Bayesian Inference of Phylogenies

Prior: H1 e H2 3:2

Data

Likelihood ratio

$$\text{Prob}\left(D \mid H_1\right) / \text{Prob}\left(D \mid H_2\right) = 1/2$$

Resultados, dados favorecem a hipótese H2

Posterior probability: 3/2*1/2=3/4

H1 e H2 3:4 Agora a hipótese H1 é menos provável do que era no prior e H2 passou a ser a hipótese mais provavel

Bayesian Inference

$$\text{Prob}(H \mid D) = \frac{\text{Prob}(H \,\&\, D)}{\text{Prob}(D)}$$

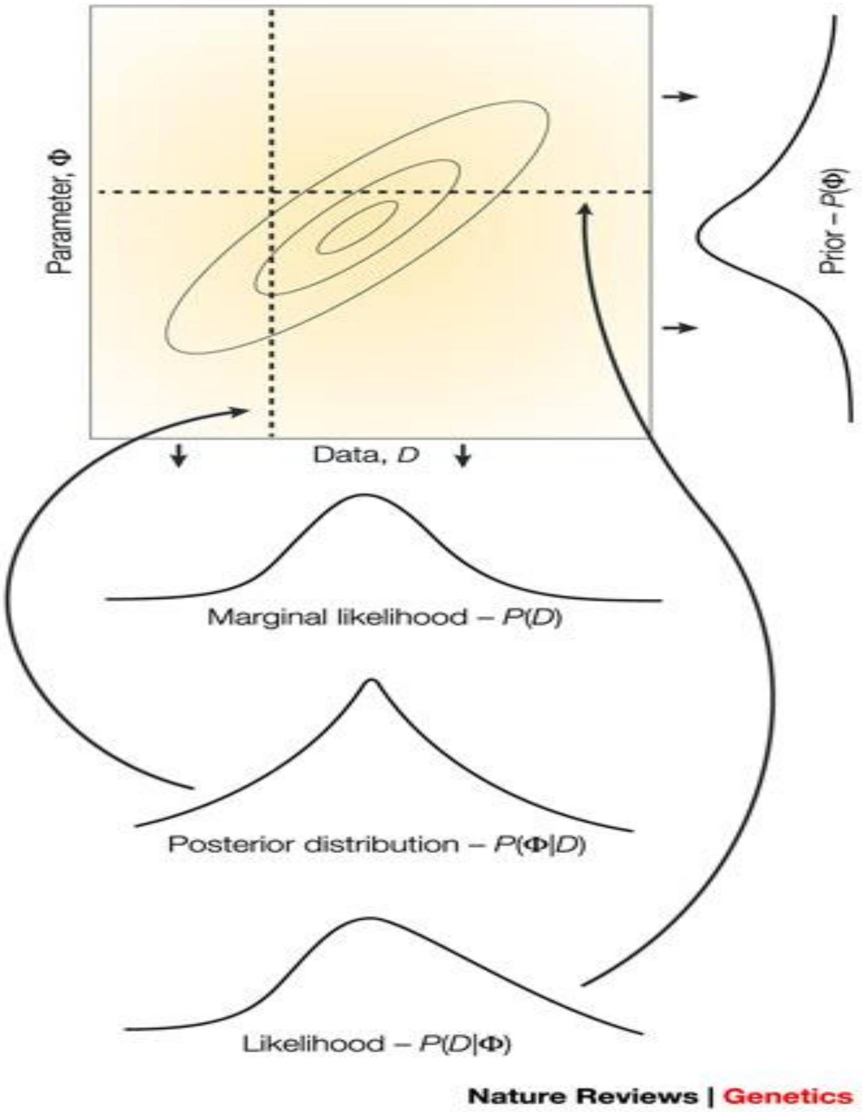
$$\text{Prob}(H \,\&\, D) = \text{Prob}(H)\ \text{Prob}(D \mid H)$$

$$\text{Prob}(H \mid D) = \frac{\text{Prob}(H)\,\text{Prob}(D \mid H)}{\text{Prob}(D)}$$

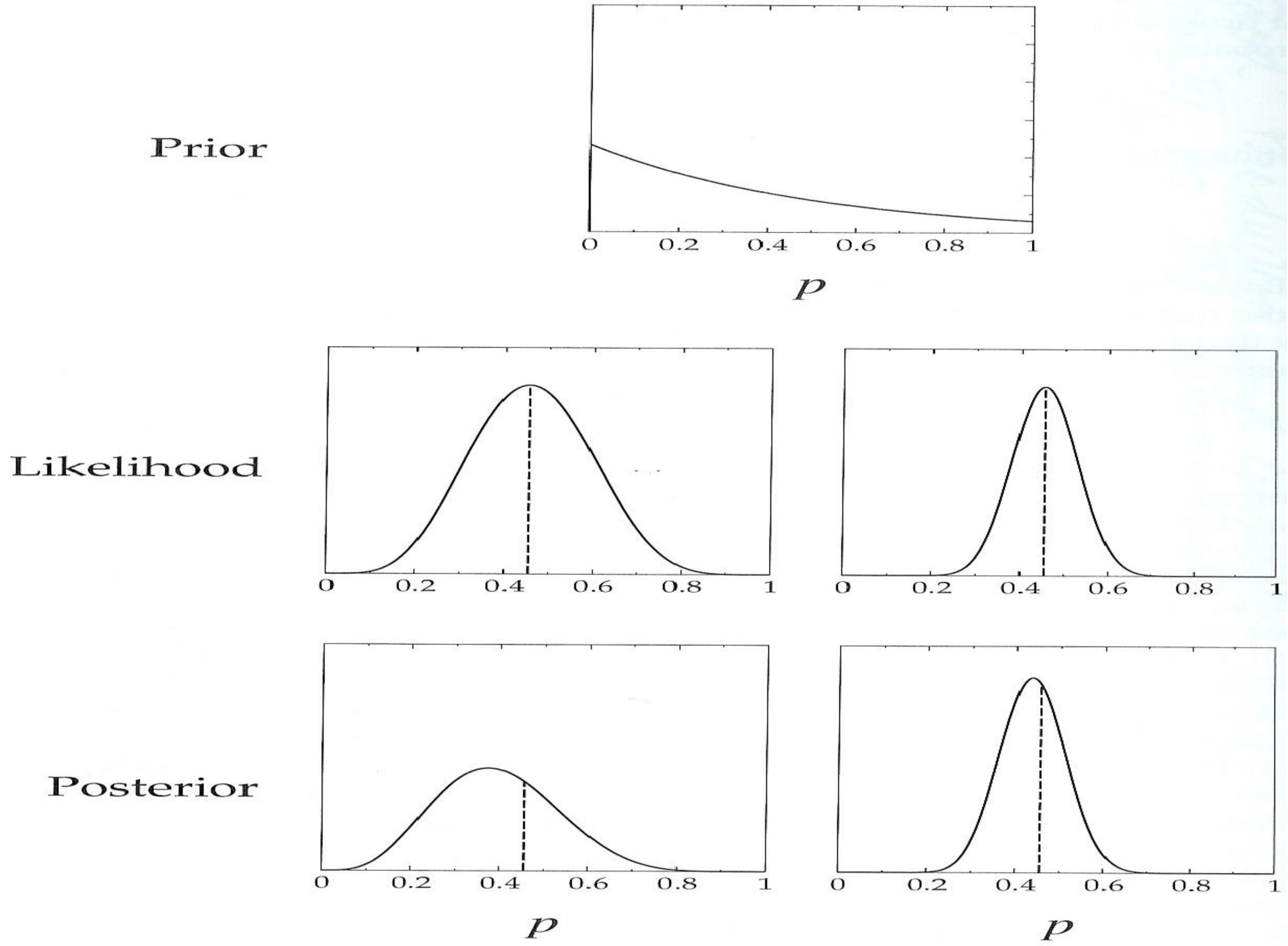
Bayesian Inference

$$\text{Prob}(H \mid D) = \frac{\text{Prob}(H)\,\text{Prob}(D \mid H)}{\sum_{H} \text{Prob}(H)\,\text{Prob}(D \mid H)}$$

