Course review

Kinematics

Material derivative

$$\frac{D(\ldots)}{Dt} = \frac{\partial(\ldots)}{\partial t} + \vec{u}\cdot\nabla(\ldots)$$

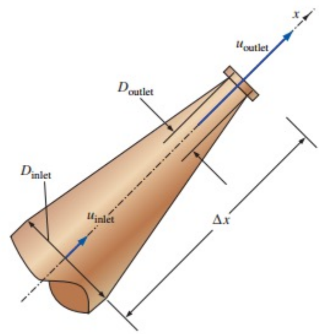
A material derivative is the time derivative of a property following a fluid.

Acceleration

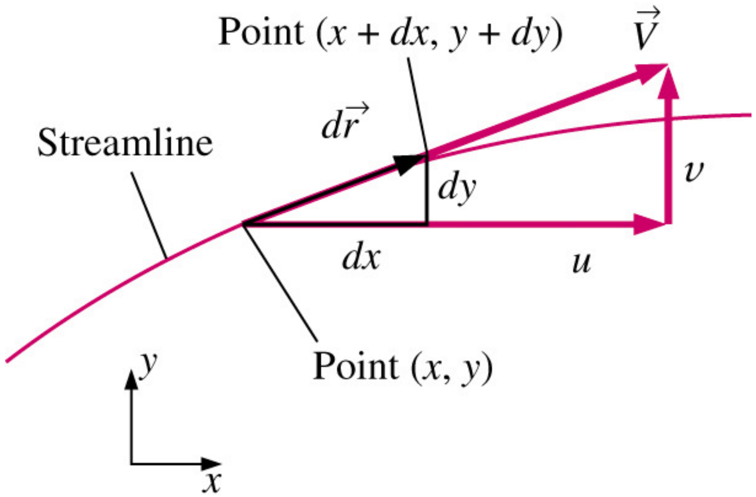
$$\vec{a} = \frac{D\vec{u}}{Dt} = \frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u}$$

$$\underbrace{\text{Steady state}}_{\Rightarrow \frac{\partial \vec{u}}{\partial t} = 0}$$

Steady state does not mean necessarily **a**=0. Ex.:

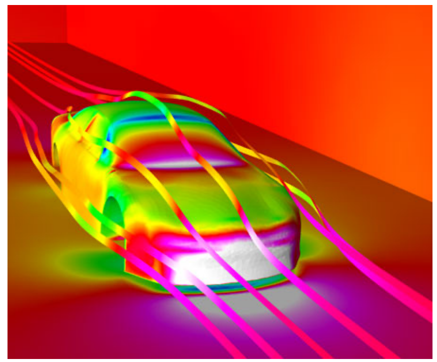


Streamline: is a curve that is everywhere tangent to the *instantaneous* local velocity vector.



$$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

NASCAR surface pressure contours and streamlines



Other ways to visualize the flow:

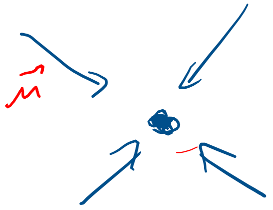
A **Pathline** is the actual path traveled by an individual fluid particle over some time period.

A **Streakline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.

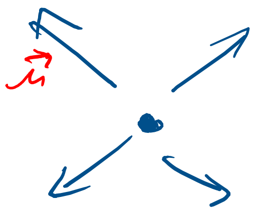
For **steady flow**, streamlines, pathlines, and streaklines are identical.

Continuity equation

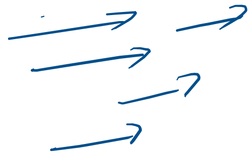
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \; , \; \text{if } \overbrace{\rho = C_{te}}^{\text{incompressivel}} \Rightarrow \nabla \cdot \vec{u} = 0$$



$\nabla \cdot \vec{u} < 0$



$\nabla \cdot \vec{u} > 0$

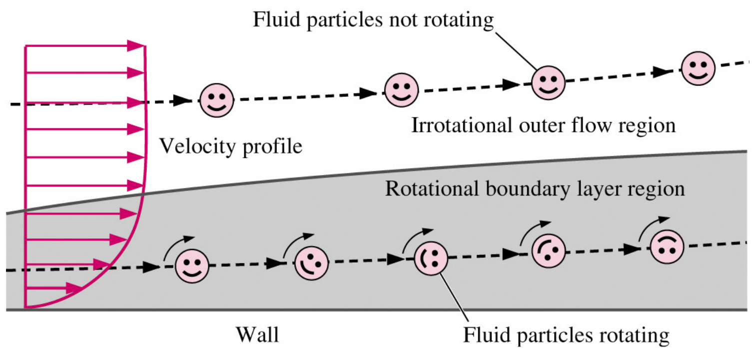


$\nabla \cdot \vec{u} = 0$

Vorticity

$$\vec{\omega} = \nabla \times \vec{u}$$

Boundary layer:



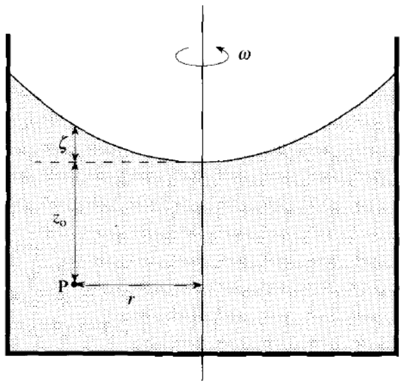
Euler equation: for incompressible and inviscid fluids.

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla p + \boldsymbol{g},$$

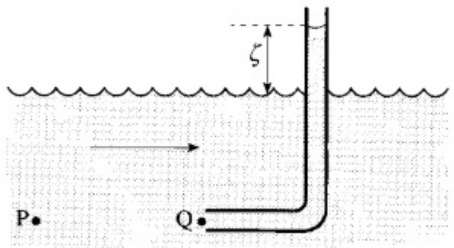
$$\frac{\partial \boldsymbol{u}}{\partial t} + \underbrace{(\nabla \wedge \boldsymbol{u})}_{=\vec{\omega}} \wedge \boldsymbol{u} = -\nabla \underbrace{\left(\frac{p}{\rho} + \tfrac{1}{2}\boldsymbol{u}^2 + \overset{gz}{\chi}\right)}_{= c_{te} \text{ if } \vec{\omega} = 0}$$

$$\rho \frac{\partial \vec{u}}{\partial t} = 0$$

$\vec{\omega} \neq 0$ (Euler)



$\vec{\omega} = 0$ (Bernoulli)



Potential flow. For irrotational flows in Euler fluids.

$$\vec{\omega} = \nabla \times \vec{u} = 0 \quad \Rightarrow \quad \vec{u} = \nabla \phi$$

