jedamzik et al (2020): https://arxiv.org/pdf/2010.04158.pdf

10. The local void

Present the idea that a local void (the fact that we may live in a local underdense region) might be the reason for the Hubble tension.

References
E. Di Valentino et al (2021) - Section 3.1: https://arxiv.org/pdf/2103.01183.pdf
W. Kenworthy et al (2019): https://arxiv.org/pdf/1901.08681.pdf
H. Y. Wu and D. Huterer (2017): https://arxiv.org/pdf/1706.09723.pdf

11. Time delays

Time delay is a strong lensing effect observed in some astrophysical systems that produces multiple images of a source. Describe the effect, deriving its central equation. Explain how it is used to constrain the Hubble parameter.

References:
E. Di Valentino et al (2021) - Section 2.2.1: https://arxiv.org/pdf/2103.01183.pdf
P. Schneider (1985): http://articles.adsabs.harvard.edu/pdf/1985A%26A...143..413S
C. Kochanek and P. Schechter (2004): https://arxiv.org/pdf/astro-ph/0306040.pdf
S. Suyu et al (2013) - example of a time-delay survey: https://arxiv.org/pdf/1208.6010.pdf

12. Standard sirens

Gravitational waves emitted by binary black holes are standard sirens that allows us to measure the luminosity distance and constrain background cosmological parameters. Present the method and explain how it is used to constrain the Hubble parameter.

References
E. Di Valentino et al (2021) - Section 2.2.2: https://arxiv.org/pdf/2103.01183.pdf
B. Schutz (2002): https://arxiv.org/pdf/gr-qc/0111095.pdf
D. Holz and S. Hughes (2005): https://arxiv.org/pdf/astro-ph/0504616.pdf
A. Palmese et al (2022) - example of a standard siren H_0 measurement: https://arxiv.org/pdf/2111.06445.pdf

13. The sigma 8 tension

Describe the σ_8 tension, a fundamental open problem in the ΛCDM model, presenting its early and late time measurements. Give an overiview of results from different probes. Present also the main systematic effects that could affect the measurements. Discuss also brifely some alternative cosmological models that could alleviate the tension.

References
L. Perivolaropolous and F. Skara (2022) - Section III.1:// https://arxiv.org/pdf/2105.05208.pdf
E. Abdalla et al (2022) - Section V: https://arxiv.org/pdf/2203.06142.pdf

14. Weak lensing measurements

Describe the method and the estimator used to measure the correlation functions of weak lensing. Explain also what are intrinsic alignments (the main factor biasing the weak lensing analyses). Explain how this method is used to constrain the S_8 parameter and address the σ_8 tension in an example survey.

References:
I. Tereno (2023) - Slides 21 (from pg 23)
M.Bartelmann and P.Schneider (2001) - Section 4.2: https://arxiv.org/pdf/astro-ph/9912508.pdf
H. Hildebrandt et al (2017) - Sections 4, 6 - example of a weak lensing survey: https://arxiv.org/pdf/1606.05338.pdf

15. ΛCDM small-scale problems and baryonic effects

Present the four classic small-scale dark matter problems of the ΛCDM model. Discuss the impact of baryonic effects on the dark matter power spectrum, and how it can help in explaining some of these problems.

References
M. Buckley and A. Peter (2018) - Section 4: https://arxiv.org/pdf/1712.06615.pdf
M. van Daalen et al (2011) - Section 3: https://arxiv.org/pdf/1104.1174.pdf

16. ΛCDM small-scale problems and recent solutions

Another important ΛCDM small-scale problem is the so-called plane of satellites problem. Explain what is this problem and why it is considered evidence against dark matter. On the other hand present a recent result where it is argued that the plane of satellites is on the contrary consistent with ΛCDM. Consider also another recent observation: In 2018 a low-mass galaxy without dark matter was observed for the first time. This immediately created a new dark matter problem, since low-mass galaxies are the ones expected to be more dark matter dominated. However, the problem was apparently solved last year with new numerical simulations. Present also this new mystery and the proposed solution to it.

References
M. Pawlowski et al. (2018): https://arxiv.org/pdf/1802.02579.pdf
T. Sawala et al (2022): https://www.nature.com/articles/s41550-022-01856-z
P. van Dokkum et al (2018): https://arxiv.org/pdf/1803.10237.pdf

J. Moreno et al (2022): https://arxiv.org/pdf/2202.05836.pdf

17. MOND

Modified Newtonian Dynamics (MOND) is a modification of Newtonian gravity suggested as an alternative to dark matter. It was proposed 40 years ago (see the celebration conference taking place this June: http://www-star.st-and.ac.uk/ hz4/MOND40Conf2023). It was originally based on empirical laws, rather than being the result of a physical theory. Present the MOND idea and how it can explain observations on different scales: rotation curves of spiral galaxies, pressure-supported systems, and cosmic structure with only baryonic matter and no dark matter. Present also the recent results from the analysis of the El Gordo galaxy cluster.