Bayesian Inference



Bayesian Inference



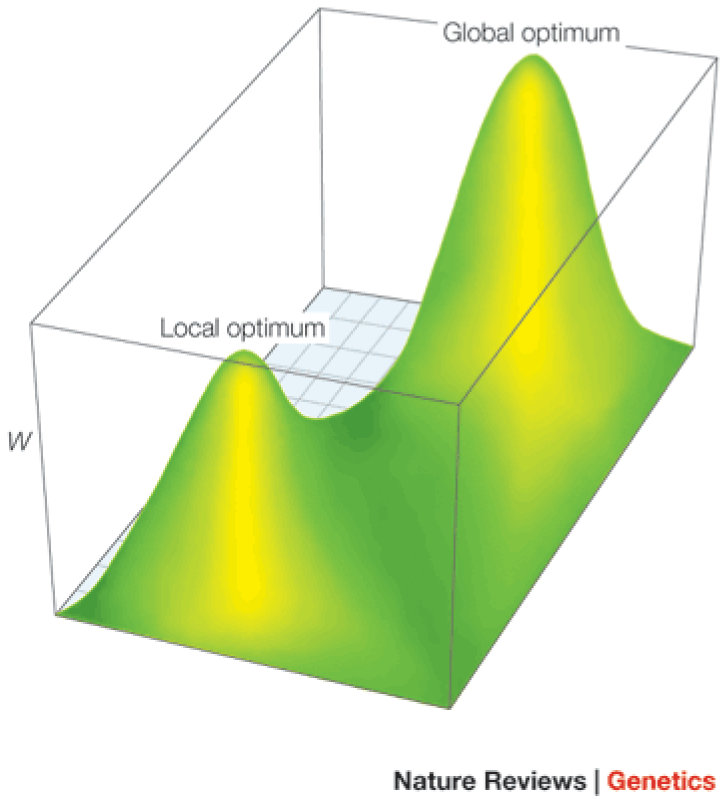
Acceptance ratio

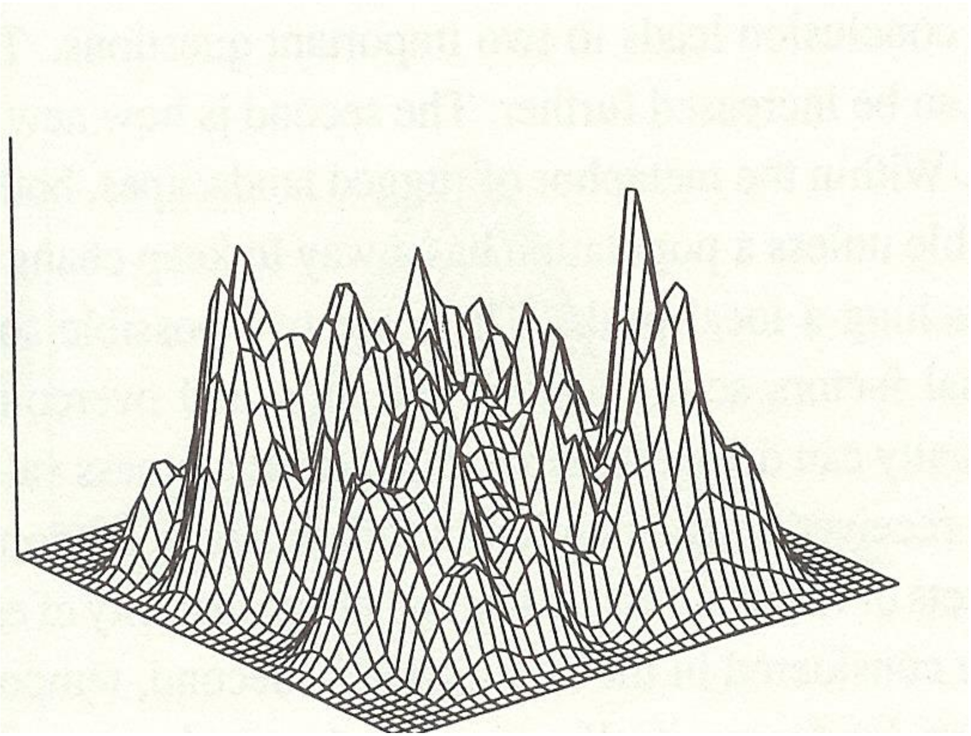
$$R = \frac{\text{Prob}(T_j)}{\text{Prob}(T_i)} \frac{\text{Prob}(D|\,T_j)}{\text{Prob}(D|\,T_i)}$$

Bayesian Inference

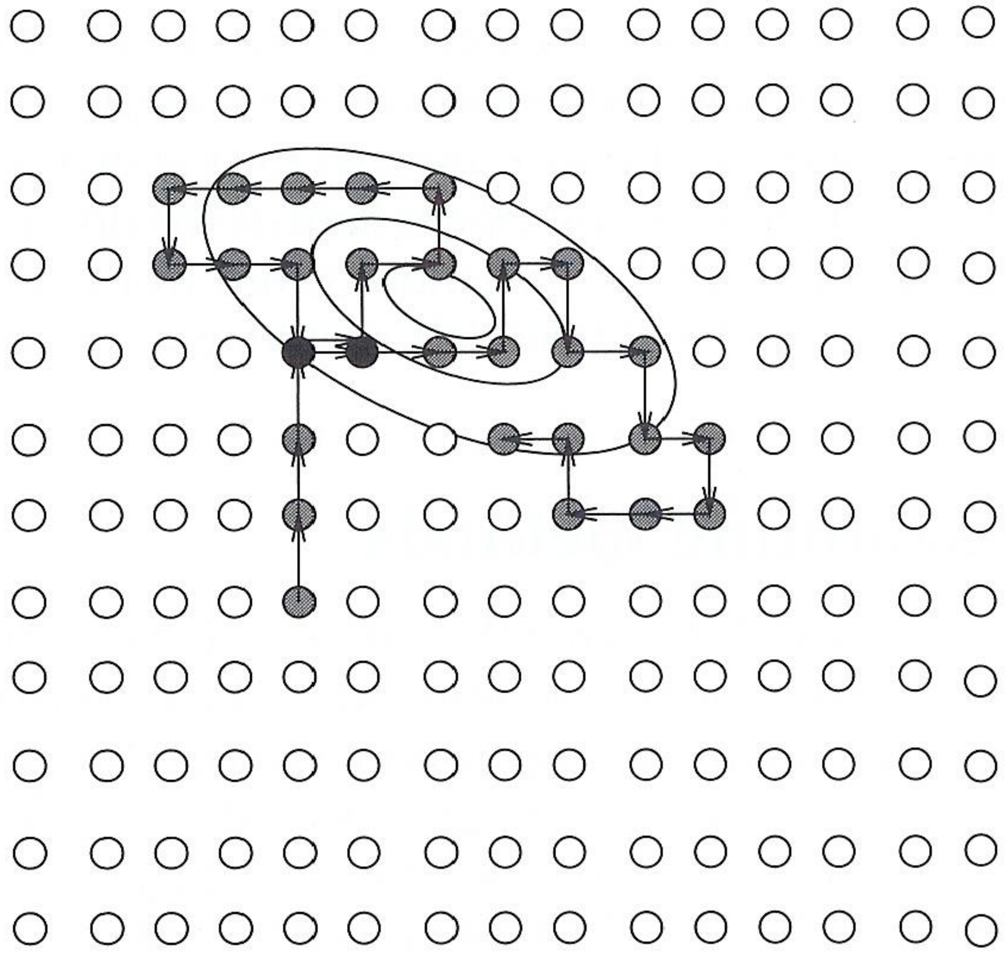
$$PP(T,\tau,\theta \mid D) = \frac{\Pr(D \mid T,\tau,\theta)}{\Pr(D)}$$

Bayesian Inference





Iterative search strategy - Markov chain Monte Carlo



Iterative search strategy - Markov chain Monte Carlo

The Metropolis algorithm involves the following steps:

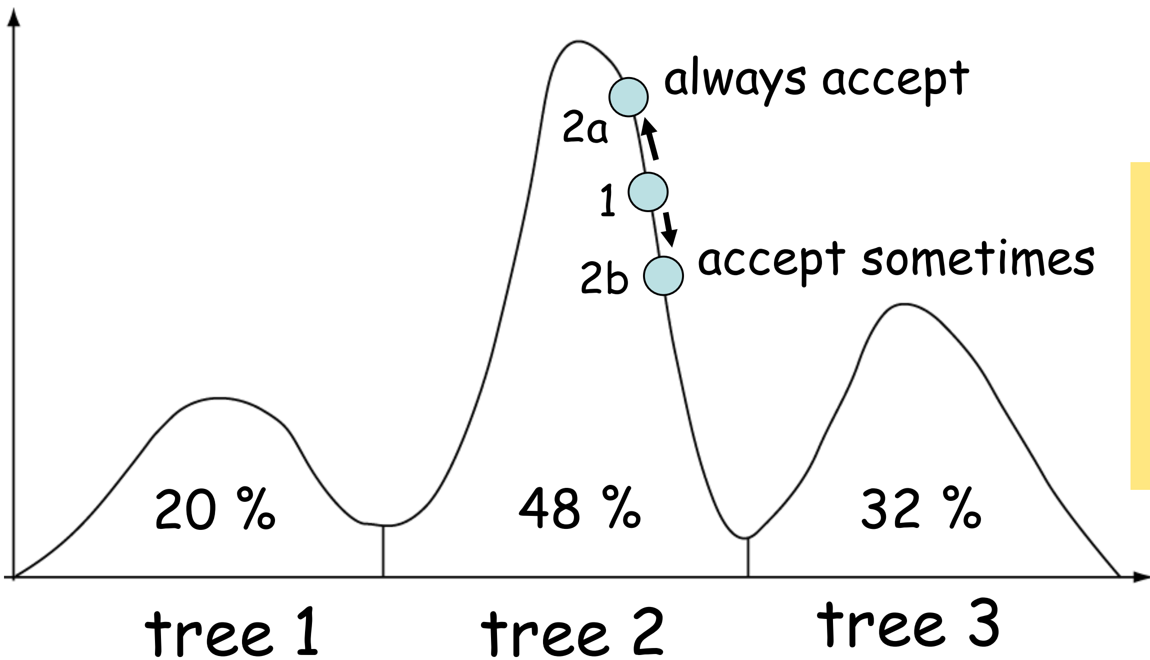
1. Start at some tree. Call this T_i.
2. Pick a tree that is a neighbor of this tree in the graph of trees. Call this the proposal T_j.
3. Compute the ratio of the probabilities (or probability density functions) of the proposed new tree and the old tree:

$$R = \frac{f(T_j)}{f(T_i)}$$

4. If $R \geq 1$, accept the new tree as the current tree.
5. If $R < 1$, draw a uniform random number (a random fraction between 0 and 1). If it is less than R, accept the new tree as the current tree.
6. Otherwise, reject the new tree and continue with tree T_i as the current tree.
7. Return to step 2.