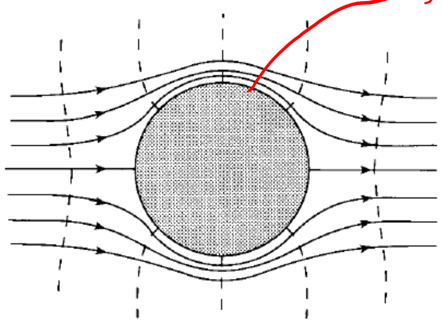
$$\nabla \cdot \vec{u} = 0 \quad \Rightarrow \nabla \cdot \nabla \phi = \nabla^2 \phi = 0$$

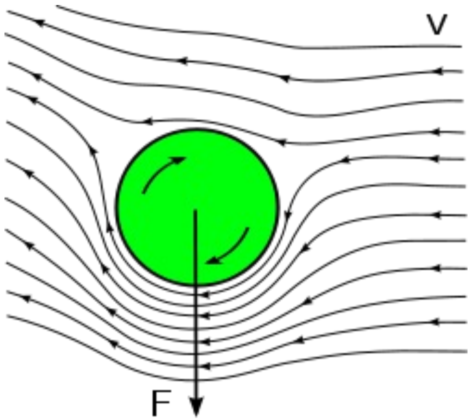
In this case, the pressure is given by the Bernoulli equation.

Kelvin circulation theorem: An ideal fluid that is vorticity free at a given instant is vorticity free at all times.

Flow around a sphere: the drag and lift forces are zero for an ideal fluid.

$u_r = 0$
$u_\theta \neq 0$





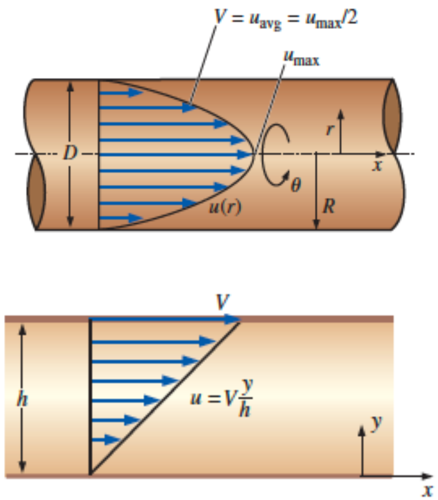
Navier-Stokes: incompressible viscous fluids.

Newtonian fluids, defined as fluids for which the shear stress is linearly proportional to the shear strain rate.

$$\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u} = -\frac{\nabla p}{\rho} + \vec{g} + \nu\nabla^2\vec{u}$$

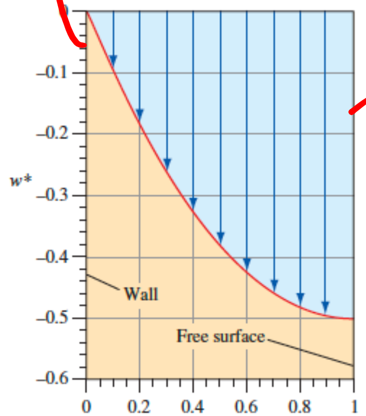
Boundary conditions. 1) no-slip: at the surface, the velocity of the liquid and solid are the same. 2) Interface BC: at the interface, the velocity and the shear-stress of the two fluid are the same. 3) Free surface BC: at the free surface, the shear stress is zero.

1)



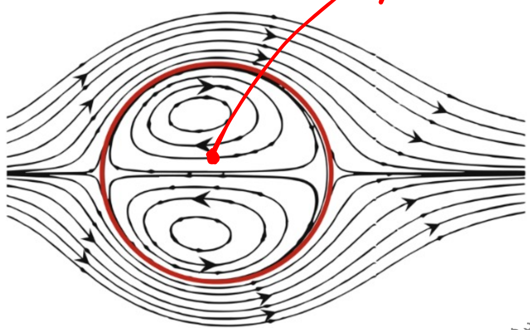
3) $\vec{u} = 0$

$\tau = 0$



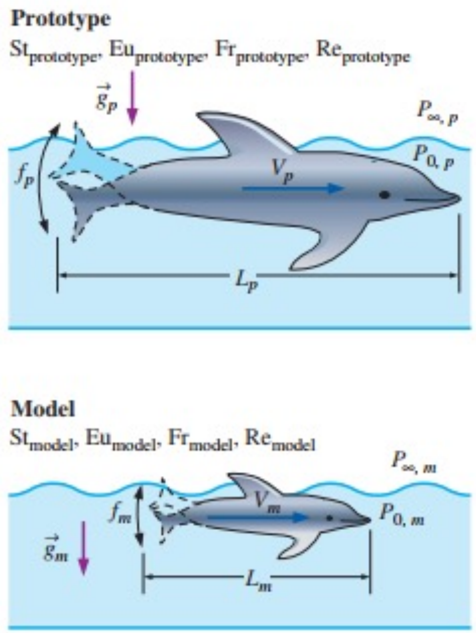
2) $f(r) = \frac{A}{r} + \dots$ $A=0$

$\vec{u}$: finite



Nondimensionalized Navier–Stokes:

$$[\mathrm{St}]\frac{\partial \vec{V}^*}{\partial t^*} + (\vec{V}^*\cdot\vec{\nabla}^*)\vec{V}^* = -[Eu]\vec{\nabla}^* P^* + \left[\frac{1}{\mathrm{Fr}^2}\right]\vec{g}^* + \left[\frac{1}{\mathrm{Re}}\right]\nabla^{*2}\vec{V}^*$$



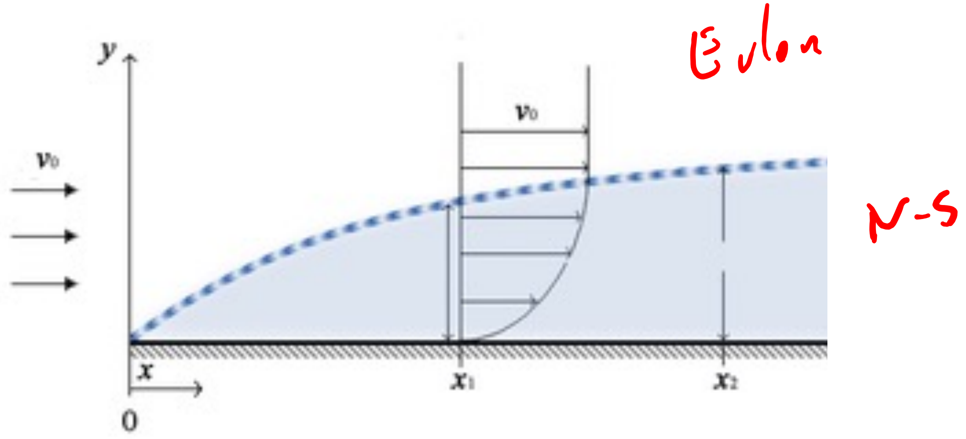
Since there are four dimensionless parameters, dynamic similarity between a model and a prototype requires all four of these to be the same for the model and the prototype ($\mathrm{St}_{\text{model}} = \mathrm{St}_{\text{prototype}}$, $\mathrm{Eu}_{\text{model}} = \mathrm{Eu}_{\text{prototype}}$, $\mathrm{Fr}_{\text{model}} = \mathrm{Fr}_{\text{prototype}}$, and $\mathrm{Re}_{\text{model}} = \mathrm{Re}_{\text{prototype}}$).

