Universo Primitivo

2024-2025 (1º Semestre)

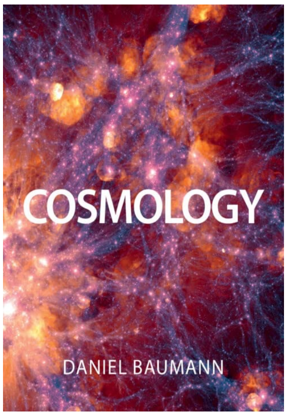
Mestrado em Física - Astronomia

Chapter 7

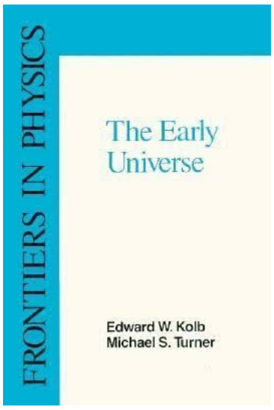
7 Recombination and Decoupling

- Initial conditions;
- Equilibrium abundances: the Saha equation;
- Hydrogen recombination;
- Photon - electron decoupling;
- Electron freeze-out

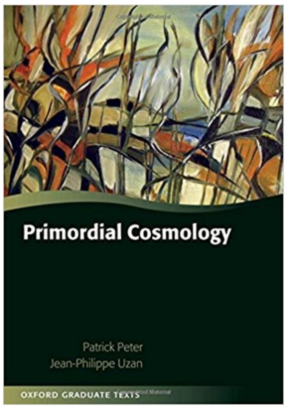
References



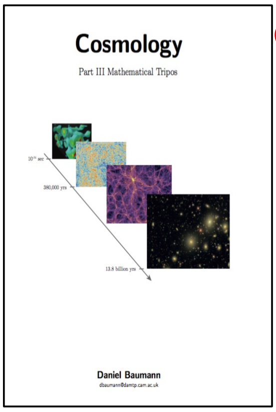
Ch. 3.2.5



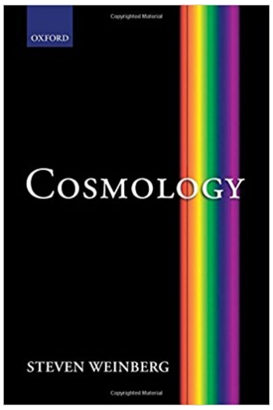
Ch. 5



Ch. 4.3



Ch. 3.3.3



Ch. 3.2

Recombination and decoupling

Initial conditions

Soon after neutron freeze-out and the production of the light nuclei by Big-Bang nucleosynthesis, $T \approx 0.1$ MeV, the universe:

- consisted of a plasma with **photons**, **electrons**, **protons** and **atomic nuclei** (and neutrinos that are decoupled from the plasma).
- **Electrons, protons and atomic nuclei are non-relativistic** (note that $m_e = 0.5\text{MeV}$)
- Electrons are **tightly coupled** to photons due to **Compton scattering** and interact with protons and atomic nuclei via **Coulomb scattering**;
- **Protons**, **electrons** and **atomic nuclei** remain in equilibrium, through **electromagnetic interactions** such as:

$$e^- + p^+ \leftrightarrow \mathrm{H} + \gamma$$

- **Very little amounts** of **neutral matter (atoms) exist** because the **lowest ionization energy is only 13.6 eV** (for the Hydrogen atom) **but the plasma is still too hot** for electrons to be totally captured by nuclei;

Recombination and decoupling

Equilibrium abundances: the Saha equation

Let us now look at this interaction: $e^- + p^+ \quad \leftrightarrow \quad \mathrm{H} + \gamma$
At temperatures smaller than the mass of electrons, protons and hydrogen nuclei, these species have non-relativistic equilibrium abundances given by (Ex. 5.1, sheet 2):

$$n_i^{\mathrm{eq}} = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_i - m_i}{T}\right)$$

Where, $i = \{e, p, \mathrm{H}\}$ and $\mu_p + \mu_e = \mu_{\mathrm{H}}$ because photons have 0 chemical potential. With these densities we can compute the following ratio:

$$\left(\frac{n_{\mathrm{H}}}{n_e n_p}\right)_{\mathrm{eq}} = \frac{g_{\mathrm{H}}}{g_e g_p}\left(\frac{m_{\mathrm{H}}}{m_e m_p}\frac{2\pi}{T}\right)^{3/2} e^{(m_p + m_e - m_{\mathrm{H}})/T}$$

That involves the densities of the species in the reaction and is independent of their chemical potentials.