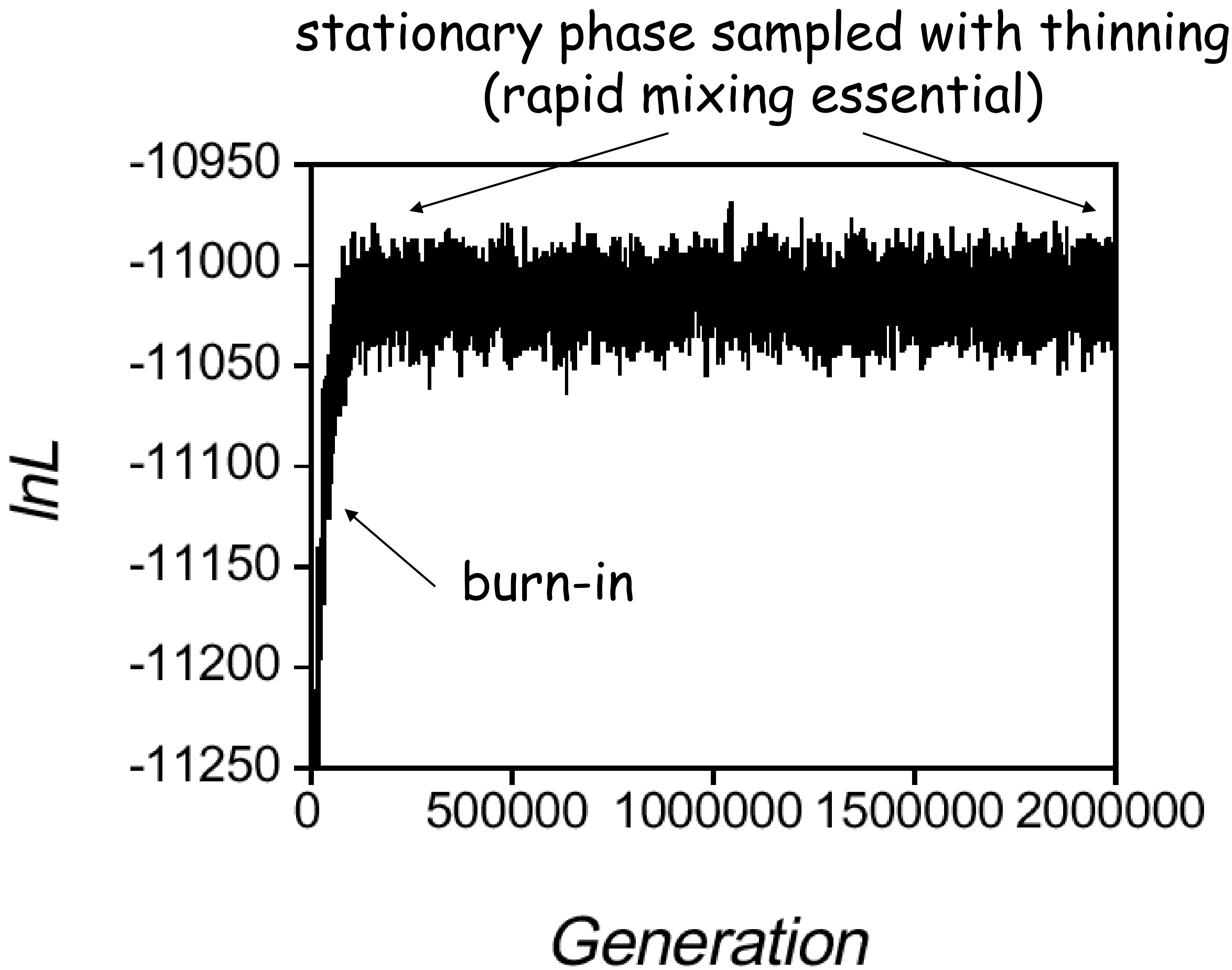
Markov chain Monte Carlo

1. Start at an arbitrary point
2. Make a small random move
3. Calculate height ratio (*r*) of new state to old state:
    1. r > 1 -> new state accepted
    2. r < 1 -> new state accepted with probability *r*. If new state not accepted, stay in the old state
4. Go to step 2



The proportion of time the MCMC procedure samples from a particular parameter region is an estimate of that region's posterior probability density

Burn-in and stationarity



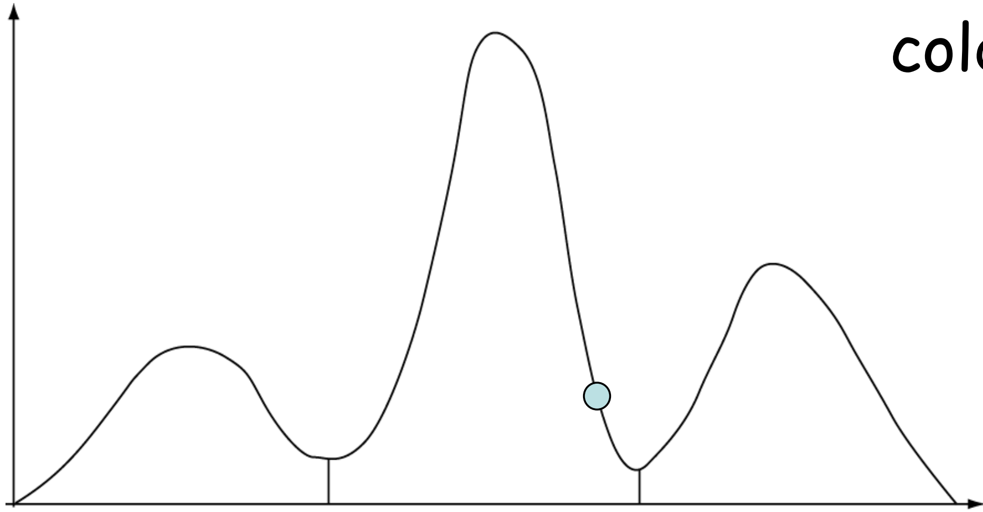
Metropolis-
coupled
Markov chain
Monte Carlo

a. k. a.

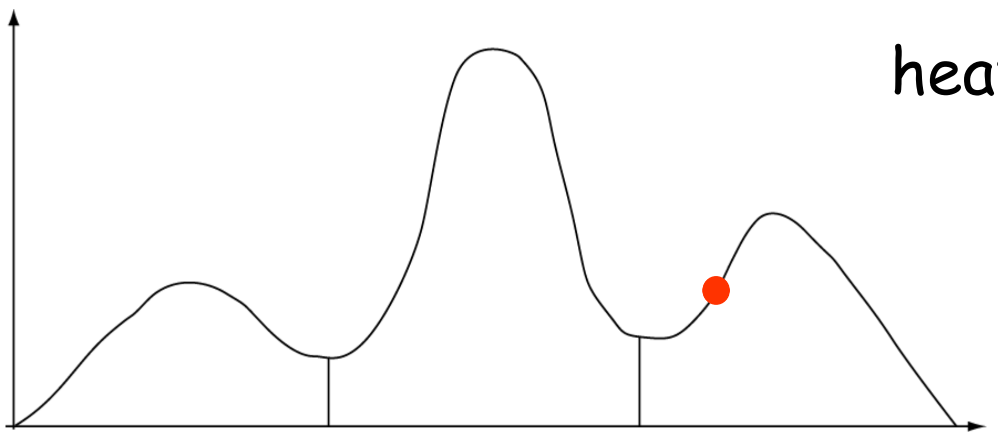
MCMCMC

a. k. a.

