6th Series of Exercises

Neutrons, moderation and nuclear fission

Useful data: $m_n c^2 = 939.57\,\text{MeV}$, $m_n = 1.675 \times 10^{-27}\,\text{kg}$, $k = 8.617 \times 10^{-5}\,\text{eV/K}$, $h = 6.626 \times 10^{-34}\,\text{J s}$, $N_A = 6.022 \times 10^{23}\,\text{mol}^{-1}$, $1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$, $1\,\text{b} = 10^{-24}\,\text{cm}^2$. Mass excesses and nuclear data are given at the end of the series.

Problem 1 *Thermal neutrons.*

a) Calculate the most probable kinetic energy ($E = kT$), the speed and the de Broglie wavelength of a thermal neutron at 20 °C.

b) Repeat for neutrons in thermal equilibrium with the water of a pressurised-water reactor, at 300 °C.

c) Compare the wavelengths with the distance between atoms in a crystal ($\sim$ 2–3 Å). What does this make possible?

Problem 2 *Elastic collisions and moderation.* A neutron collides elastically with a nucleus of mass number A at rest.

a) Calculate the minimum ratio E_f/E_i, $\alpha = \left(\frac{A-1}{A+1}\right)^2$, and the average logarithmic energy decrement ξ for hydrogen, deuterium, beryllium ($A = 9$), carbon ($A = 12$) and uranium ($A = 238$).

b) How many collisions are needed, on average, to bring a 2 MeV fission neutron down to 0.025 eV in each material?

c) A 2.0 MeV neutron collides head-on with a carbon nucleus. What is its energy after the collision? And after a glancing collision?

d) Why is uranium itself a useless moderator, and why does it matter that the neutrons slow down *outside* the fuel?

Problem 3 *Mean free paths.*

a) Calculate the scattering mean free path of thermal neutrons in graphite ($\rho = 1.70\,\text{g/cm}^3$, $\sigma_s = 4.7\,\text{b}$).

b) Calculate it in heavy water ($\rho = 1.105\,\text{g/cm}^3$, $M = 20.03\,\text{g/mol}$, $\sigma_s(\text{D}) = 3.4\,\text{b}$, $\sigma_s(\text{O}) = 3.8\,\text{b}$).

c) In light water, $\Sigma_s \approx 1.46\,\text{cm}^{-1}$ (see the lecture), but the absorption cross section of hydrogen is 0.332 b (that of oxygen is negligible). Calculate Σ_a and the mean free path for absorption. What fraction of a thermal beam crosses 1 cm of water without any interaction?

Problem 4 *A neutron beam through a plutonium foil.* A thermal neutron beam of 10^6 neutrons/s crosses a ^{239}Pu foil of areal density $0.05\,\text{kg/m}^2$. The relevant cross sections are $\sigma_f = 742\,\text{b}$ (fission), $\sigma_\gamma = 270\,\text{b}$ (radiative capture) and $\sigma_{\text{el}} = 10\,\text{b}$ (elastic scattering).

a) What is the intensity of the beam after the foil?

b) How many fissions and how many captures occur per second?

c) What is the flux of scattered neutrons 2 m from the foil, outside the beam, assuming isotropic scattering?

d) What power is released in the foil by fission (200 MeV per fission)?

Problem 5 *Fissile or not?* For each of the following targets, calculate the energy Q released when a neutron at rest is captured, and compare it with the activation energy E_a of the compound nucleus (data table at the end). Decide whether the nucleus is fissile with thermal neutrons and relate your result to the number of neutrons N of the target.

I) ^{233}U II) ^{240}Pu III) ^{241}Pu IV) ^{232}Th

Problem 6 *Energy of a fission event.* Consider again the fission $n + {}^{235}\mathrm{U} \to {}^{90}\mathrm{Rb} + {}^{143}\mathrm{Cs} + 3n$ ($Q = 171.8\,\mathrm{MeV}$, 4th series).

a) Estimate the energy released in a fission of ^{235}U from the binding energy per nucleon: $B/A \approx 7.6\,\mathrm{MeV}$ for uranium and $\approx 8.5\,\mathrm{MeV}$ for the fragments.

b) Assume the two fragments share 168 MeV of kinetic energy and the neutrons carry negligible momentum. Calculate the kinetic energy of each fragment, and its speed as a fraction of c.

c) Where does the remaining energy ($\approx 30\,\mathrm{MeV}$ of the usual 200 MeV) go?

Problem 7 *The Portuguese research reactor.* The RPI research reactor at Sacavém (1961–2016) operated at a thermal power of 1 MW.

a) How many fissions per second occurred at full power? (200 MeV per fission.)

b) What mass of ^{235}U was fissioned per day, and per year of continuous operation?

c) Taking into account that, of the thermal neutrons absorbed in ^{235}U, a fraction $\sigma_\gamma/(\sigma_f + \sigma_\gamma)$ are captured without fission ($\sigma_f = 584\,\mathrm{b}$, $\sigma_\gamma = 99\,\mathrm{b}$), how much ^{235}U was *consumed* per year?

