# 5th Series of Exercises

Cross sections, reaction rates and activation

A target with $N$ nuclei in a uniform flux $\Phi$ (particles $\mathrm{cm^{-2}\,s^{-1}}$) reacts at the rate $R = N\sigma\Phi$ (thin target). If the product is radioactive, with decay constant $\lambda = \ln 2/t_{1/2}$, its activity during an irradiation that starts at $t = 0$ is $\mathcal{A}(t) = R\,(1 - e^{-\lambda t})$, and it decays as $e^{-\lambda t'}$ after the irradiation ends. $N_A = 6.023 \times 10^{23}\,\mathrm{mol^{-1}}$, $1\,\mathrm{b} = 10^{-24}\,\mathrm{cm^2}$, $1\,\mathrm{Ci} = 3.7 \times 10^{10}\,\mathrm{Bq}$.

**Problem 1** Silver ($M = 107.9\,\mathrm{g/mol}$, $\rho = 10.49\,\mathrm{g/cm^3}$) is made up of two stable isotopes. $^{107}$Ag makes up 51.8% of natural silver. A small silver foil, of area $2\,\mathrm{cm^2}$ and thickness 1 mm, is bombarded for half an hour with a thermal neutron beam of flux $8 \times 10^9\,\mathrm{neutrons\,cm^{-2}\,s^{-1}}$, producing $^{108}$Ag. This reaction has a cross section of 37.6 b. $^{108}$Ag is unstable, with $t_{1/2} = 2.37\,\mathrm{min}$.

a) What is the maximum activity of $^{108}$Ag reached in this experiment?

b) What is the activity of that isotope 10 minutes after the irradiation ends?

c) The other silver isotope, $^{109}$Ag (48.2%), absorbs neutrons with a cross section of 86.3 b, producing unstable $^{110}$Ag, with $t_{1/2} = 24.6\,\mathrm{s}$. In the same experiment, what is the maximum activity of $^{110}$Ag?

d) How many $^{110}$Ag nuclei were produced during the irradiation?

**Problem 2** A very thin gold foil ($^{197}$Au, $M = 196.97\,\mathrm{g/mol}$, $\rho = 19.3\,\mathrm{g/cm^3}$), with an area of $4\,\mathrm{cm^2}$ and a thickness of 0.1 mm, is bombarded with thermal neutrons, with a flux of $3 \times 10^{10}\,\mathrm{neutrons\,cm^{-2}\,s^{-1}}$, for 15 hours. The $^{198}$Au produced decays by $\beta^-$ with $t_{1/2} = 2.7\,\mathrm{d}$. The maximum activity measured (at the end of the irradiation) is 27.7 mCi.

a) What is the activity 7 days after the end of the irradiation?

b) What is the thermal neutron capture cross section of gold?

c) What would be the maximum activity of an identical indium foil ($M = 114.8\,\mathrm{g/mol}$, $\rho = 7.31\,\mathrm{g/cm^3}$; the product decays by $\beta^-$ with $t_{1/2} = 54\,\mathrm{min}$), for the same flux and irradiation time, given that the thermal neutron cross section of indium is 160 b?

**Problem 3** An indium foil ($\rho = 7.31\,\mathrm{g/cm^3}$, $M = 114.8\,\mathrm{g/mol}$), of area $3.0\,\mathrm{cm^2}$ and thickness 2.0 mm, is bombarded for 8 hours with thermal neutrons, with a flux of $2 \times 10^7\,\mathrm{neutrons\,cm^{-2}\,s^{-1}}$. The isotope $^{115}$In (95.7%) absorbs a neutron and forms $^{116}$In, which decays by $\beta^-$ with $t_{1/2} = 54\,\mathrm{min}$. Measurements only start 3 hours after the end of the irradiation, when the activity is 0.191 27 mCi.

a) What was the maximum activity reached in this experiment?

b) What is the cross section of the process $^{115}\mathrm{In} + n \to {}^{116}\mathrm{In}$?

c) How many electrons are emitted per second two hours after the end of the irradiation?

d) With a longer irradiation, what is the maximum activity that could be reached in this experiment?