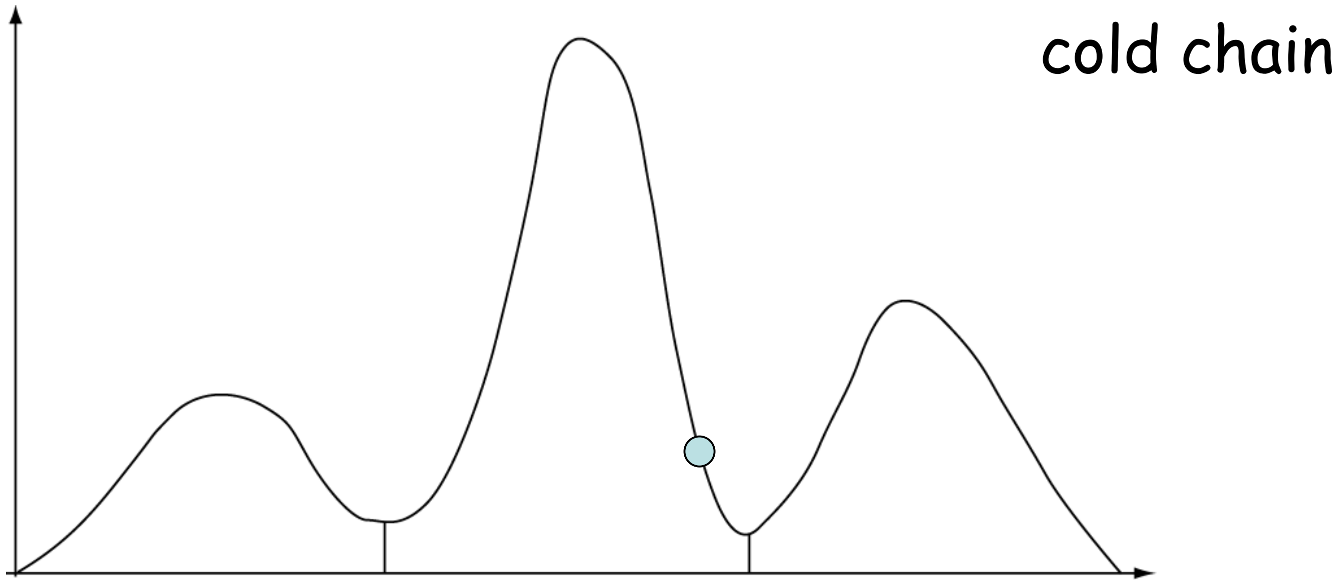
$(MC)^3$

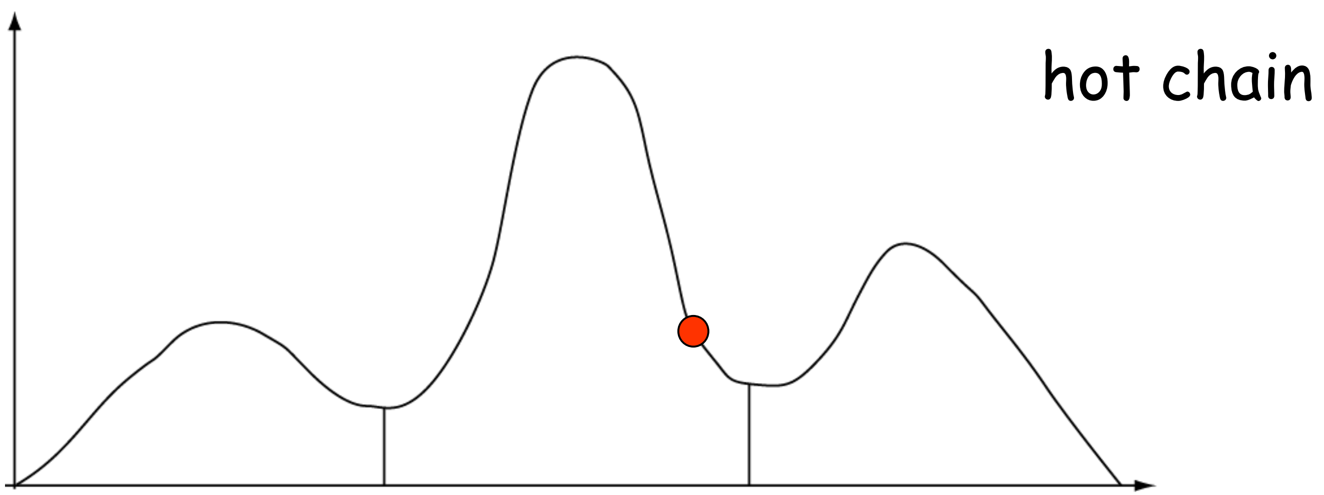


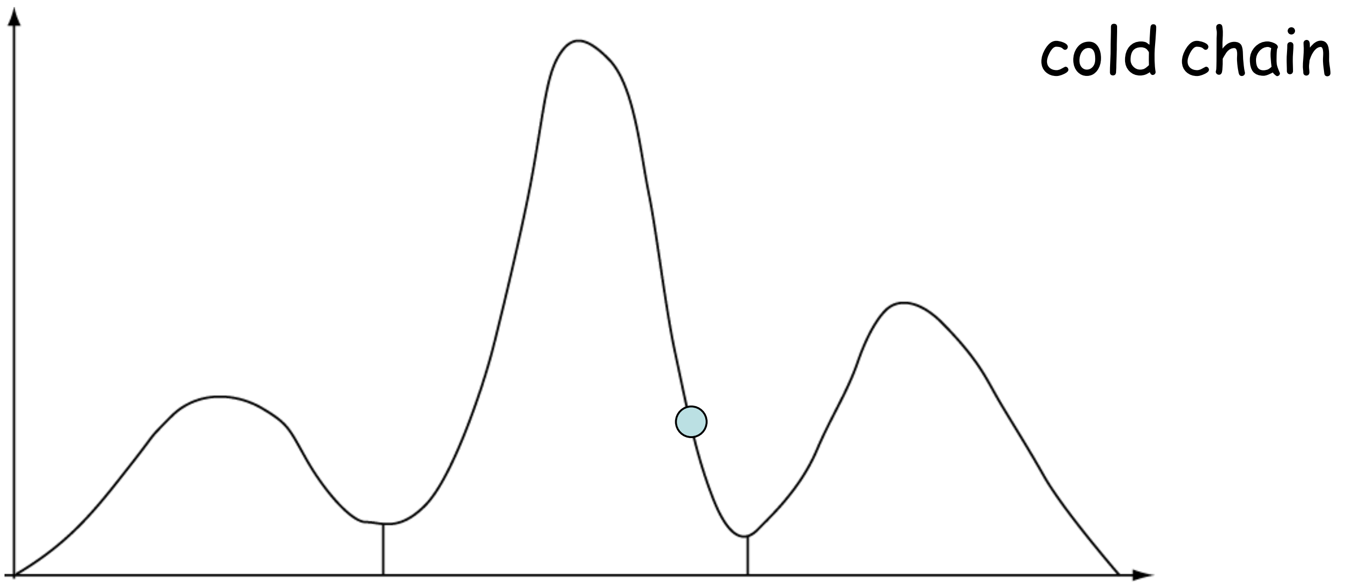
cold chain

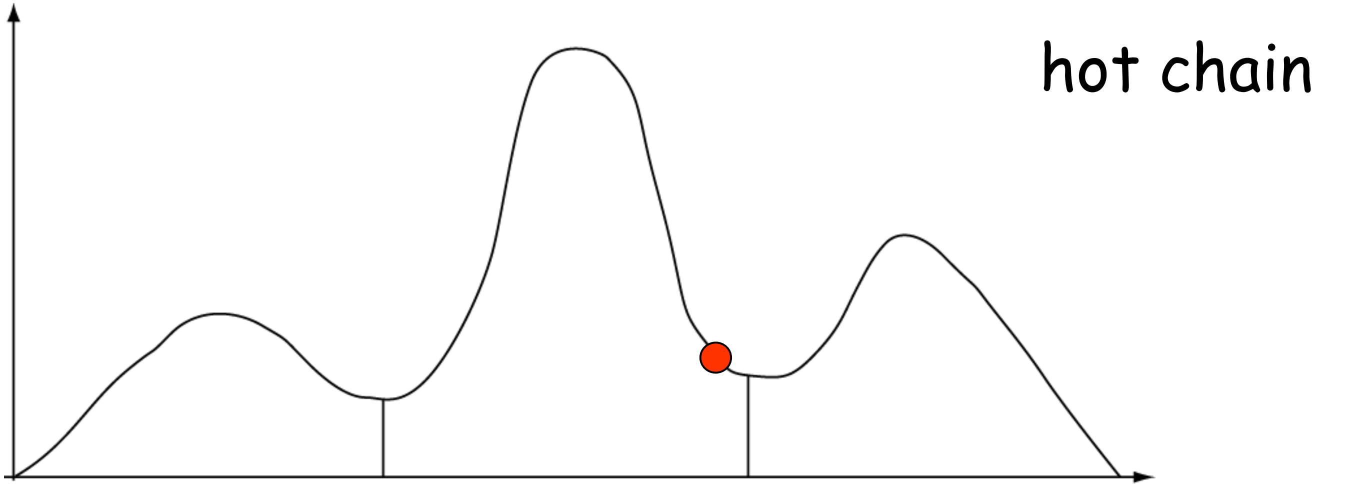


heated chain


cold chain


hot chain