Approximate Navier–Stokes equation for creeping flow: $\vec{\nabla} P \cong \mu \nabla^2 \vec{V}$ $Re \ll 1!$

Drag force on a sphere in creeping flow: $F_D = 3\pi\mu V D$

Reversibility of the Stokes equation and the swimming at the microscale.

Boundary layer. Separates viscous and inviscid flows close to a solid surface.



Assumptions to obtain the BL equations

Boundary layer equations:

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \\ u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = U\dfrac{dU}{dx} + \nu\dfrac{\partial^2 u}{\partial y^2} \end{cases}$$

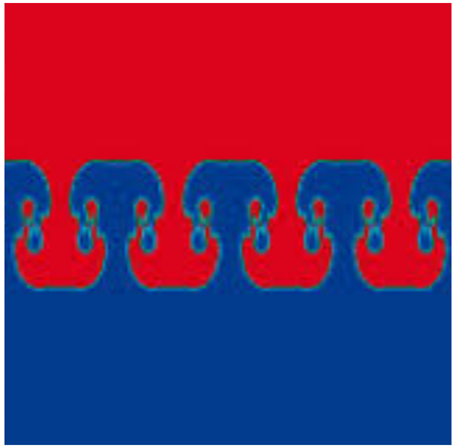
Boundary conditions in the flat plate problem.

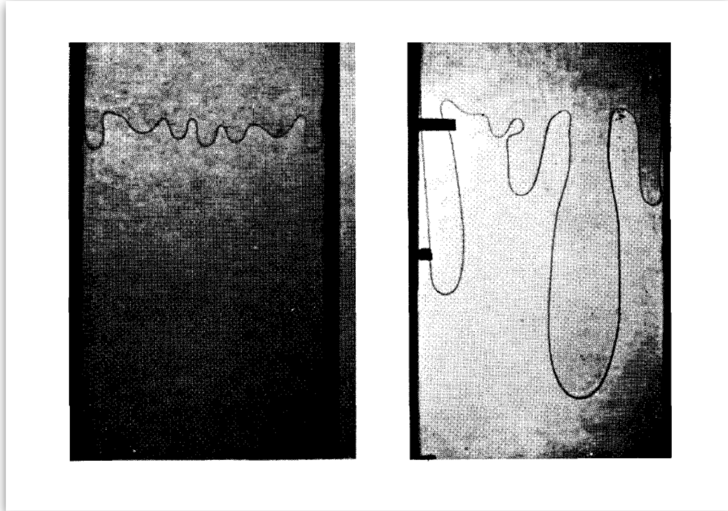
How to calculate the vorticity equation and its interpretation in simple cases

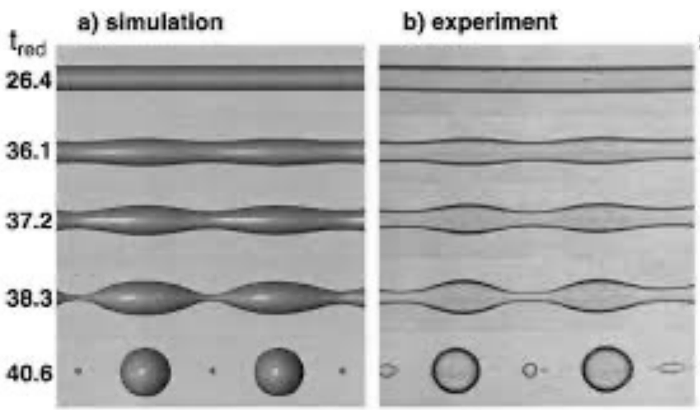
To find what difference viscosity makes, we need to repeat the above analysis using the Navier–Stokes equation as our starting point, rather than the Euler equation. The viscous term on the left-hand side of (6.25) is $-\eta\nabla \wedge \boldsymbol{\Omega}$, and the curl of this, since $\nabla\cdot\boldsymbol{\Omega} = 0$, is $\eta\nabla^2\boldsymbol{\Omega}$. Hence we now have

$$\frac{D\boldsymbol{\Omega}}{Dt} = (\boldsymbol{\Omega}\cdot\nabla)\boldsymbol{u} + \frac{\eta}{\rho}\nabla^2\boldsymbol{\Omega}. \qquad (7.3)$$

Instabilities







How to find the critical conditions for the instability (marginal instability) and which mode grows faster.

Why does the instabilities happen in each case? Ex.: physical mechanism in the Rayleigh-Taylor instability.

Exame tipo

Questão 1

Euler equation: for incompressible and inviscid fluids.

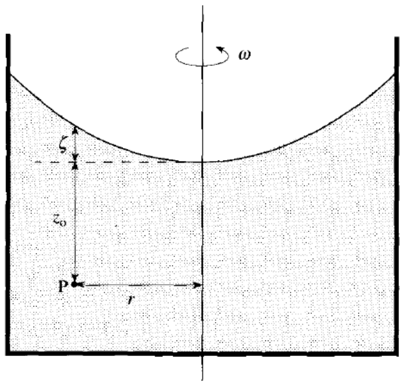
$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \boldsymbol{g},$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \underbrace{(\nabla \wedge \boldsymbol{u})}_{=\vec{\omega}} \wedge \boldsymbol{u} = -\nabla \underbrace{\left(\frac{p}{\rho} + \tfrac{1}{2}\boldsymbol{u}^2 + \overset{gz}{\chi}\right)}_{= C_{te} \text{ if } \vec{\omega} = 0}$$

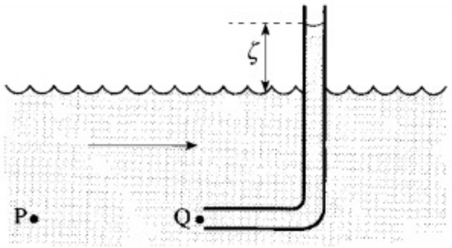
$$\rho \frac{\partial \vec{u}}{\partial t} = 0$$

$$\Rightarrow \vec{u} \cdot \nabla H = 0$$

$\vec{\omega} \neq 0$ (Euler)