**Problem 4** Gold was used at the ITN research reactor (Sacavém) to measure the neutron flux. In one experiment, a gold foil ($\rho = 19.3\,\mathrm{g/cm^3}$, $M = 196.97\,\mathrm{g/mol}$), of area $5.0\,\mathrm{cm^2}$ and thickness 1.0 mm, is bombarded for 14 hours by thermal neutrons with an unknown flux. In this reaction, with a cross section of 98.65 b, the isotope $^{198}$Au is formed; it decays by $\beta^-$ to $^{198}$Hg, with $t_{1/2} = 2.695\,\mathrm{d}$. Five hours after the end of the irradiation, the activity is 51.909 8 mCi.

a) What was the maximum activity reached in this experiment?

b) What is the thermal neutron flux of the reactor?

c) How many $^{198}$Au nuclei were formed during the irradiation?

d) How many $^{198}$Au nuclei are present at the end of the irradiation? Explain the difference from the previous result.

e) How long would the irradiation need to be to reach 99% of the maximum possible activity?

**Problem 5** Mercury ($M = 200.59\,\mathrm{g/mol}$, $\rho = 13.53\,\mathrm{g/cm^3}$) has several stable isotopes. The most abundant, $^{202}$Hg, makes up about 30% of natural mercury. A small container with 5 mL of mercury is bombarded with a thermal neutron beam of flux $7 \times 10^9$ neutrons $\mathrm{cm^{-2}\,s^{-1}}$ for 12 hours, producing $^{203}$Hg, which decays by $\beta^-$ to $^{203}$Tl with $t_{1/2} = 46.61\,\mathrm{d}$. Seven hours after the end of the irradiation the measured activity is 412.635 8 µCi.

a) What was the maximum activity of $^{203}$Hg reached in this experiment?

b) Determine the thermal neutron capture cross section of $^{202}$Hg.

c) What irradiation time is needed to reach an activity of 1 mCi?

d) Another isotope, $^{201}$Hg (about 13% of natural mercury), has a neutron capture cross section of 7.8 b and produces stable $^{202}$Hg. How many $^{202}$Hg nuclei were produced this way during the irradiation?

**Problem 6** The PET isotope $^{18}$F ($t_{1/2} = 110\,\mathrm{min}$) is produced in cyclotrons via the reaction

$$p + {}^{18}\mathrm{O} \to {}^{18}\mathrm{F} + n,$$

with a cross section of 0.22 b. The target is water ($\rho = 1.0\,\mathrm{g/cm^3}$) enriched to 85% in $^{18}$O (so its molar mass is $\approx 19.7\,\mathrm{g/mol}$).

a) Determine the reaction rate $R$ for a target of volume $V = 2\,\mathrm{mL}$ and a proton flux $\Phi = 7 \times 10^{12}$ protons $\mathrm{cm^{-2}\,s^{-1}}$.

b) Show that, for an isotope produced at a constant rate $R$, the activity during production is $\mathcal{A}(t) = R\,(1 - e^{-\lambda t})$, where $\lambda$ is the decay constant.

c) When the $^{18}$F is processed for injection, about 30% of it is lost, and the synthesis of the radiopharmaceutical (fluorodeoxyglucose, FDG) takes about 45 minutes. How long must the target be irradiated to obtain an FDG batch with an activity of 30 GBq ready for administration?

d) What is the maximum activity that could be reached under these conditions, and the corresponding activity of the final product?

(To learn more: `https://www.ijedr.org/papers/IJEDR1704099.pdf`)

**Problem 7** Cobalt-60 ($t_{1/2} = 5.27\,\mathrm{yr}$), used in radiotherapy and for sterilisation, is produced by irradiating natural cobalt (100% $^{59}$Co, $M = 58.93\,\mathrm{g/mol}$) in a reactor: $^{59}\mathrm{Co}(n,\gamma)^{60}\mathrm{Co}$, $\sigma = 37.2\,\mathrm{b}$.

a) A 1.00 g cobalt pellet is irradiated in a flux of $1.0 \times 10^{13}\,\mathrm{neutrons\,cm^{-2}\,s^{-1}}$ for one year. What is its $^{60}$Co activity at the end, in Bq and in Ci?

b) What is the saturation activity? How long would the irradiation need to be to reach half of it?

c) Check that the depletion of $^{59}$Co during the irradiation can be neglected.

**Problem 8** Cadmium is used to absorb thermal neutrons (in shielding, control rods and to separate thermal from faster neutrons). Natural cadmium ($M = 112.41\,\mathrm{g/mol}$, $\rho = 8.65\,\mathrm{g/cm^3}$) has a thermal absorption cross section of 2 520 b per atom.

a) Calculate the macroscopic absorption cross section $\Sigma_a$ and the mean free path for absorption of a thermal neutron.

b) What thickness of cadmium reduces a thermal neutron beam to $10^{-3}$ of its intensity?

c) What fraction of the beam is transmitted by a 1 mm cadmium sheet? Why is 1 mm of cadmium nevertheless almost transparent to neutrons above $\approx 0.5\,\mathrm{eV}$?