cold chain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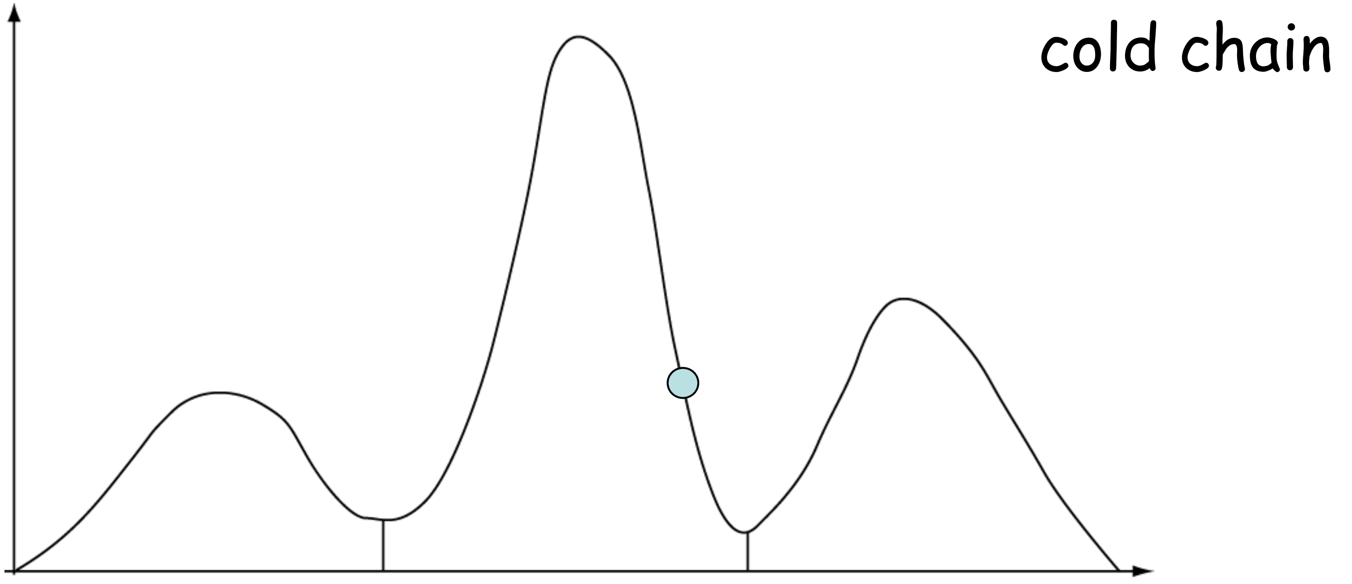
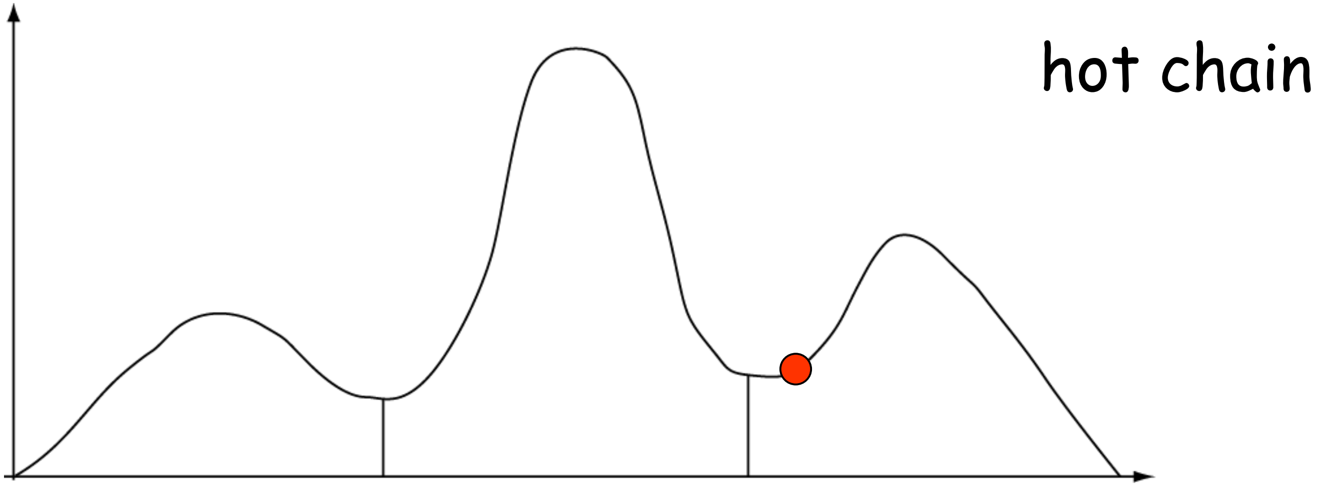
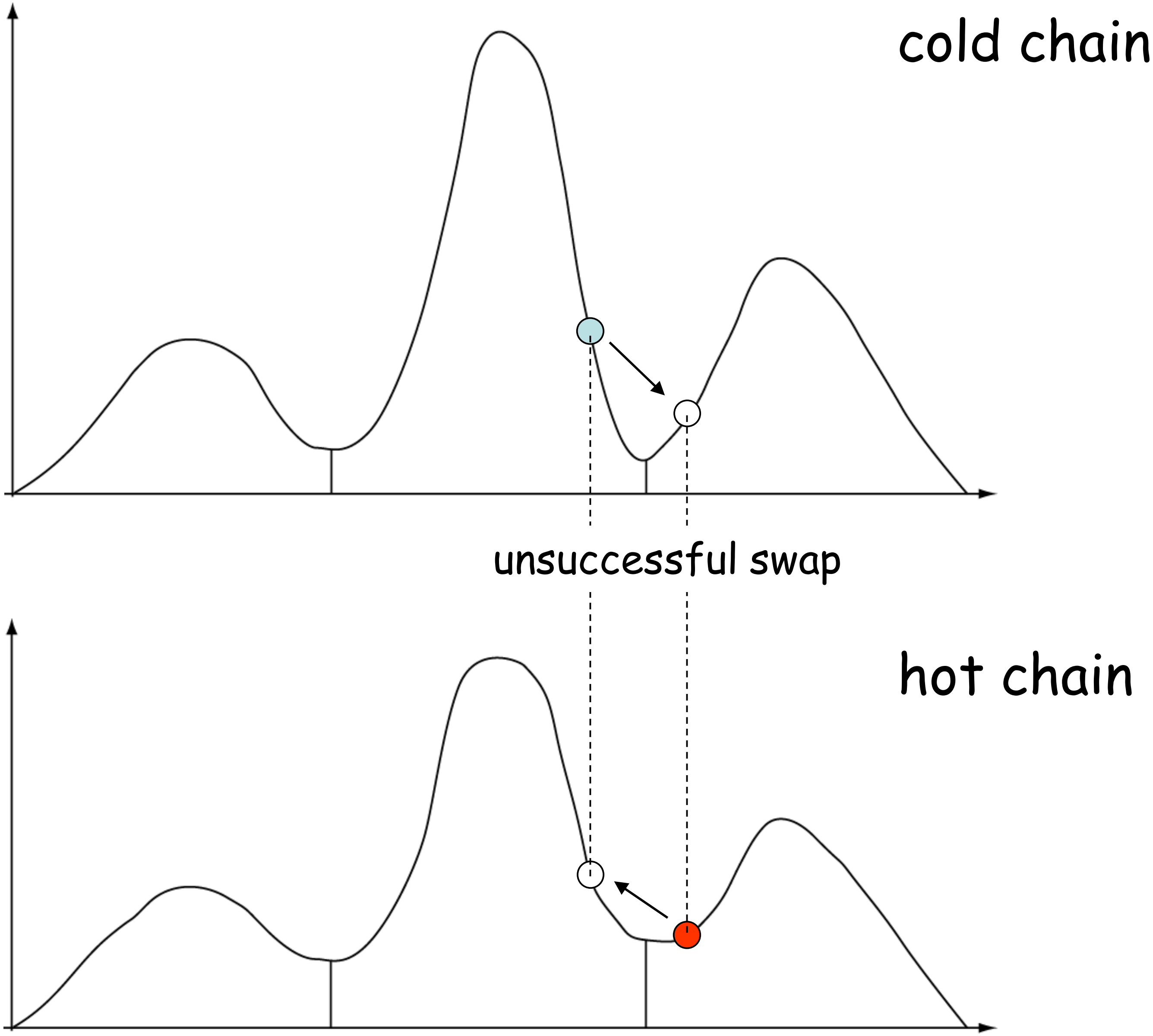
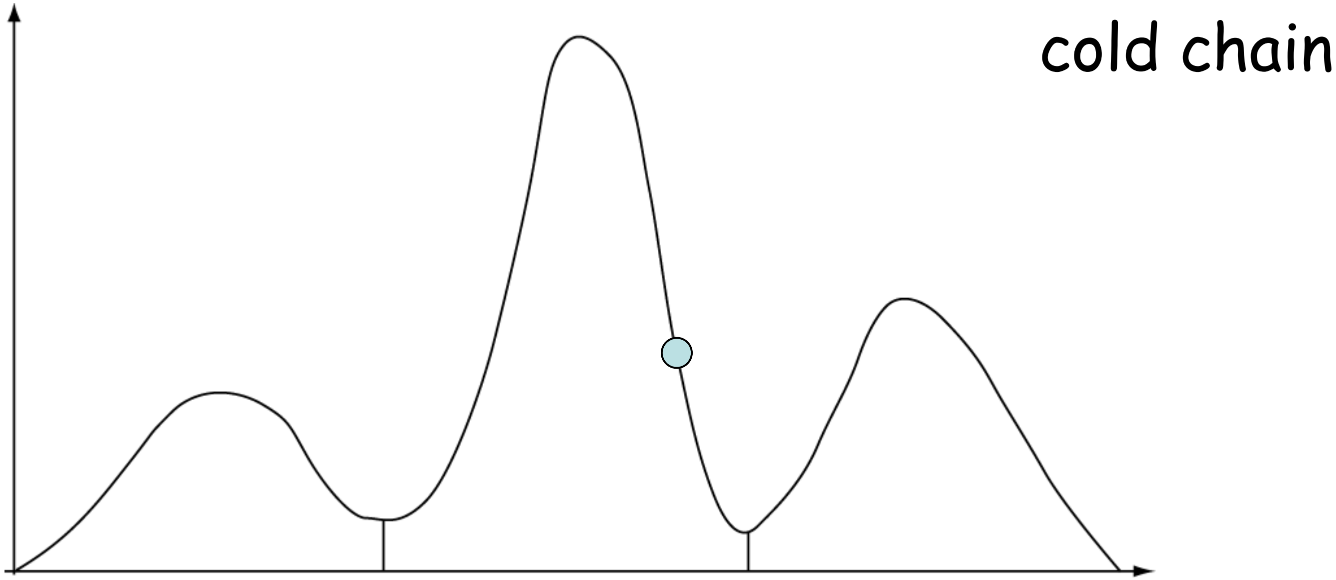
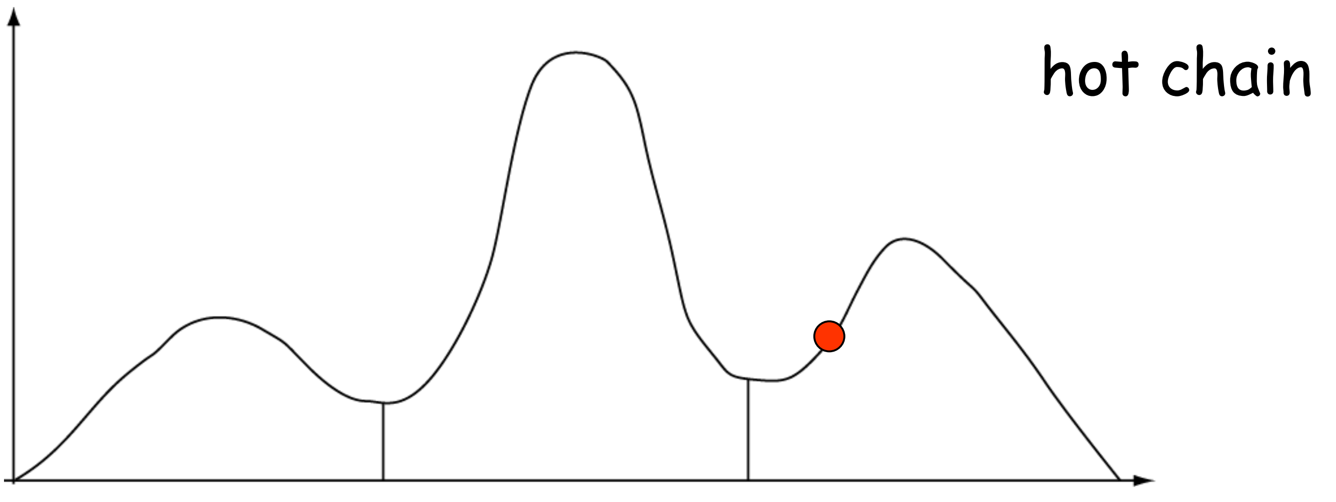

