4th Series of Exercises

Energy balances of nuclear decays and reactions

Use the table of atomic mass excesses at the end of the series, $\Delta = [M_{\text{atom}} - A\,u]c^2$. Because atomic masses include the electrons, the Q-values are

$$Q_{\beta^-} = \Delta_P - \Delta_D, \qquad Q_\alpha = \Delta_P - \Delta_D - \Delta(^4\text{He}), \qquad Q_{\beta^+} = \Delta_P - \Delta_D - 2m_e c^2, \qquad Q_{\text{EC}} = \Delta_P - \Delta_D,$$

and, for a reaction, $Q = \sum \Delta(\text{initial}) - \sum \Delta(\text{final})$. Use $m_e c^2 = 0.511\,\text{MeV}$.

Problem 1

a) Using a chart of nuclides (decay modes), complete the following decays: I) $^{103}\text{Ru} \rightarrow$ _____ II) $^{215}\text{Fr} \rightarrow$ _____ III) $^{191}\text{Os} \rightarrow$ _____ IV) $^{209}\text{Po} \rightarrow$ _____

b) Calculate the energy released in each of these decays.

Problem 2

a) State, justifying your answer, the type of each of the following decays: I) $^{82}\text{Br} \rightarrow {}^{82}\text{Kr}$ II) $^{212}\text{Rn} \rightarrow {}^{208}\text{Po}$ III) $^{233}\text{Pa} \rightarrow {}^{233}\text{U}$

b) Calculate the energy released in each of these decays.

Problem 3 The radioactive isotopes carbon-11, nitrogen-13, oxygen-15 and fluorine-18 are used in PET (positron emission tomography) scans.

a) What type of decay is used in PET?

b) Calculate the energy released in this type of decay for each of these isotopes.

Problem 4 State, justifying your answer with calculations, exactly what type of decay $^{72}\text{Se} \rightarrow {}^{72}\text{As}$ is.

Problem 5 Potassium-40, present in the environment, can decay by electron capture to ^{40}Ar. The neutrino and the recoiling nucleus share an energy of $1.500\,8\,\text{MeV}$. Using the mass-excess table, estimate the binding energy of the innermost-shell (K-shell) electrons of potassium. Consider only the transition between ground states.

Problem 6

a) State, justifying your answer, the type of each of the following decays: I) $^{107}\text{Pd} \rightarrow {}^{107}\text{Ag}$ II) $^{146}\text{Sm} \rightarrow {}^{142}\text{Nd}$

b) Calculate the energy released in each of these decays.

Problem 7

a) State, justifying your answer, the type of each of the following decays: I) $^{166}\mathrm{Ho} \to {}^{166}\mathrm{Er}$ II) $^{150}\mathrm{Gd} \to {}^{146}\mathrm{Sm}$

b) Calculate the energy released in each of these decays.

Problem 8 Calculate the Q-value of the following reactions, stating whether they are exoergic or endoergic: I) $n + {}^{235}\mathrm{U} \to {}^{90}\mathrm{Rb} + {}^{143}\mathrm{Cs} + 3n$ II) ${}^{2}\mathrm{H} + {}^{3}\mathrm{H} \to \alpha + n$

Problem 9 Calculate the Q-value of the following reactions, stating whether they are exoergic or endoergic:

I) $^{37}\mathrm{Ar} + n \to {}^{34}\mathrm{S} + \alpha$ II) $^{7}\mathrm{Be} + n \to {}^{7}\mathrm{Li} + p$ III) $^{27}\mathrm{Al} + p \to {}^{27}\mathrm{Si} + n$
IV) $^{4}\mathrm{He} + {}^{9}\mathrm{Be} \to {}^{12}\mathrm{C} + n$ V) $^{14}\mathrm{N} + \alpha \to {}^{17}\mathrm{O} + p$ VI) $p + {}^{18}\mathrm{O} \to {}^{18}\mathrm{F} + n$

Problem 10 Radium-226 decays by α emission to radon-222.

a) Calculate the energy released in the decay.

b) Assuming the radium nucleus is at rest and the radon is left in its ground state, calculate the kinetic energies of the α particle and of the recoiling radon nucleus. (Non-relativistic kinematics; use mass numbers instead of masses.)

c) Why do α particles from a given decay have a single, well-defined energy, while β particles have a continuous spectrum?

Problem 11 Copper-64 ($t_{1/2} = 12.7\,\mathrm{h}$), used in medical imaging and therapy, is one of the few nuclides that decays in three different ways.

a) Write the three decays: β^-, β^+ and electron capture, identifying the daughters.

b) Calculate the Q-value of each.

c) Explain, using the semi-empirical mass formula, why an odd–odd nucleus such as $^{64}\mathrm{Cu}$ can decay to *both* of its even–even neighbours.