$\vec{\omega} = 0$ (Bernoulli)



Questão 2

Uniform (free) stream

Pot. de escoamento

função corrente

Uniform stream: $u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} = V \qquad v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} = 0$

$$\phi = Vx + f(y) \quad \rightarrow \quad v = \frac{\partial \phi}{\partial y} = f'(y) = 0 \quad \rightarrow \quad f(y) = \text{constant}$$

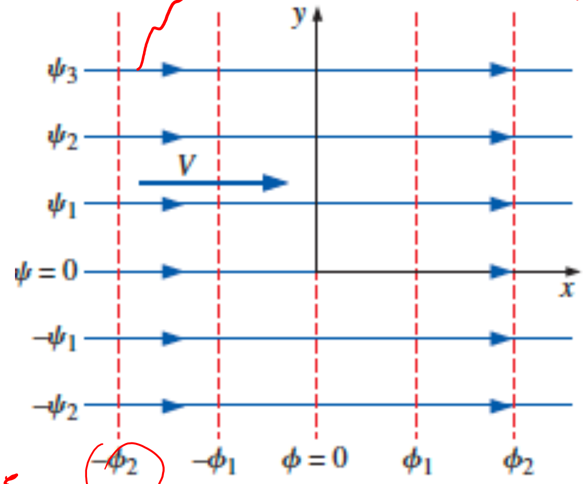
Velocity potential function for a uniform stream: $\phi = Vx = C_{te} \Rightarrow x = \frac{C_{te}}{V}$

Stream function for a uniform stream: $\psi = Vy = C_2 \Rightarrow y = \frac{C_2}{V}$

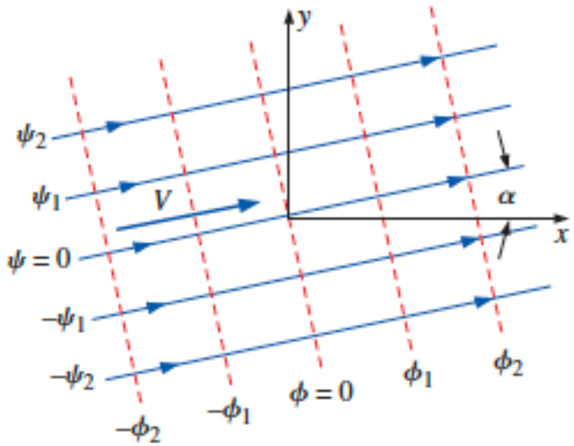
Uniform stream: $\phi = Vr\cos\theta \qquad \psi = Vr\sin\theta$

linha de corrente

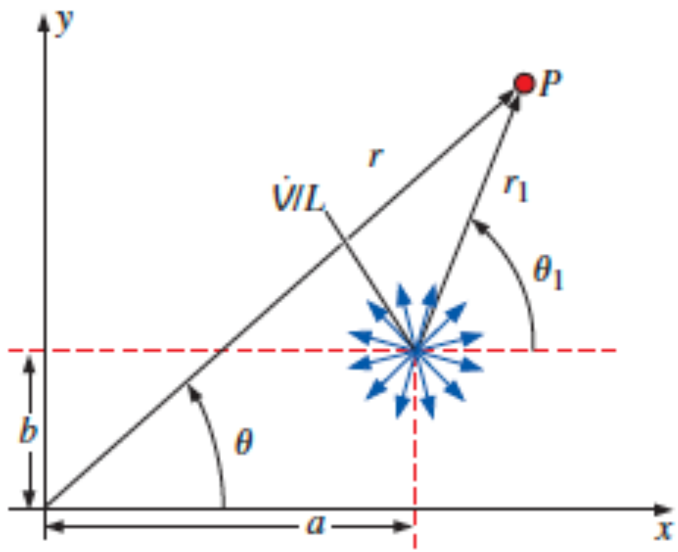
$\psi = C_{te}$



Uniform stream inclined at angle α: $\begin{cases} \phi = V(x\cos\alpha + y\sin\alpha) \\ \psi = V(y\cos\alpha - x\sin\alpha) \end{cases}$



Line source or sink at an arbitrary point



Line source at point (a, b):

$$\phi = \frac{\dot{V}/L}{2\pi} \ln r_1 = \frac{\dot{V}/L}{2\pi} \ln \sqrt{(x-a)^2 + (y-b)^2}$$

$$\psi = \frac{\dot{V}/L}{2\pi} \theta_1 = \frac{\dot{V}/L}{2\pi} \arctan \frac{y-b}{x-a}$$

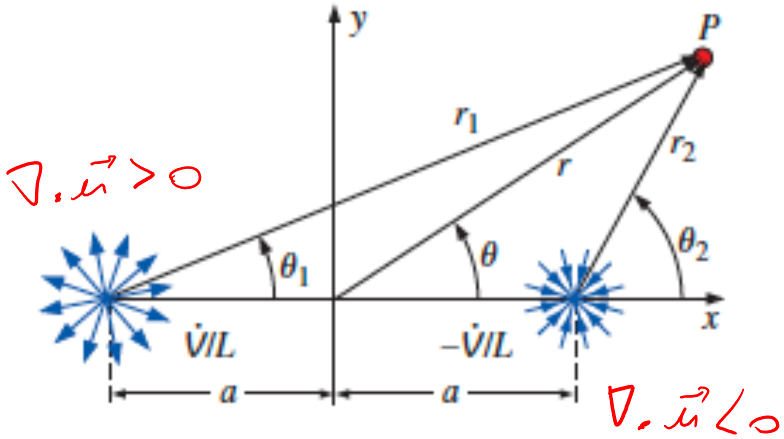
Superposition of a source and sink of equal strength

Line source at $(-a, 0)$: $\psi_1 = \frac{\dot{V}/L}{2\pi}\theta_1$ where $\theta_1 = \arctan\frac{y}{x+a}$

Similarly for the sink,

Line sink at $(a, 0)$: $\psi_2 = \frac{-\dot{V}/L}{2\pi}\theta_2$ where $\theta_2 = \arctan\frac{y}{x-a}$

Composite stream function: $\psi = \psi_1 + \psi_2 = \frac{\dot{V}/L}{2\pi}(\theta_1 - \theta_2)$



Final result, Cartesian coordinates: $\psi = \frac{-\dot{V}/L}{2\pi}\arctan\frac{2ay}{x^2+y^2-a^2}$