cold chain

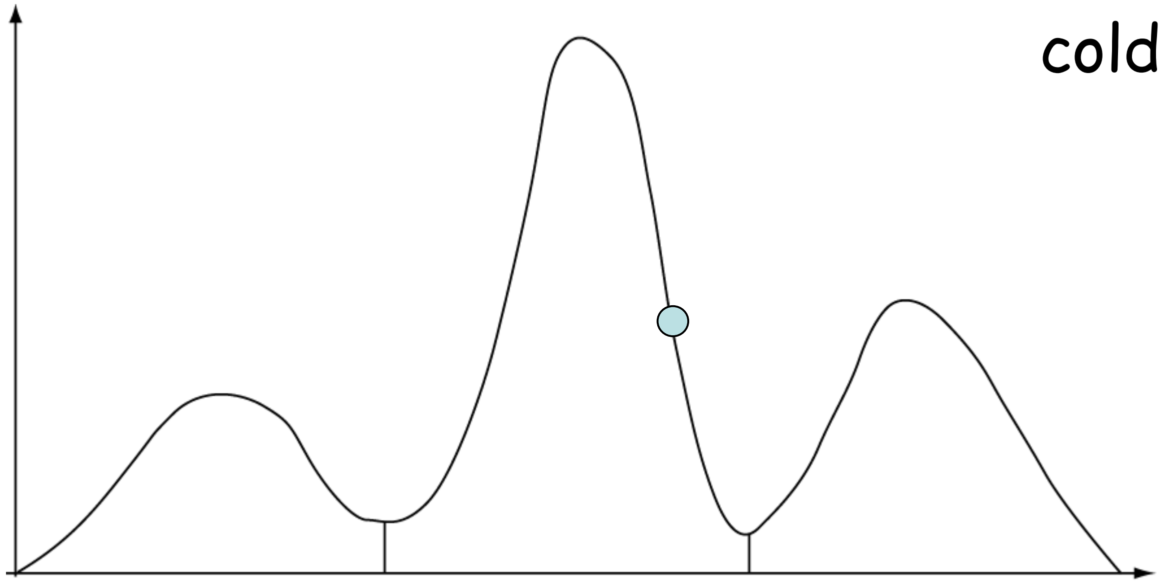

hot chain


cold chain
unsuccessful swap
hot chain

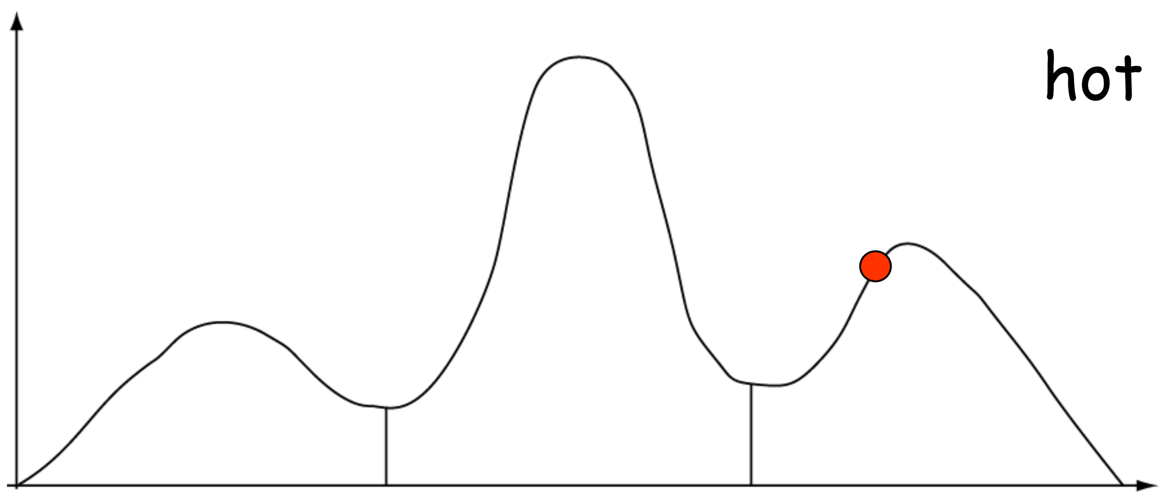