In the exponential pre-factor, it is safe to assume $m_H \approx m_p$, but the very small difference of mass between hydrogen and the proton is crucial. It gives the **binding (or ionization) energy** of the hydrogen atom

$$B_{\mathrm{H}} \equiv m_p + m_e - m_{\mathrm{H}} = 13.6 \text{ eV}$$

Recombination and decoupling

Equilibrium abundances: the Saha equation

The number of effective degrees of freedom of the hydrogen atom is $g_H = 4$. This arises because the spins of the electron and the proton can be aligned or anti-aligned, giving rise to 4 states. If we assume **the Universe is not electrically charged** electrons and protons should have the same initial density, $\boldsymbol{n_e = n_p}$. So the previous ratio can be simplified as:

$$\left(\frac{n_{\mathrm{H}}}{n_e^2}\right)_{\mathrm{eq}} = \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_{\mathrm{H}}/T}$$

The ratio on the left-hand side of this equation can be related to the electron fraction abundance,

$$\boxed{X_e \equiv \frac{n_e}{n_b}}$$

Where the baryon density is, $n_b = \eta\, n_\gamma = \eta \times \frac{2\zeta(3)}{\pi^2} T^3$ and η is the baryon to photon ration.

Since the majority of baryons is in the form of hydrogen and free protons, one may assume that $n_b \approx n_p + n_H = n_e + n_H$, and one can derive that:

$$\frac{1 - X_e}{X_e^2} = \frac{n_{\mathrm{H}}}{n_e^2} n_b$$

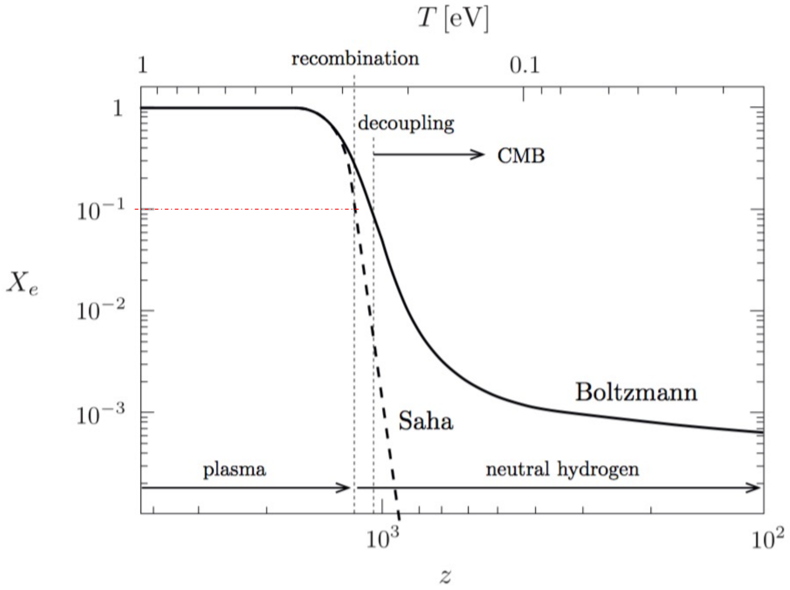
Recombination and decoupling

Equilibrium abundances: the Saha equation

Combining these expressions we obtain the so-called Saha equation.

$$\left(\frac{1-X_e}{X_e^2}\right)_{\rm eq} = \frac{2\zeta(3)}{\pi^2}\,\eta\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_{\rm H}/T}$$

Which allows one to compute the free electrons fraction in equilibrium



Recombination and decoupling

Hydrogen recombination

A way to **define the epoch of recombination is to consider** the moment when the free electron fraction reduces below a given amount, usually $\boldsymbol{X_e = 0.1}$.
Using this in the Saha equation one can compute the temperature at recombination:

$$\left(\frac{1-0.1}{0.1^2}\right)_{eq} = \frac{2\zeta(3)}{\pi}\,\eta\left(\frac{2\pi T_{rec}}{m_e}\right)^{3/2} e^{B_H/T_{rec}} \quad \Leftrightarrow \quad T_{rec} \approx 0.3\,\mathrm{eV} \simeq 3600\,\mathrm{K}$$

If one uses the redshift – temperature relation derived in Chapter 3,

$$T_{rec} = T_0(1+z_{rec})$$

one obtains that

$$z_{rec} \approx 1320$$

This redshift is lower than the redshift of matter radiation equality (exercise in exer. sheet 1), **so recombination occurred during the matter dominated era**, where the time dependence of the scale factor is $\boldsymbol{a(t) \propto t^{2/3}}$. Using this scaling in the temperature – redshift relation one concludes that:

$$t_{rec} = \frac{t_0}{(1+z_{rec})^{3/2}} \sim 290\,000\,\mathrm{yrs}$$

Recombination and decoupling

Photon – electron decoupling

Photons and electron are strongly coupled mostly due to Compton scattering

$$e^- + \gamma \quad \leftrightarrow \quad e^- + \gamma$$

The **Compton scattering interaction rate** can e approximated by $\Gamma_\gamma \approx n_e \sigma_T$ where the Thompson cross section is $\sigma_T \approx 2 \times 10^{-3}\,\mathrm{MeV}^{-2}$.