Final result, cylindrical coordinates: $\psi = \frac{-\dot{V}/L}{2\pi}\arctan\frac{2ar\sin\theta}{r^2-a^2}$

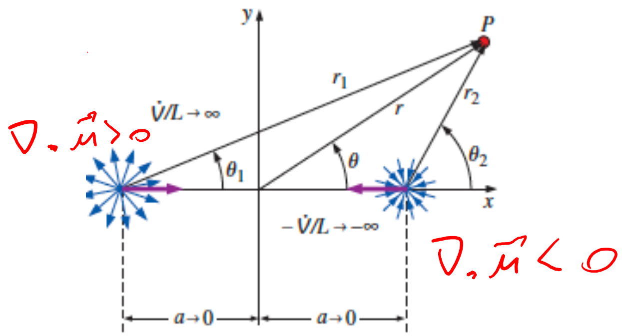
Using

$$\arctan(u) \pm \arctan(v) = \arctan\left(\frac{u \pm v}{1 \mp uv}\right) \pmod{\pi}, \quad uv \neq 1.$$

Doublet: line source and sink close to origin

We have seen before that

Composite stream function: $$\psi = \frac{-\dot{V}/L}{2\pi} \arctan \left[\frac{2ar\sin\theta}{r^2 - a^2} \right]$$



By Taylor expanding the arctan around zero:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$tg^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots$$

$a \ll r$

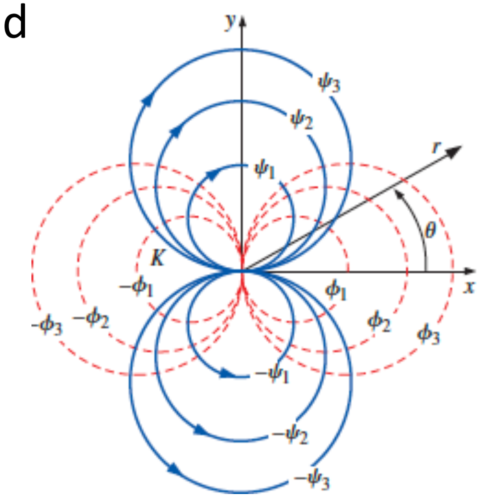
Stream function as $a \to 0$: $$\boxed{\psi \to \frac{-a(\dot{V}/L)r\sin\theta}{\pi(r^2 - a^2)}}$$

Doublet: line source and sink close to origin

Let a tend to zero at constant doublet strength K, to find

Doublet along the x-axis: $$\psi = \frac{-a(\dot{V}/L)}{\pi}\frac{\sin\theta}{r} = -K\frac{\sin\theta}{r}$$

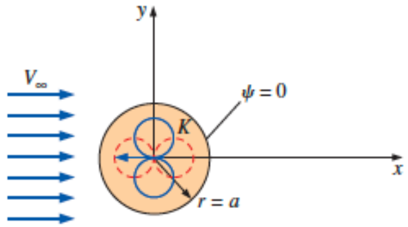
Doublet along the x-axis: $$\phi = K\frac{\cos\theta}{r}$$



Streamlines (solid) and equipotential lines (dashed) for a doublet of strength K located at the origin in the xy-plane and aligned with the x-axis.

Superposition of a uniform stream and a doublet: Flow over a circular cylinder

Superposition: $\psi = V_\infty r \sin\theta - K\dfrac{\sin\theta}{r}$

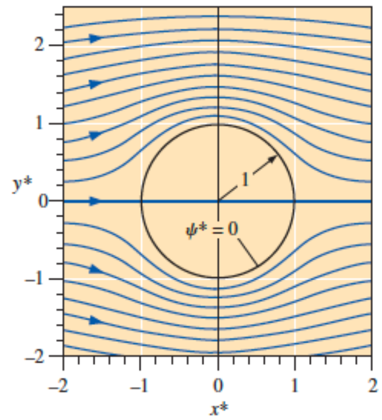


For convenience we set $\psi = 0$ when $r = a$

Doublet strength: $K = V_\infty a^2$

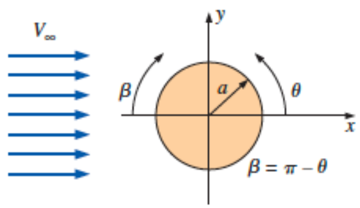
Alternate form of stream function: $\psi = V_\infty \sin\theta\left(r - \dfrac{a^2}{r}\right)$

$$\psi^* = \sin\theta\left(r^* - \frac{1}{r^*}\right)$$



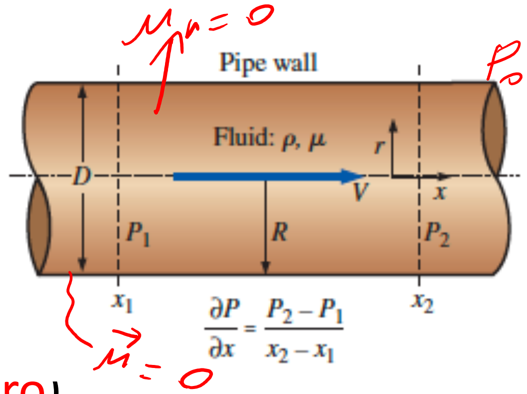
Nondimensional streamlines: $r^* = \dfrac{\psi^* \pm \sqrt{(\psi^*)^2 + 4\sin^2\theta}}{2\sin\theta}$

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = V_\infty \cos\theta\left(1 - \frac{a^2}{r^2}\right) \qquad u_\theta = -\frac{\partial\psi}{\partial r} = -V_\infty \sin\theta\left(1 + \frac{a^2}{r^2}\right)$$



Questão 3

Flow in a round pipe: Poiseuille



1 The pipe is infinitely long in the x-direction.

2 The flow is steady (all partial time derivatives are zero).

3 This is a parallel flow (the r-component of velocity, u_r, is zero).

4 The fluid is incompressible and Newtonian with constant properties, and the flow is laminar. $Re < 2000$

5 A constant pressure gradient is applied in the x-direction such that pressure changes linearly with respect to x.

6 The velocity field is axisymmetric with no swirl, implying that $u_\theta = 0$ and all partial derivatives with respect to θ are zero.

7 We ignore the effects of gravity.

8 The first boundary condition comes from imposing the no slip condition at the pipe wall: (1) at $r = R, \vec{V} = 0$.

9 The second boundary condition comes from the fact that the centerline of the pipe is an axis of symmetry: (2) at $r = 0$, $\frac{\partial u}{\partial x} = 0$. Alternatively: the velocity is finite at the center.