Problem 8 *Reactor kinetics.* In a thermal reactor the prompt-neutron generation time is $\tau \approx 10^{-4}\,\mathrm{s}$; a fraction $\beta = 0.65\%$ of the fission neutrons of ^{235}U are delayed, with mean delay $\bar{t} \approx 12.5\,\mathrm{s}$.

a) If only prompt neutrons existed, by what factor would the neutron population (and the power) grow in one second with $K = 1.001$?

b) Using the effective generation time $\tau_{\mathrm{eff}} \approx \tau + \beta\bar{t}$, by what factor does it actually grow in one second? How long does it take to double?

c) What happens if K reaches 1.007? Estimate the time scale of the power growth in this case.

Problem 9 *Spontaneous fission.* ^{238}U decays by α emission ($t_{1/2} = 4.47 \times 10^9\,\mathrm{yr}$) and, very rarely, by spontaneous fission ($t_{1/2}^{\mathrm{SF}} \approx 8.2 \times 10^{15}\,\mathrm{yr}$), emitting on average 2.0 neutrons per fission.

a) For 1 kg of ^{238}U, calculate the α activity, the number of spontaneous fissions per second and the number of neutrons emitted per second.

b) What is the branching fraction for spontaneous fission?

c) Why is spontaneous fission important for starting up a reactor — and a problem for plutonium weapons containing ^{240}Pu?

Problem 10 *Critical mass.*

a) The critical mass of a bare sphere of ^{235}U metal ($\rho = 18.7\,\mathrm{g/cm^3}$) is about 52 kg. What is its radius?

b) The critical radius is proportional to the neutron mean free path, which is inversely proportional to the density. Show that the critical mass scales as $M_c \propto 1/\rho^2$.

c) By what factor does the critical mass change if the metal is compressed to twice its normal density? What is the new critical mass?

d) Why does a natural-uranium lump never become critical with fast neutrons, however large?

Data

Atomic mass excesses (AME2020), $\Delta = [M_{\mathrm{atom}} - A\,u]c^2$ in MeV:

Nuclide	Δ	Nuclide	Δ
n	8.0713	^{235}U	40.9188
^{232}Th	35.4483	^{236}U	42.4446
^{233}Th	38.7336	^{239}Pu	48.5884
^{233}U	36.9194	^{240}Pu	50.1259
^{234}U	38.1459	^{241}Pu	52.9567
		^{242}Pu	54.7184

Activation energies of the compound nucleus ($A+1$), from K. S. Krane, *Introductory Nuclear Physics* (^{233}Th: approximate value):

Compound nucleus	E_a (MeV)
^{233}Th	≈ 6.5
^{234}U	6.5
^{236}U	6.2
^{241}Pu	6.3
^{242}Pu	6.0

Answers

1. a) 0.025 3 eV, 2 200 m/s, 1.80 Å b) 0.049 4 eV, 3 070 m/s, 1.29 Å c) Same order: neutron diffraction.

2. a) α: 0, 0.111, 0.640, 0.716, 0.983; ξ: 1, 0.725, 0.207, 0.158, 0.0084. b) 18, 25, 88, 115, $\approx$ 2200. c) 1.43 MeV; $\approx$ 2.0 MeV.

3. a) 2.5 cm b) 2.8 cm c) $\Sigma_a = 0.022\,\mathrm{cm^{-1}}$, $\lambda_a = 45$ cm; 23%.

4. a) $9.872 \times 10^5\,\mathrm{s^{-1}}$ (1.28% interact) b) 9.29×10^3 fissions/s, 3.38×10^3 captures/s c) $2.5\,\mathrm{m^{-2}\,s^{-1}}$ d) 3.0×10^{-7} W

5. I) $Q = 6.84$ MeV $> E_a = 6.5$ MeV: fissile. II) $Q = 5.24$ MeV < 6.3 MeV: not fissile. III) $Q = 6.31$ MeV > 6.0 MeV: fissile. IV) $Q = 4.79$ MeV < 6.5 MeV: not fissile (fertile). Odd-N targets are fissile.

6. a) $\approx$ 210 MeV b) $T(\mathrm{Rb}) = 103$ MeV ($v \approx 0.050c$), $T(\mathrm{Cs}) = 65$ MeV ($v \approx 0.031c$) c) Prompt neutrons and γ rays, and the β decays of the fragments (including antineutrinos).

7. a) $3.1 \times 10^{16}\,\mathrm{s^{-1}}$ b) 1.05 g/day; 384 g/yr c) 450 g/yr

8. a) $e^{10} \approx 2.2 \times 10^4$ b) $\tau_{\mathrm{eff}} \approx 0.081$ s; factor 1.012; doubling in $\approx$ 56 s c) Prompt critical: period $\approx$ 0.2 s.

9. a) $\mathcal{A}_\alpha = 1.24 \times 10^7$ Bq; 6.8 fissions/s; 14 neutrons/s. b) 5.5×10^{-7}. c) Supplies stray neutrons.

10. a) 8.7 cm b) $M_c \propto \rho R_c^3 \propto \rho\rho^{-3}$ c) Factor 1/4: 13 kg d) Too little ^{235}U; ^{238}U absorbs and slows the neutrons.