UNIVERSO PRIMITIVO

Mestrado em Física Astronomia 2024-2025

Exercise Sheet 2

1. Convert kilogram (kg), second (s), meter (m) and kelvin (K) to giga-electron volt (GeV), assuming natural units $c = \hbar = k_B = 1$. Use your findings to express your weight, age, height and body temperature in GeV.

2. Derive expressions for the number density, energy density and pressure of a gas of ultra-relativistic particles in thermal equilibrium with vanishing chemical potential.

3. Show that the energy density and pressure of non-relativistic particles with vanishing chemical potential is given by $\rho = (m + \frac{3}{2}T)\, n$ and $P = nT$, respectively. Explain why in these conditions one has $P \ll \rho$.

4. Consider a thermal equilibrium distribution of relativistic particles with non-vanishing chemical potential $\mu$. Compute the number density, energy and pressure for:
   4.1. Degenerate fermions with $\mu \gg T$;
   4.2. $\mu < 0$ and $|\mu| < T$
   [Hint: in 4.1 assume that for degenerate fermions all energy states are occupied up to a maximum energy equal to $\mu$.]

5. Consider now the case of the non-relativistic limit with a non-vanishing chemical potential. Prove the expressions below. The overbar denotes densities for anti-particles. Assume that particles and anti-particles are in chemical equilibrium. (Regarding 5.2 and 5.3: note that in general there can be excess of particles over antiparticles):
   5.1. $n = g\left(\frac{mT}{2\pi}\right)^{3/2} \exp(-\frac{m-\mu}{T})$
   5.2. $n - \bar{n} = 2g\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{T}\right) \sinh\left(\frac{\mu}{T}\right)$
   5.3. $\rho + \bar{\rho} = 2gm\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{T}\right) \cosh\left(\frac{\mu}{T}\right)$