Problem 12 Potassium-40 ($t_{1/2} = 1.25 \times 10^9\,\mathrm{yr}$) is another odd–odd nucleus with competing decays: $^{40}\mathrm{K} \to {}^{40}\mathrm{Ca}$ by β^- (89.3%) and $^{40}\mathrm{K} \to {}^{40}\mathrm{Ar}$ by electron capture (10.7%; a β^+ branch of only 0.001% also exists).

a) Calculate Q_{β^-}, Q_{EC} and Q_{β^+}.

b) Calculate the partial decay constants of the β^- and EC branches, and the corresponding partial half-lives.

c) The human body contains about 140 g of potassium, of which 0.0117% is $^{40}\mathrm{K}$. Calculate the $^{40}\mathrm{K}$ activity of the body.

Problem 13 Rutherford's reaction $^{14}\mathrm{N}(\alpha,p)^{17}\mathrm{O}$ is endoergic (Problem 9).

a) Show that, for a reaction $a + X \to Y + b$ with $Q < 0$ and target X at rest, the minimum (threshold) kinetic energy of the projectile is, non-relativistically,

$$T_{\mathrm{th}} = -Q\left(1 + \frac{m_a}{m_X}\right).$$

b) Calculate the threshold energy for Rutherford's reaction. Could the 7.7 MeV α particles from ^{214}Po used by Rutherford induce it?

c) Why is T_{th} larger than $|Q|$?

Problem 14 Tritium (^{3}H, $t_{1/2} = 12.3\,\mathrm{yr}$) decays by β^- to ^{3}He.

a) Calculate the energy released.

b) What is the maximum kinetic energy of the emitted electron? (Neglect the recoil of the ^{3}He nucleus.)

c) The KATRIN experiment studies the shape of this spectrum very close to its end point. Why does this tell us about the neutrino mass, and why is tritium a good choice?

Problem 15 Compare the energy released per unit mass of fuel in

a) the fission reaction of Problem 8 (I) and

b) the D–T fusion reaction of Problem 8 (II).

c) In D–T fusion, how is the energy shared between the α particle and the neutron? (Neglect the initial kinetic energies.)

Problem 16 Lithium-6 is used in fusion reactor blankets to breed tritium, and boron-10 is used in neutron detectors and control rods. Both reactions are induced by thermal neutrons:

$$\text{I)}\ \ n + {}^6\mathrm{Li} \to {}^3\mathrm{H} + \alpha \qquad\qquad \text{II)}\ \ n + {}^{10}\mathrm{B} \to {}^7\mathrm{Li} + \alpha$$

a) Calculate the Q-value of each reaction.

b) For reaction II, calculate the kinetic energies of the α particle and of the ^{7}Li nucleus when ^{7}Li is left (i) in its ground state; (ii) in its first excited state, at 0.478 MeV (94% of the cases). Neglect the momentum of the thermal neutron.

c) Why are these reactions so useful for *detecting* slow neutrons?

Data: atomic mass excesses (AME2020)

$\Delta = [M_{\mathrm{atom}} - A\,u]c^2$, in MeV. $\quad u = 931.494\,\mathrm{MeV/c^2}$, $\ m_e c^2 = 0.511\,\mathrm{MeV}$, $\ N_A = 6.022 \times 10^{23}\,\mathrm{mol^{-1}}$.

Nuclide	Δ	Nuclide	Δ	Nuclide	Δ	Nuclide	Δ
n	8.0713	^{14}N	2.8634	^{64}Cu	−65.4242	^{166}Ho	−63.0703
^{1}H	7.2890	^{15}N	0.1014	^{64}Ni	−67.0990	^{166}Er	−64.9251
^{2}H	13.1357	^{15}O	2.8556	^{64}Zn	−66.0036	^{191}Os	−36.3952
^{3}H	14.9498	^{17}O	−0.8088	^{72}Se	−67.8682	^{191}Ir	−36.7102
^{3}He	14.9312	^{18}O	−0.7828	^{72}As	−68.2301	^{205}Pb	−23.7701
^{4}He	2.4249	^{18}F	0.8731	^{82}Br	−77.4986	^{208}Po	−17.4692
^{6}Li	14.0869	^{27}Al	−17.1968	^{82}Kr	−80.5895	^{209}Po	−16.3660
^{7}Li	14.9071	^{27}Si	−12.3845	^{90}Rb	−79.3655	^{211}At	−11.6472
^{7}Be	15.7690	^{34}S	−29.9317	^{103}Ru	−87.2672	^{212}Rn	−8.6592
^{9}Be	11.3484	^{37}Ar	−30.9477	^{103}Rh	−88.0262	^{215}Fr	0.3186
^{10}B	12.0506	^{40}Ar	−35.0399	^{107}Pd	−88.3726	^{222}Rn	16.3736
^{11}B	8.6677	^{40}K	−33.5355	^{107}Ag	−88.4067	^{226}Ra	23.6691
^{11}C	10.6494	^{40}Ca	−34.8464	^{142}Nd	−85.9490	^{233}Pa	37.4895
^{12}C	0			^{143}Cs	−67.6758	^{233}U	36.9191
^{13}C	3.1250			^{146}Sm	−80.9962	^{235}U	40.9188
^{13}N	5.3455			^{150}Gd	−75.7640		