**Problem 9** *Neutron activation analysis.* A sample containing an unknown amount of sodium (100% $^{23}$Na, $M = 22.99\,\mathrm{g/mol}$) is irradiated for 2 hours in a thermal flux of $1.0\times10^{12}\,\mathrm{neutrons\,cm^{-2}\,s^{-1}}$. The reaction $^{23}\mathrm{Na}(n,\gamma)^{24}\mathrm{Na}$ has $\sigma = 0.53\,\mathrm{b}$, and $^{24}$Na decays with $t_{1/2} = 14.96\,\mathrm{h}$. One hour after the end of the irradiation, the $^{24}$Na activity is $5.0 \times 10^4\,\mathrm{Bq}$.

a) Calculate the production rate $R$.

b) What mass of sodium is in the sample?

c) Why is the method so sensitive? Give one advantage over chemical analysis.

**Problem 10** *How thin is a "thin" target?* The formula $R = N\sigma\Phi$ assumes the flux is the same throughout the target. In reality the beam is attenuated as it crosses it.

a) Show that, for a beam of $\Phi$ (per cm$^2$ per s) hitting a slab of area $S$ and thickness $x$, with $n$ nuclei per cm$^3$, the reaction rate is $R = \Phi S\,(1 - e^{-n\sigma x})$, and that it reduces to $N\sigma\Phi$ when $n\sigma x \ll 1$.

b) Calculate $n\sigma x$ for the 1.0 mm gold foil of Problem 4 and for the 0.1 mm foil of Problem 2 ($\sigma = 98.65\,\mathrm{b}$). By what factor does the thin-target formula overestimate the reaction rate in each case?

c) Correct the flux obtained in Problem 4.

## Answers

**1.** a) $1.824 \times 10^9\,\mathrm{Bq}$ b) $9.79 \times 10^7\,\mathrm{Bq}$ c) $3.897 \times 10^9\,\mathrm{Bq}$ d) $7.01 \times 10^{12}$ nuclei

**2.** a) $1.70 \times 10^8\,\mathrm{Bq}$ (4.6 mCi) b) 97.6 b c) $7.36 \times 10^9\,\mathrm{Bq}$ (0.20 Ci)

**3.** a) $7.13 \times 10^7\,\mathrm{Bq}$ b) 162 b c) $1.53 \times 10^7\,\mathrm{s^{-1}}$ d) $7.15 \times 10^7\,\mathrm{Bq}$

**4.** a) $2.026 \times 10^9\,\mathrm{Bq}$ b) $5.00 \times 10^9\,\mathrm{cm^{-2}\,s^{-1}}$ c) $7.33 \times 10^{14}$ d) $6.81 \times 10^{14}$ e) 17.9 d

**5.** a) $1.533 \times 10^7\,\mathrm{Bq}$ b) 4.85 b c) 29.1 h d) $6.23 \times 10^{13}$

**6.** a) $8.0 \times 10^{10}\,\mathrm{s}^{-1}$ c) 3.3 h d) $8.0 \times 10^{10}$ Bq (2.2 Ci); $4.2 \times 10^{10}$ Bq after losses and synthesis.

**7.** a) $4.7 \times 10^{11}$ Bq (12.7 Ci) b) $3.8 \times 10^{12}$ Bq (103 Ci); one half-life, 5.27 yr c) only 1.2% of the $^{59}$Co is used up.

**8.** a) $\Sigma_a = 117\,\mathrm{cm}^{-1}$, $\lambda = 86\,\mu\mathrm{m}$ b) 0.59 mm c) $8.5 \times 10^{-6}$; the cross section drops sharply above the 0.18 eV resonance ("cadmium cut-off").

**9.** a) $5.9 \times 10^{5}\,\mathrm{s}^{-1}$ b) 43 µg c) Counting single decays; non-destructive, multi-element.

**10.** b) 1 mm: $n\sigma x = 0.58$, the true rate is 0.76 of $N\sigma\Phi$; 0.1 mm: 0.058, factor 0.97. c) $\Phi \approx 6.6 \times 10^{9}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$.