References
R. Sanders and S. McGaugh (2002) - Sections 1-4, and Section 6:
https://arxiv.org/pdf/astro-ph/0204521.pdf
E. Ascencio et al (2021): https://arxiv.org/pdf/astro-ph/2012.03950.pdf

18. Very large large-scale structures

Observation of very large structures have an implication for the scale of the homogeneity and the validity of the cosmological principle. In addition, there are also several observations of so-called anomalies, i.e. alignments and assymetries in the cosmological sky. Give a brief overview of those anomalies, and focus in more detail on the homogeneity scale and the detection of giant arcs in the Universe.

References
P. K. Aluri et al (2023) - mainly Sections I, II, VII: https://arxiv.org/pdf/2207.05765.pdf
A. M. Lopez et al (2022): https://arxiv.org/pdf/2201.06875.pdf

III. Parameter estimation

19. Dependence on cosmological parameters

Use CLASS to plot the matter power spectrum and the CMB power spectrum for several values of the cosmological parameters. In particular, observe and explain the following behaviours: i) shift in scale of the matter power spectrum peak when increasing the value of Ω_m, ii) shift of the first peak of CMB when changing w_{DE}, iii) change in position or amplitude of the first peak of the CMB when increasing the value of H_0, iv) change in relative amplitude of second and third peaks of CMB when changing Ω_b, v) change in amplitude of the first peak of the CMB when changing the value of Ω_m, vi) effect of changing n_s.

Make also plots of the luminosity distance $D_L(z)$ or $D_L(a)$ for some values of Ω_m, to see how it changes, in preparation of the next part of the work, which is to constrain ΛCDM with supernovae (JLA) data. To find the constrains you need to:

- Download the JLA supernova data into Monte Python (see instructions in the reference).
- Run Monte Python with the Metropolis-Hastings sampling method (MCMC) with the

JLA SN likelihood. The free parameters that vary in the chains will be Ω_m and Ω_Λ (i.e. the curvature is not zero but $1 - \Omega_\mathrm{m} - \Omega_\Lambda$), plus the nuisance parameters of JLA.

- Make the resulting contour plots with the Monte Python tools.
- Run again MCMC but now also fixing Ω_Λ together with the other standard cosmological parameters. Make the contour plots to compare with the previous ones.
- You can make several runs with various number of points. For example with N=1 000, N=10 000, N=100 000, to compare the results. They should run in a few minutes in your laptop.
- You can also run again Monte Python but now using nested sampling. Again, make the contours plots for comparison.
- You can make other comparisons that you think of.

References:
D. Castelão (2022): "CLASS and Monte Python guide"

20. Fisher matrix

Present the Fisher matrix method, an analytical procedure that allows us to find the confidence ellipses in the parameters space, valid for Gaussian likelihoods. Explain also what are maginalized and non-marginalized results and how their confidence contours are computed from the Fisher matrix.

After this, solve the following two exercises:

1) Consider a very small supernova survey that detected 3 supernovae and measured their distance modulus with uncorrelated errors. The data vector obtained is: distance modulus $\mu = 42.10 \pm 0.25$ for the object SN 2002dc at z=0.475; $\mu = 43.14 \pm 0.21$ for SN 2003bd at z=0.67; and $\mu = 44.25 \pm 0.14$ for SN 2001hb at z=1.0. (These values are taken from the SNLS survey).

1.1) The distance modulus, $\mu = 5 \log_{10} D_L + 25$, depends on the cosmological parameters through the luminosity distance D_L. In the following, consider the approximate expression for D_L:

$$D_L = \frac{c}{H_0} \left[z + \frac{z^2}{2} \left(1 + \Omega_\Lambda - \frac{\Omega_\mathrm{m}}{2} \right) \right].$$

a) Is the observable μ more sensitive to Ω_m or Ω_Λ?

b) In a flat model (and considering only a fluid with two components) these 2 parameters are not independent: are they correlated or anti-correlated?

c) μ depends on a combination of the 2 parameters, i.e., the measurement of μ correlates the parameters. The estimated parameter values become correlated or anti-correlated?

