

# Project Topics

## 1. Geometric Ensemble Analysis

**Scope:** Explore the geometric, coordinate-free approach to ensembles (Liouville measure, invariant phase-space volume).

**Tasks:**

- Derive microcanonical density using geometric arguments.
- Compare entropy definitions (Boltzmann vs. Gibbs) and their geometric interpretations.
- Demonstrate ensemble equivalence via a toy system (e.g., ideal gas or oscillator).

**Deliverables:** Analytical exposition, geometric diagrams, short simulation.

## 2. Statistical Thermodynamics with Curvature

**Scope:** Apply statistical thermodynamics in curved (gravitational) settings, leveraging insights from Chapter 5.

**Tasks:**

- Derive temperature and entropy for systems in an external gravitational field (e.g., hydrostatic equilibrium).
- Compute Tolman's law (temperature gradient in gravitational potential).
- Contrast with flat-space results.

**Deliverables:** Analytical write-up, discussion of astrophysical relevance (e.g., stellar atmospheres, redshifted black hole emission).

## 3. Black Hole Thermodynamics: Statistical Perspective

**Scope:** Although more advanced, students can apply chapter 5's treatment of gravity to black hole thermodynamics.

**Tasks:**

- Model entropy and temperature of Schwarzschild black hole using statistical/gravitational arguments.
- Connect to the geometric interpretation and the role of horizon area.

**Deliverables:** Analytical derivation of Bekenstein–Hawking relations, discussion on ensemble interpretation.

#### 4. Random Processes & Langevin Dynamics

**Scope:** Explore stochastic differential equations and the Langevin approach (Chapter 6).

**Tasks:**

- Derive Fokker–Planck equation from Langevin dynamics for Brownian motion in a potential.
- Compute equilibrium distribution and relaxation times.
- Simulate 1D Langevin dynamics (Euler–Maruyama scheme) and compare to analytics.

**Deliverables:** Derivations, simulation scripts, plots of relaxation behavior.

#### 5. Fluctuation–Dissipation Theorem (FDT)

**Scope:** Use random process framework to derive and test FDT.

**Tasks:**

- Derive FDT connecting correlation functions and response functions.
- Numerically simulate a Langevin particle in harmonic potential: measure fluctuations and responses to small perturbations.

**Deliverables:** Mathematical derivation, simulation data verifying FDT.

#### 6. Statistical Mechanics of Self-Gravitating Systems

**Scope:** Apply chapters 4 and 5 concepts on systems with long-range gravity.

**Tasks:**

- Analyze the thermodynamics of a simple self-gravitating system (e.g., Plummer sphere, isothermal sphere).
- Discuss issues like negative specific heat, gravothermal catastrophe.

**Deliverables:** Analytical modeling, qualitative discussion of astrophysical phenomena.

#### 7. Equations of State from Virial Expansion

**Scope:** Explore the virial expansion as a systematic correction to the ideal gas law.

**Tasks:**

- Derive the virial expansion up to third order from the cluster expansion.
- Compute second and third virial coefficients for simple model interactions (hard sphere, Lennard–Jones).
- Compare predictions to experimental data for a real gas (e.g., nitrogen, CO<sub>2</sub>).

**Deliverables:** Analytical derivations, short numerical evaluation of virial coefficients, plots comparing equations of state.

## 8. Transport Coefficients from Green–Kubo Relations

**Scope:** Connect microscopic dynamics to macroscopic transport properties.

**Tasks:**

- Derive Green–Kubo relations for diffusion, viscosity, and thermal conductivity.
- Implement a short simulation (e.g., molecular dynamics of Lennard–Jones particles).
- Compute autocorrelation functions and extract transport coefficients.

**Deliverables:** Derivations, numerical simulation data, plots of correlation decay and coefficient estimates.

## 9. Kinetic Theory and the Boltzmann Equation

**Scope:** Bridge kinetic theory with macroscopic transport equations.

**Tasks:**

- Derive the Boltzmann equation and Chapman–Enskog expansion.
- Obtain expressions for shear viscosity and thermal conductivity of a dilute hard-sphere gas.
- Compare with experimental values or numerical simulations.

**Deliverables:** Step-by-step derivation, calculation of transport coefficients, discussion of accuracy.

## 10. Critical Phenomena and Equations of State Near Phase Transitions

**Scope:** Study how equations of state behave near critical points.

**Tasks:**

- Review the van der Waals equation of state and locate its critical point.
- Derive critical exponents (mean-field level).
- Compare to experimental data for CO<sub>2</sub> or water near criticality.

**Deliverables:** Analytical derivations, phase diagrams, plots of isotherms showing the critical region.

## 11. Cosmological Phase Transitions and Topological Defects

**Scope:** Explore the thermodynamics of the early universe during symmetry-breaking transitions.

**Tasks:**

- Study the Kibble–Zurek mechanism
- Estimate defect densities, and connect to cosmic strings/domain walls.

**Deliverables:** Analytical derivations plus simple numerical estimates of correlation lengths.

## 12. Dark Energy & Effective Equations of State

**Scope:** Model dark energy with different EOS forms (e.g., constant ( $w$ ), quintessence, Chaplygin gas).

**Tasks:**

- Compare predictions for cosmic expansion history with standard  $\Lambda$ CDM.