cold chain

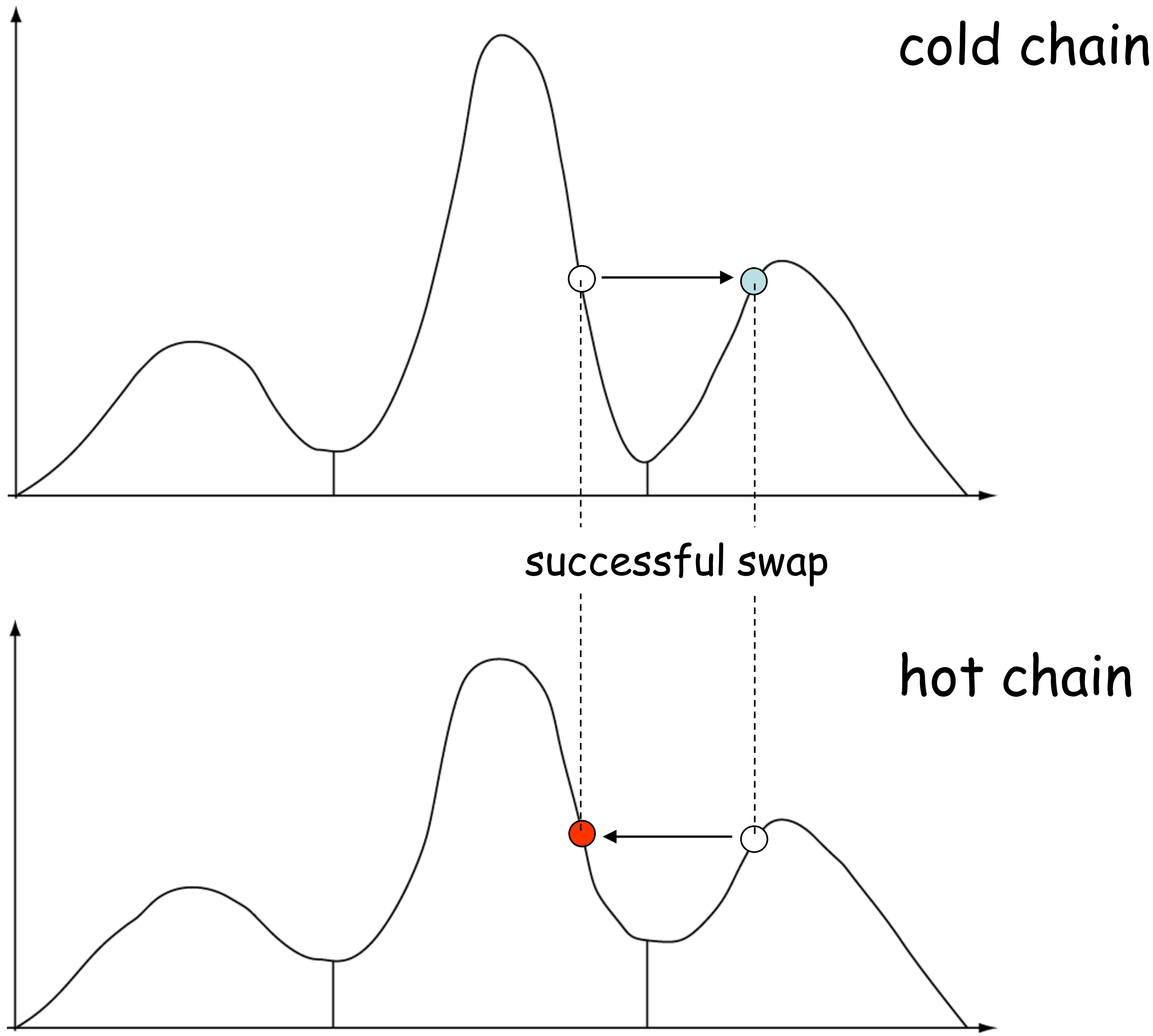

hot chain

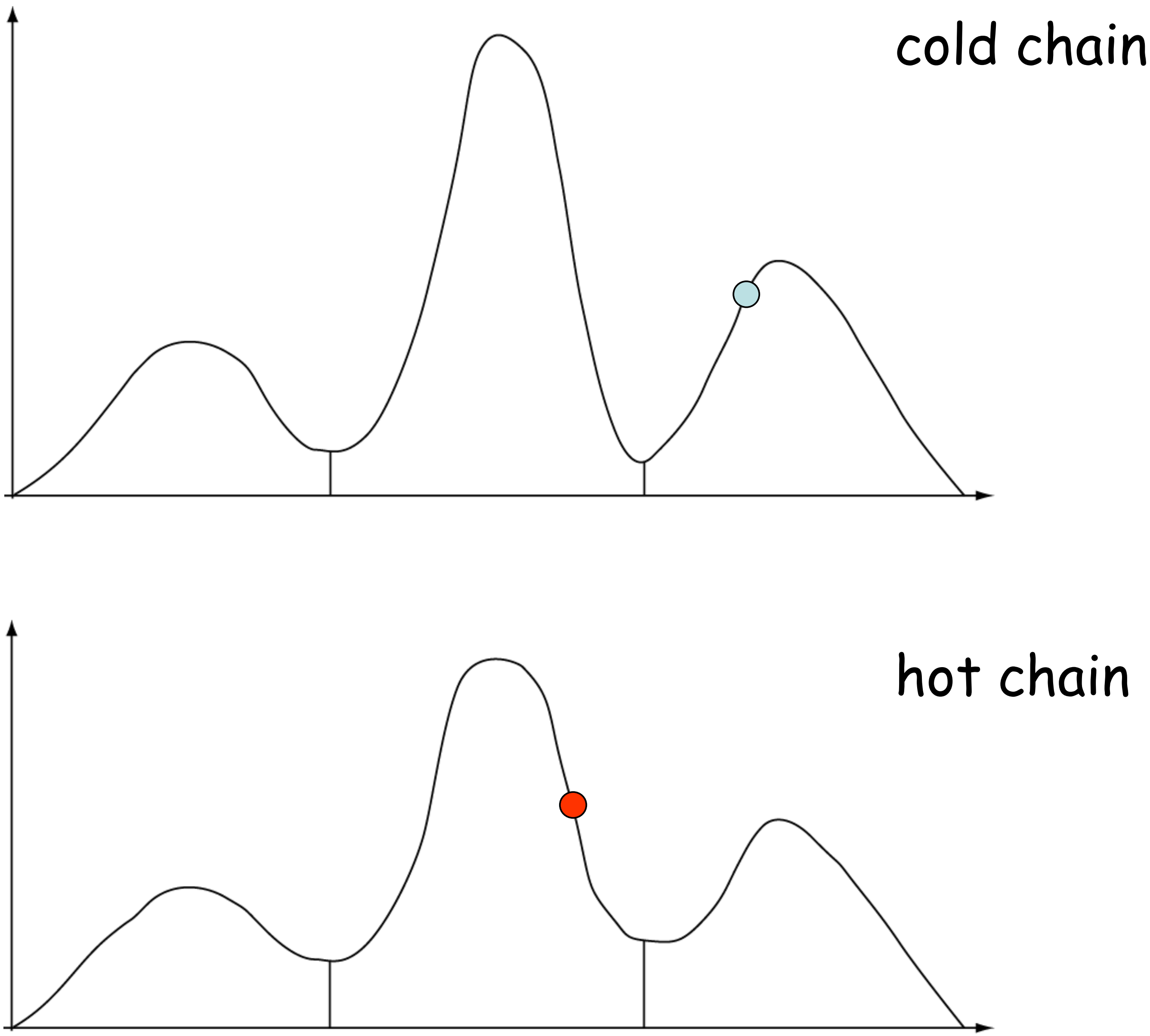
cold chain

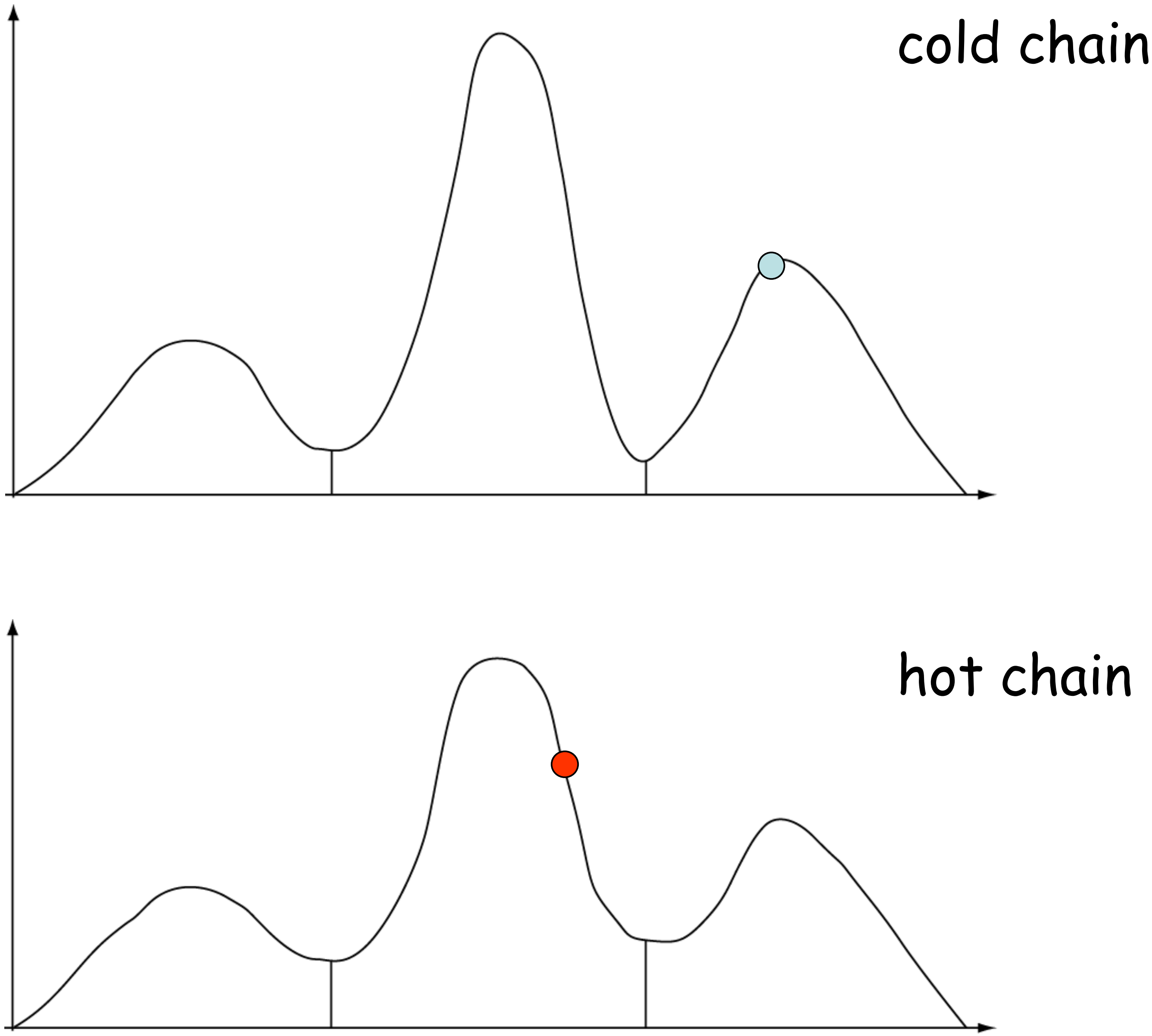