1.2) Consider the Fisher matrix for the distance modulus, which is given by

$$F_{ij} = \sum_z \left[\left(\frac{\partial \mu_z}{\partial p_i} \right)_\mathrm{fid} \frac{1}{\sigma_z^2} \left(\frac{\partial \mu_z}{\partial p_j} \right)_\mathrm{fid} \right]$$

a) Compute the Fisher matrix of this survey, assuming $\Omega_{\rm m} = 0.3$ and $\Omega_\Lambda = 0.7$ as the fiducial model. (Note that we do not need to give a fiducial value to the Hubble constant, because the Fisher matrix will be independent of H_0.)

b) Notice that the Fisher matrix just obtained is singular (which does not happen when using the full integral expression for D_L). What is the feature in the $D_L(z)$ approximate formula that is responsable for this behaviour?

c) Being singular, the associated contour ellipse stretches to infinity, and we cannot get finite constraints on the individual parameters. This means that μ only has information about one effective parameter: the combination $\Omega = \Omega_\Lambda - \Omega_{\rm m}/2$. Use the Fisher matrix method in just one dimension for the effective parameter Ω, to find the constraint on that parameter. Write the result in the form: $\Omega = \Omega_{\rm fid} \pm \sigma_\Omega$.

d) The constraint obtained for Ω with this small survey is weak (i.e., the uncertainty you just found was quite large). Consider that the main contribution to the error bars on μ is Poisson noise in the measured flux. Estimate the number of supernovae we would need to measure (assuming they are measured with the same signal-to-noise ratio as the three original ones) in order to obtain an uncertainty of 1% on Ω.

2) Consider again the same supernova survey of three supernovae presented before, but now we will consider a bias. The bias correlates the measurements of the 3 supernovae which imply that the errors are now given by a non-diagonal covariance matrix:

$$C = \begin{matrix} (0.25)^2 & 0.007 & 0.003 \\ 0.007 & (0.21)^2 & 0.005 \\ 0.003 & 0.005 & (0.14)^2 \end{matrix}$$

2.1) Let us consider first the stretch bias. This implies that the μ estimator needs to be corrected with an additive parameter, i.e.,

$$\mu = 5 \log_{10} D_L + 25 + \alpha\,(s-1).$$

Consider that the 3 SNe all have the same stretch factor s, and so the additive term is a constant bias parameter, $b = \alpha\,(s-1)$.

a) The impact of the bias a larger at low or at high redshift?

b) Compute the Fisher matrix, for the biased SNe survey on the (Ω, b) parameter space, where $\Omega = \Omega_\Lambda - \Omega_{\rm m}/2$, using again the second-order approximation formula for D_L.

Hint: Remember the Fisher matrix is now given by its full form to account for the cross-correlations:

$$F_{ij} = \left(\frac{\partial \mu_z}{\partial p_i}\right)^T_{\rm fid} C^{-1}_{zz'} \left(\frac{\partial \mu_{z'}}{\partial p_j}\right)_{\rm fid}$$

c) Compute the marginalized uncertainty of Ω, i.e. marginalize the Fisher matrix over the bias parameter b.

d) Compare this result with the constraint obtained in the unbiased case. Did the uncertainty on the cosmological parameter Ω increase or decrease by correcting the bias? Is this the expected behaviour? Why?

References

I. Tereno (2023) - Slides 17 (from pg 41)

L. Amendola and S. Tsujikawa (2010) "Dark Energy"[book] - Section 13.3

21. Dynamical dark energy

CLASS already contains the basic dynamical dark energy model, defined by the equation of state (eos) $w(a) = w_0 + w_a(1 - a/a_0)$ (known as the CLP parameterization). In this project you should add a new parameterization (that we will call CF for Cosmologia Fisica) defined as $w(a) = w_0 + w_b(1 - a/a_0)^2$. For this you need to modify CLASS as follows: (more detailed instructions given in the appendix of the reference).

- background.c - in this module of CLASS look for the function background_w_fld. In this function search for CLP and copy-paste the lines associated to it, to define your new parameterization CF. CLP appears three times in this module, defining the eos, its derivative and its integral. Introduce the new functions (you need to analytically derive and integrate to find the functions to write in the code).
- background.h - search for CLP and copy-paste it to insert the equivalent CF information (for example the wb_fld instead of a wa_fld used in CLP).
- input.c - again search for CLP and copy-paste and modify what is there for the case of CF

You now have your own modified version of CLASS. You can now procced with the next steps of the project:

- Compile and run CLASS and make a plot of the background quantities w(z) and $D_L(z)$ for the two parameterizations CLP and CF (using standard values for the cosmological parameters).
- Download the JLA supernova data into Monte Python (see instructions in the reference).
- Run Monte Python with the Metropolis-Hastings sampling method (MCMC) with the JLA SN likelihood for the two dark energy parameterizations. The free parameters that vary in the chains will be $\Omega_{\rm cdm}$, w_0, w_a (or w_b for CF), plus the nuisance parameters of JLA.
- Make the contour plots with the Monte Python tools.
- Run again MCMC but now also fixing $\Omega_{\rm cdm}$ together with the other standard cosmological parameters. Make the contour plots to compare with the previous ones.
- You can make several runs with various number of points. For example with N=1 000, N=10 000, N=100 000, to compare the results. They should run in a few minutes in your laptop.
- You can also run again Monte Python but now with different sampling methods (nested sampling and Fisher matrix). Again, make the contours plots for comparison.

- You can make other comparisons that you think of.

<u>*References*:</u>

D. Castelão (2022): "CLASS and Monte Python guide"