Continuity: $\nabla \cdot \vec{u} = 0$

$u_r = 0 \qquad u_\theta = 0$

$$\frac{1}{\cancel{r}}\frac{\partial(r\cancel{u_r})}{\partial r} + \frac{1}{\cancel{r}}\frac{\partial(\cancel{u_\theta})}{\partial \theta} + \frac{\partial u}{\partial x} = 0 \quad \rightarrow \quad \frac{\partial u}{\partial x} = 0$$

Result of continuity: $\underline{u = u(r)}$ only

NS u:

$0 \qquad u_r = 0 \qquad u_\theta = 0 \qquad \vec{g} = 0$

$$\rho\left(\frac{\partial \cancel{u}}{\partial t} + u_r\frac{\partial \cancel{u}}{\partial r} + \frac{u_\theta}{\cancel{r}}\frac{\partial \cancel{u}}{\partial \theta} + u\frac{\partial \cancel{u}}{\partial x}\right) = -\frac{\partial P}{\partial x} + \underbrace{\rho g_x} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{\cancel{r^2}}\frac{\partial^2 \cancel{u}}{\partial \theta^2} + \frac{\partial^2 \cancel{u}}{\partial x^2}\right)$$

$\frac{\partial}{\partial \theta}(\ldots) = 0$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{1}{\mu}\frac{\partial P}{\partial x}$$

NS p:

r-momentum: $\frac{\partial P}{\partial r} = 0$

Result of r-momentum: $P = P(x)$ only $\Rightarrow P = \frac{\partial P}{\partial x} \cdot x + P_0$

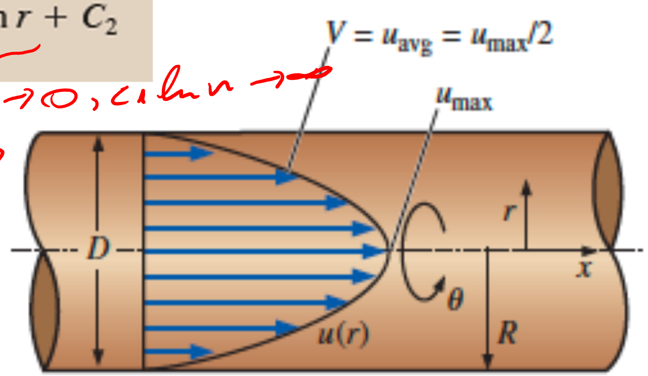
Integration of NS for u:

$$r\frac{du}{dr} = \frac{r^2}{2\mu}\frac{dP}{dx} + C_1 \qquad u = \frac{r^2}{4\mu}\frac{dP}{dx} + \underbrace{C_1 \ln r} + C_2$$

$u(r = R) = 0$

se $r \to 0$, $C_1 \ln r \to \infty$

$\Rightarrow C_1 = 0$



Axial velocity: $$u = \frac{1}{4\mu}\frac{dP}{dx}(r^2 - R^2)$$

Poiseuille's law for the flow rate

Maximum axial velocity: $$u_{\max} = -\frac{R^2}{4\mu}\frac{dP}{dx}$$

$$\dot{V} = \int_{\theta=0}^{2\pi}\int_{r=0}^{R} ur\,dr\,d\theta = \frac{2\pi}{4\mu}\frac{dP}{dx}\int_{r=0}^{R}(r^2 - R^2)r\,dr = -\frac{\pi R^4}{8\mu}\frac{dP}{dx}$$

Average axial velocity: $$V = \frac{\dot{V}}{A} = \frac{(-\pi R^4/8\mu)\,(dP/dx)}{\pi R^2} = -\frac{R^2}{8\mu}\frac{dP}{dx}$$

Viscous shear force

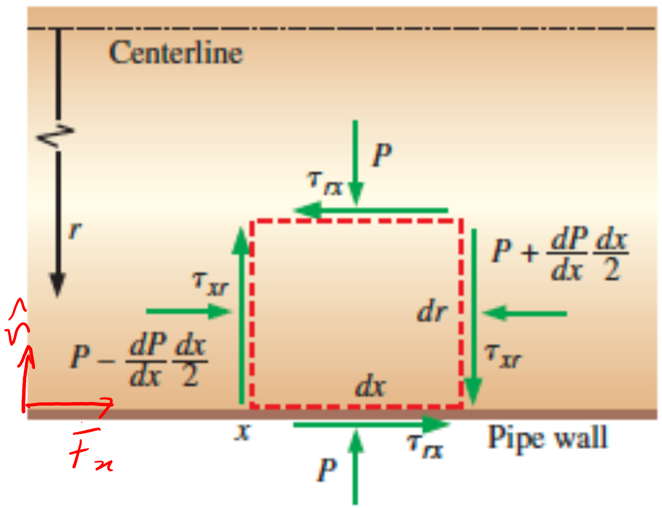
The stress tensor is

$$\tau_{ij} = \begin{pmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rx} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta x} \\ \tau_{xr} & \tau_{x\theta} & \tau_{xx} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \mu \dfrac{\partial u}{\partial r} \\ 0 & 0 & 0 \\ \mu \dfrac{\partial u}{\partial r} & 0 & 0 \end{pmatrix}$$

Force on $\hat{x}$

Viscous shear stress at the pipe wall: $\tau_{rx} = \mu \dfrac{du}{dr} = \dfrac{R}{2}\dfrac{dP}{dx}$