The epoch of decoupling between photons and electrons can be estimated by equating

$$\boxed{\Gamma_\gamma(T_{dec}) \sim H(T_{dec})}$$

Now:

$$\begin{cases} \Gamma_\gamma(T_{dec}) = n_b X_e(T_{dec})\, \sigma_T = \dfrac{2\zeta(3)}{\pi^2}\, \eta\, \sigma_T\, X_e(T_{dec}) T_{dec}^3\ , \\ H(T_{dec}) = H_0 \sqrt{\Omega_m} \left(\dfrac{T_{dec}}{T_0}\right)^{3/2} . \end{cases}$$

Using these in the previous equation gives:

$$X_e(T_{dec}) T_{dec}^{3/2} \sim \frac{\pi^2}{2\zeta(3)} \frac{H_0\sqrt{\Omega_m}}{\eta \sigma_T T_0^{3/2}}$$

Recombination and decoupling

Photon decoupling

Using this result with the Saha equation one obtains:

$$T_{dec} \sim 0.27\,\mathrm{eV}$$

Which is a slightly lower temperature than the recombination temperature.

Using this value in the temperature-redshift relation and the time-redshift (or time-temperature) relations one obtains the following estimates for the redshift and time of decoupling

$$z_{dec} \sim 1100\ ,$$
$$t_{dec} \sim 380\,000\,\mathrm{yrs}\ .$$

Recombination and decoupling

Electron Freeze-out

To compute the non-equilibrium abundance of free electrons one should rely on the Boltzmann equation for the reaction that describes the capture of free electrons by protons:

$$e^- + p^+ \leftrightarrow \mathrm{H} + \gamma$$

To a reasonable approximation one may assume that:

- The hydrogen density is $n_\mathrm{H} \approx n_\mathrm{H}^{\mathrm{eq}}$
- The universe is neutral, i.e. $n_e = n_p$

Under these assumptions, the Boltzmann equation derived in Chapter 4 (slides 26-28) for the above interaction reduces to:

$$\frac{1}{a^3}\frac{d(n_e a^3)}{dt} = -\langle\sigma v\rangle\left[n_e^2 - (n_e^{\mathrm{eq}})^2\right]$$

The thermally averaged cross recombination section $< \sigma v >$ can be approximated by:

$$\langle\sigma v\rangle \simeq \sigma_T\left(\frac{B_\mathrm{H}}{T}\right)^{1/2}$$

Where B_H is the binding energy of the hydrogen atom.

Recombination and decoupling

Electron Freeze-out

Using this result, writing $n_e = n_B X_e$ and using the fact that $n_b a^3 = constant$, one can approximate the Boltzmann equation as:

$$\frac{dX_e}{dx} = -\frac{\lambda}{x^2}\left[X_e^2 - (X_e^{\mathrm{eq}})^2\right]$$

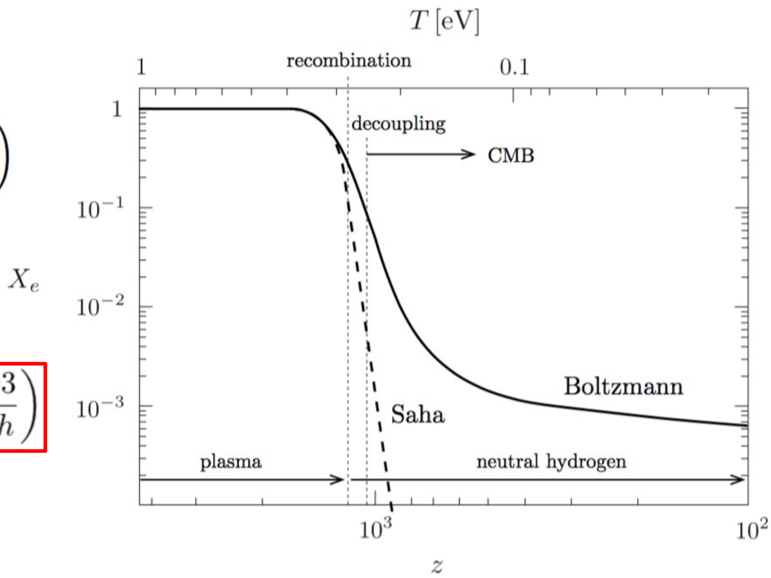
(same solution as the Ricatti equation)

Where $x \equiv B_\mathrm{H}/T$ and

$$\lambda \equiv \left[\frac{n_b\langle\sigma v\rangle}{xH}\right]_{x=1} = 3.9\times 10^3\left(\frac{\Omega_b h}{0.03}\right)$$

This equation has the same type of (approximate) solution as the Ricatti equation (see Chap. 5):

$$X_e^\infty \simeq \frac{x_f}{\lambda} = 0.9\times 10^{-3}\left(\frac{x_f}{x_{rec}}\right)\left(\frac{0.03}{\Omega_b h}\right)$$



Which is a good approximation to the full Boltzmann Integration (the solid line in the figure). This yields a present (global) free electron fraction in the Universe of about 0.1% $(x_f/x_{rec}\sim 1)$.