Answers

1. a) $^{103}\mathrm{Ru}\xrightarrow{\beta^-}{}^{103}\mathrm{Rh}$; $^{215}\mathrm{Fr}\xrightarrow{\alpha}{}^{211}\mathrm{At}$; $^{191}\mathrm{Os}\xrightarrow{\beta^-}{}^{191}\mathrm{Ir}$; $^{209}\mathrm{Po}\xrightarrow{\alpha}{}^{205}\mathrm{Pb}$. b) 0.759 MeV; 9.541 MeV; 0.315 MeV; 4.979 MeV.

2. a) β^-; α; β^-. b) 3.091 MeV; 6.385 MeV; 0.570 MeV.

3. a) β^+. b) ^{11}C: 0.960 MeV; ^{13}N: 1.199 MeV; ^{15}O: 1.732 MeV; ^{18}F: 0.634 MeV.

4. Electron capture only: $Q_{\mathrm{EC}} = 0.362\,\mathrm{MeV} < 2m_ec^2$, so β^+ is forbidden ($Q_{\beta^+} = -0.660\,\mathrm{MeV}$).

5. $Q_{\mathrm{EC}} = 1.504\,4\,\mathrm{MeV}$; $B_K \approx Q_{\mathrm{EC}} - 1.5008 \approx 3.6\,\mathrm{keV}$.

6. a) β^-; α. b) 0.034 MeV; 2.528 MeV.

7. a) β^-; α. b) 1.855 MeV; 2.807 MeV.

8. I) $Q = 171.8\,\mathrm{MeV}$, exoergic (fission). II) $Q = 17.59\,\mathrm{MeV}$, exoergic (D–T fusion).

9. I) +4.630; II) +1.644; III) −5.595; IV) +5.702; V) −1.192; VI) −2.438 (all in MeV; positive = exoergic, negative = endoergic).

10. a) $Q = 4.871\,\mathrm{MeV}$. b) $T_\alpha = 4.784\,\mathrm{MeV}$, $T_{\mathrm{Rn}} = 86\,\mathrm{keV}$. c) Two-body vs three-body final state.

11. a) $^{64}\mathrm{Cu}\xrightarrow{\beta^-}{}^{64}\mathrm{Zn}$; $^{64}\mathrm{Cu}\xrightarrow{\beta^+,\,\mathrm{EC}}{}^{64}\mathrm{Ni}$. b) $Q_{\beta^-} = 0.579\,\mathrm{MeV}$; $Q_{\mathrm{EC}} = 1.675\,\mathrm{MeV}$; $Q_{\beta^+} = 0.653\,\mathrm{MeV}$. c) Pairing term.

12. a) 1.311 MeV; 1.504 MeV; 0.482 MeV. b) $\lambda_{\beta^-} = 4.95 \times 10^{-10}\,\mathrm{yr}^{-1}$ ($t_{1/2} = 1.40 \times 10^{9}\,\mathrm{yr}$); $\lambda_{\mathrm{EC}} = 5.93 \times 10^{-11}\,\mathrm{yr}^{-1}$ ($t_{1/2} = 1.17 \times 10^{10}\,\mathrm{yr}$). c) $\approx 4.3\,\mathrm{kBq}$.

13. b) $T_{\mathrm{th}} = 1.53\,\mathrm{MeV}$; yes. c) Momentum conservation: the centre-of-mass motion cannot be converted.

14. a) $Q = 18.6\,\mathrm{keV}$. b) $T_{e,\max} \approx 18.6\,\mathrm{keV}$. c) End point shifted by $m_\nu c^2$; low Q, simple atoms, short half-life.

15. a) 0.728 MeV/u ($7.0 \times 10^{13}\,\mathrm{J/kg}$). b) 3.52 MeV/u ($3.4 \times 10^{14}\,\mathrm{J/kg}$); about 4.8 times more. c) $T_n = 14.1\,\mathrm{MeV}$, $T_\alpha = 3.5\,\mathrm{MeV}$.

16. a) I) 4.78 MeV; II) 2.79 MeV. b) (i) $T_\alpha = 1.78\,\mathrm{MeV}$, $T_{\mathrm{Li}} = 1.01\,\mathrm{MeV}$; (ii) $T_\alpha = 1.47\,\mathrm{MeV}$, $T_{\mathrm{Li}} = 0.84\,\mathrm{MeV}$. c) Large Q and cross section, charged products.