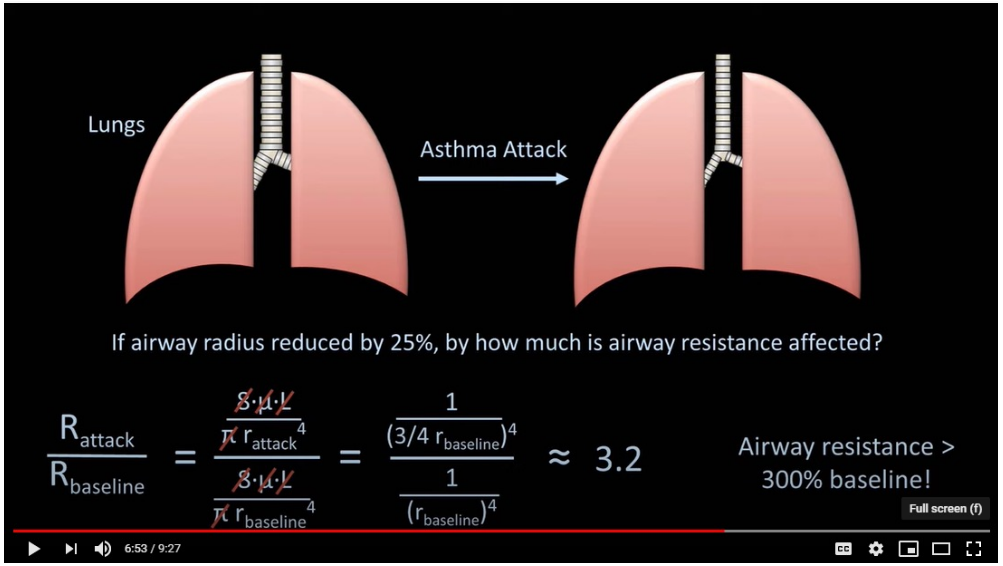
normal $\hat{n}$

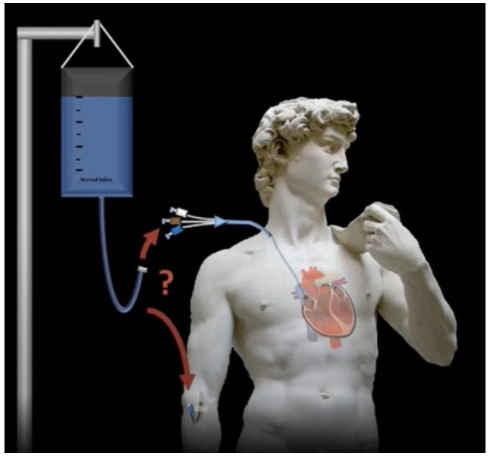


For flow from left to right, dP/dx is negative, so the viscous shear stress on the bottom of the fluid element at the wall is in the direction opposite to that indicated in the figure. (This agrees with our intuition since the pipe wall exerts a retarding force on the fluid.) The shear force per unit area on the wall is equal and opposite to this; hence,

Viscous shear force per unit area acting on the wall: $\dfrac{\vec{F}}{A} = -\dfrac{R}{2}\dfrac{dP}{dx}\vec{i}$

Viscosity and Poiseuille's Law:
https://www.youtube.com/watch?v=wTnI_kfPBhQ





Force balance

Navier-Stokes equation

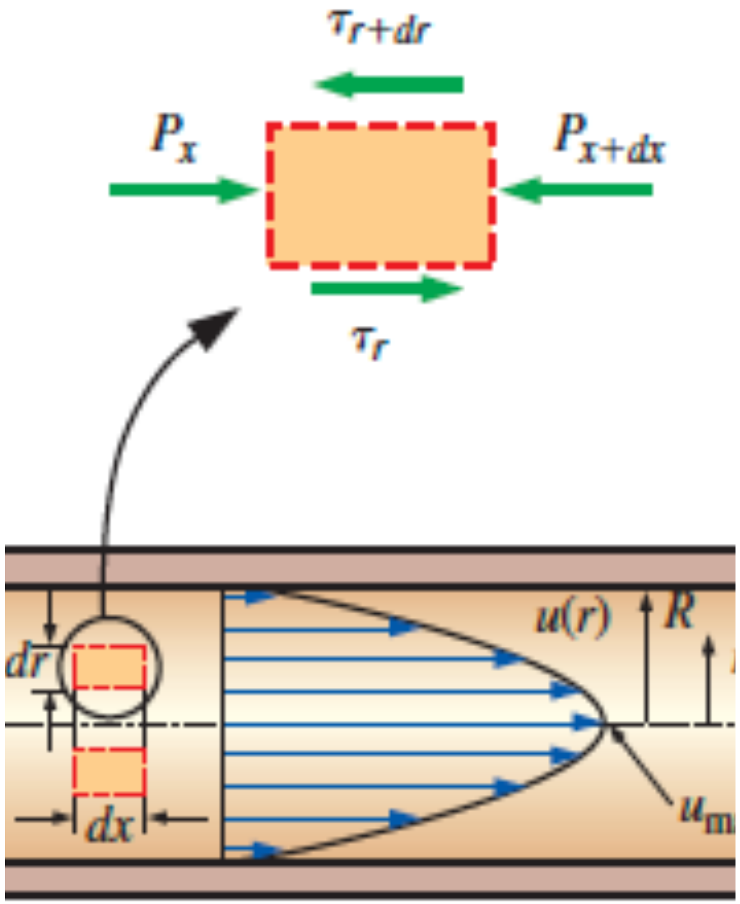
$$\underbrace{\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u}}_{=0} = -\frac{\nabla p}{\rho} + \vec{g} + \nu \nabla^2 \vec{u}$$

In most of the previous examples, the acceleration of the fluid elements is zero. It means that the viscous force balance the external force (e.g., gravity) or pressure gradients in such a way that the sum of forces acting on a fluid element is zero.

Alternative derivation for flow in a circular pipe

Obtain the momentum equation by applying a momentum balance to a differential volume element, and we obtain the velocity profile by solving it.

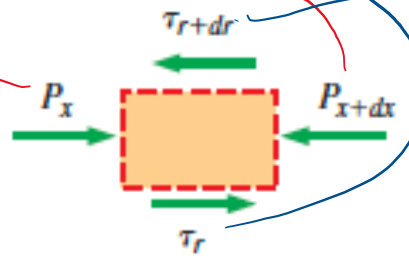
Free-body diagram of a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with a horizontal pipe in fully developed laminar flow.

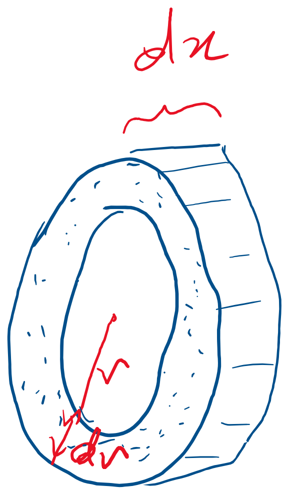


In fully developed laminar flow the axial velocity is, u= u(r). There is no motion in the radial direction. There is no acceleration (check: calculate the acceleration and verify that it is zero).