**Deliverables:** Analytical FRW solutions, plots of scale factor evolution, comparison with observational constraints.

## 13. Cosmic Microwave Background (CMB) Fluctuations

**Scope:** Study statistical properties of the CMB.

**Tasks:**

- Derive angular power spectrum scaling in the Sachs–Wolfe approximation
- Explore Gaussian vs. non-Gaussian statistics.

**Deliverables:** Analytical derivations, coding to generate mock CMB maps with different statistical assumptions.

## 14. Bose–Einstein Condensation and Finite-Size Effects

**Scope:** Go beyond the thermodynamic limit to see how condensation appears in trapped atomic gases.

**Tasks:**

- Derive critical temperature shifts for harmonic traps
- Explore fluctuations in small- $N$  systems.

**Deliverables:** Analytical derivations + simple numerical plots of condensate fraction vs. temperature.

## 15. Superfluid Hydrodynamics & Quantized Vortices

**Scope:** Study vortex dynamics in superfluid helium or BECs.

**Tasks:**

- Derive quantization of circulation
- Simulate vortex precession or lattice formation under rotation.

**Deliverables:** Analytical vortex solutions + basic Gross–Pitaevskii (GP) simulations.

## 16. Quantum Phase Transitions in Lattice Models

**Scope:** Explore transitions in Bose–Hubbard (Mott insulator  $\leftrightarrow$  superfluid) or Ising-like models.

**Tasks:**

- Derive critical point using mean-field
- Compute excitation spectrum
- Compare to finite-size numerics.

**Deliverables:** Analytical derivations, phase diagrams, small-scale simulations.

## Summary Table

| Project                      | Focus                   | Difficulty | Coding Effort | Balance                   | Best For                                   |
|------------------------------|-------------------------|------------|---------------|---------------------------|--------------------------------------------|
| 1. Geometric Ensembles       | Ensembles via geometry  | ★☆☆        | Low           | Math-heavy                | Foundation builders                        |
| 2. Grav. Thermodynamics      | Gravity in stat mech    | ★★☆        | None          | Analytical                | Relativity/astro focus                     |
| 3. Black Hole Thermodynamics | BH entropy & T          | ★★★        | None          | Conceptual-heavy          | Ambitious theorists                        |
| 4. Langevin Dynamics         | Random processes        | ★★☆        | Medium        | Balanced                  | Coding + math students                     |
| 5. Fluctuation–Dissipation   | FDT testing             | ★★☆        | Medium        | Balanced                  | Data/analysis oriented                     |
| 6. Self-Gravitating Systems  | Long-range interactions | ★★★        | Optional      | Math-heavy                | Stat mech + astrophysics                   |
| 7. Virial Expansion & EOS    | Equations of state      | ★★☆        | Low–Medium    | Analytical + short coding | Students interested in gases & real fluids |
| 8. Green–Kubo Transport      | Transport coefficients  | ★★☆        | Medium–High   | Theory + simulation       | Students keen on MD and                    |

|                                                 |                                    |     |             |                                 |                                                                  |
|-------------------------------------------------|------------------------------------|-----|-------------|---------------------------------|------------------------------------------------------------------|
| <b>9. Boltzmann &amp; Chapman–Enskog</b>        | Kinetic theory transport           | ★★☆ | Low–Medium  | Derivation-heavy                | statistical dynamics<br>Students who like bridging micro ↔ macro |
| <b>10. Critical EOS &amp; Phase Transitions</b> | EOS near critical points           | ★★☆ | Low–Medium  | Analytical + plotting           | Students interested in phase transitions                         |
| <b>11. Cosmological Phase Transitions</b>       | Early universe defects             | ★★★ | Low–Medium  | Theory-heavy, optional coding   | Students interested in early-universe cosmology                  |
| <b>12. Dark Energy &amp; EOS</b>                | Alt. dark energy models            | ★★☆ | Low         | Analytical + plotting           | Students interested in cosmology beyond $\Lambda$ CDM            |
| <b>13. CMB Fluctuations</b>                     | Statistical mechanics of CMB       | ★★★ | Medium      | Statistical physics + cosmology | Students keen on data/statistical analysis                       |
| <b>14. BEC &amp; Finite-Size Effects</b>        | BEC beyond thermodynamic limit     | ★★☆ | Low–Medium  | Analytical + numerics           | Students interested in ultracold atoms                           |
| <b>15. Superfluid Vortices</b>                  | Vortex dynamics in BEC/superfluids | ★★★ | Medium–High | Analytical + coding             | Students interested in fluid dynamics/quantum coherence          |
| <b>16. Quantum Phase Transitions</b>            | Bose–Hubbard or Ising criticality  | ★★★ | Medium      | Mean-field + numerics           | Students interested in condensed matter/criticality              |

The list spans:

- **Foundations & geometry** (Proj. 1–2)
- **Frontier/statistical gravitation** (Proj. 3, 6)
- **Random processes & fluctuations** (Proj. 4–5)
- **Equations of state & transport coefficients** (Proj. 7–10)
- **Astro/Cosmology** (Proj. 11–13)
- **Condensed Matter (inc BEC)** (Proj 14–16)