- Consider a ring-shaped differential volume element of radius r, thickness dr, and length dx oriented coaxially with the pipe.
- The volume element involves only pressure and viscous effects and thus the pressure and shear forces must balance each other. The pressure force acting on a submerged plane surface is the product of the pressure at the centroid of the surface and the surface area. A force balance on the volume element in the flow direction (x) gives

$$(\underbrace{2\pi r\,dr}_{dA}\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} = 0$$





Force balance implies

$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} = 0 \quad \div (2\pi\,dx\,dr)$$

$$r\,\frac{\overbrace{P_{x+dx} - P_x}^{dP}}{dx} + \frac{\overbrace{(r\tau)_{r+dr} - (r\tau)_r}^{d(r\tau)}}{dr} = 0$$

$$r\,\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

and substituting the stress (component rz): $\tau_{rz} = -\mu(du/dr)$ we find

$$\boxed{\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}}$$

Same equation obtained with NS: $\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{1}{\mu}\frac{\partial P}{\partial x}$

Recall

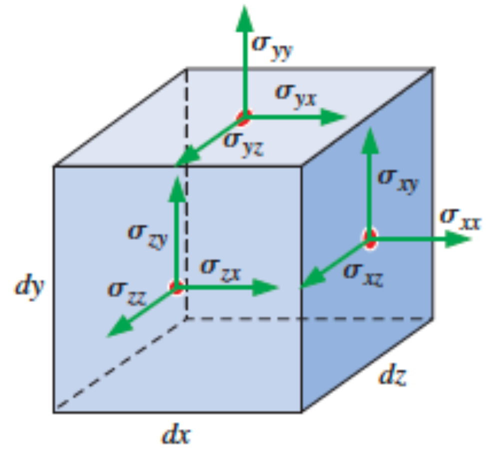
Deviatoric stress tensor

$$\tau_{ij} = \begin{pmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{pmatrix}$$

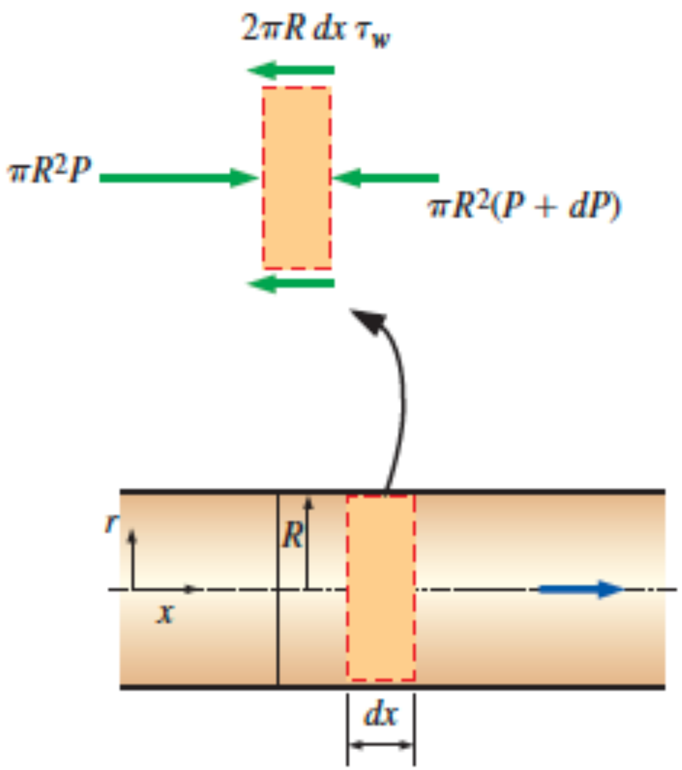
$$= \begin{pmatrix} 2\mu \dfrac{\partial u_r}{\partial r} & \mu\left[r\dfrac{\partial}{\partial r}\left(\dfrac{u_\theta}{r}\right) + \dfrac{1}{r}\dfrac{\partial u_r}{\partial \theta}\right] & \mu\left(\dfrac{\partial u_r}{\partial z} + \dfrac{\partial u_z}{\partial r}\right) \\ \mu\left[r\dfrac{\partial}{\partial r}\left(\dfrac{u_\theta}{r}\right) + \dfrac{1}{r}\dfrac{\partial u_r}{\partial \theta}\right] & 2\mu\left(\dfrac{1}{r}\dfrac{\partial u_\theta}{\partial \theta} + \dfrac{u_r}{r}\right) & \mu\left(\dfrac{\partial u_\theta}{\partial z} + \dfrac{1}{r}\dfrac{\partial u_z}{\partial \theta}\right) \\ \mu\left(\dfrac{\partial u_r}{\partial z} + \dfrac{\partial u_z}{\partial r}\right) & \mu\left(\dfrac{\partial u_\theta}{\partial z} + \dfrac{1}{r}\dfrac{\partial u_z}{\partial \theta}\right) & 2\mu \dfrac{\partial u_z}{\partial z} \end{pmatrix}$$

Stress tensor

$$\sigma_{ij} = -P\delta_{ij} + \tau_{ij}$$



Different fluid element (r from 0 to R)



Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\,dx\,\tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

Separation of variables implies that the pressure gradient is constant $\frac{dP}{dx} = -\frac{2\tau_w}{R}$

The velocity profile is obtained by integration and use of the boundary conditions:

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

(...) $u(r=R) = 0$

$= 0$ ($u(r=0)$ é finito)

$$\boxed{u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)}$$

The average velocity is

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r) r\, dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\, dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

In terms of which the profile becomes

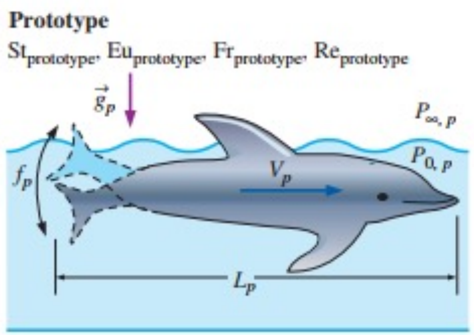
$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

$r = 0 \Rightarrow u(r) = u_{max} = 2V_{avg}$

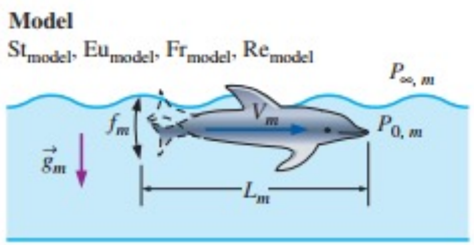
Questão 4

Nondimensionalized Navier–Stokes:

$$[\mathrm{St}]\frac{\partial \vec{V}^*}{\partial t^*} + (\vec{V}^* \cdot \vec{\nabla}^*)\vec{V}^* = -[Eu]\vec{\nabla}^* P^* + \left[\frac{1}{\mathrm{Fr}^2}\right]\vec{g}^* + \left[\frac{1}{\mathrm{Re}}\right]\nabla^{*2}\vec{V}^*$$



Since there are four dimensionless parameters, dynamic similarity between a model and a prototype requires all four of these to be the same for the model and the prototype ($\mathrm{St}_{model} = \mathrm{St}_{prototype}$, $\mathrm{Eu}_{model} = \mathrm{Eu}_{prototype}$, $\mathrm{Fr}_{model} = \mathrm{Fr}_{prototype}$, and $\mathrm{Re}_{model} = \mathrm{Re}_{prototype}$).



Approximate Navier–Stokes equation for creeping flow: $\vec{\nabla}P \cong \mu\nabla^2\vec{V}$

Drag force on a sphere in creeping flow: $F_D = 3\pi\mu VD$