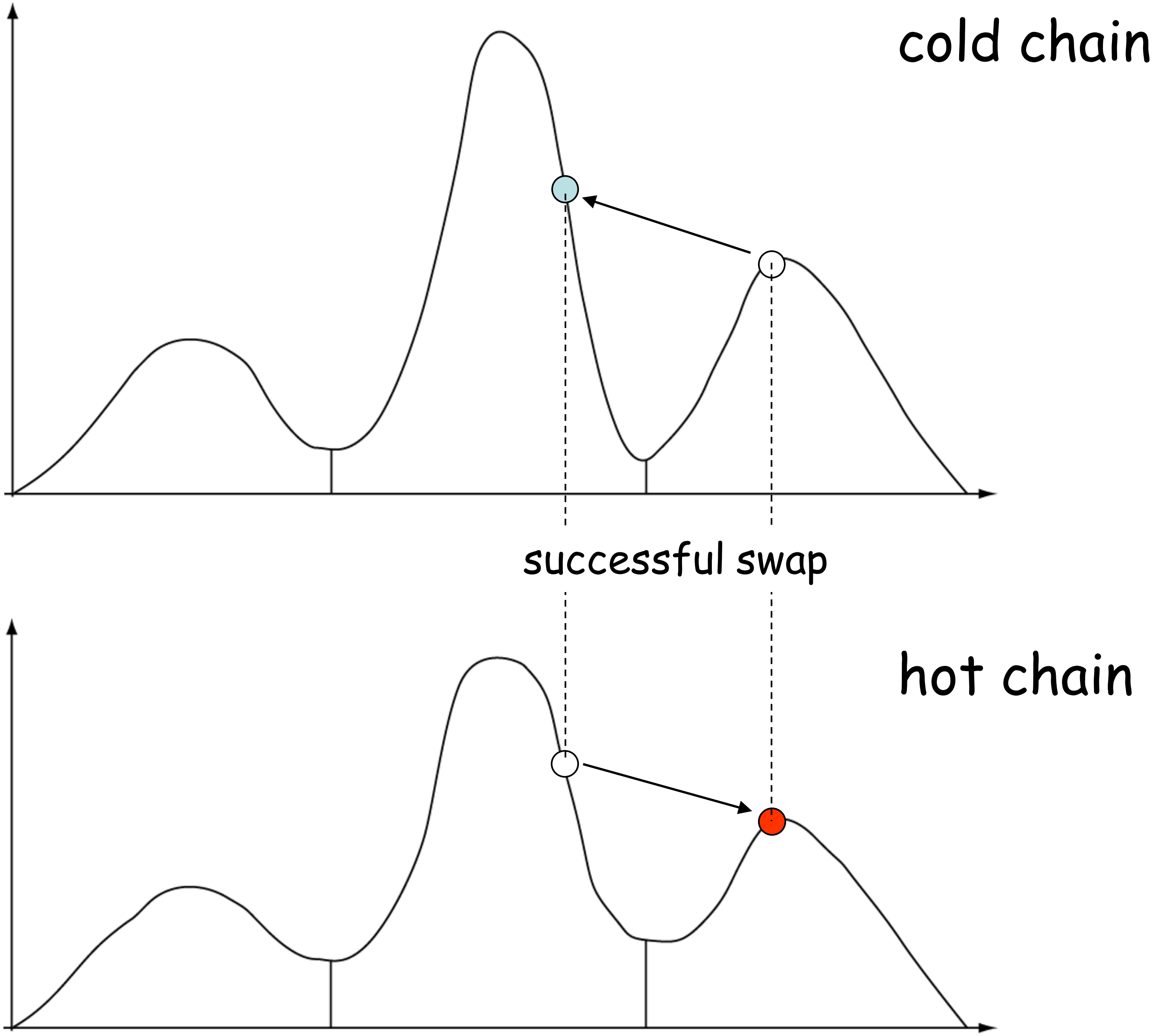
hot chain

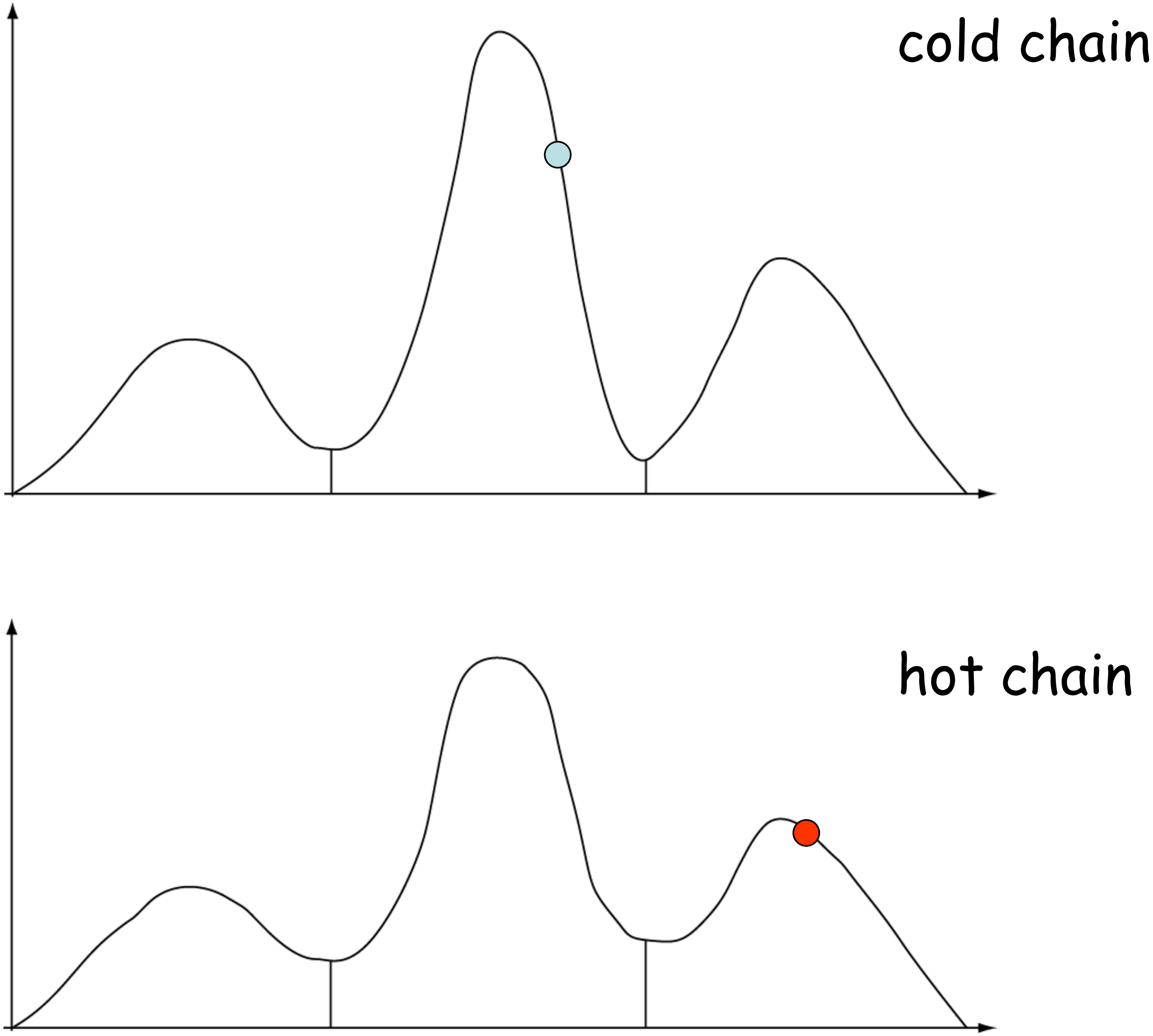



cold chain
successful swap
hot chain

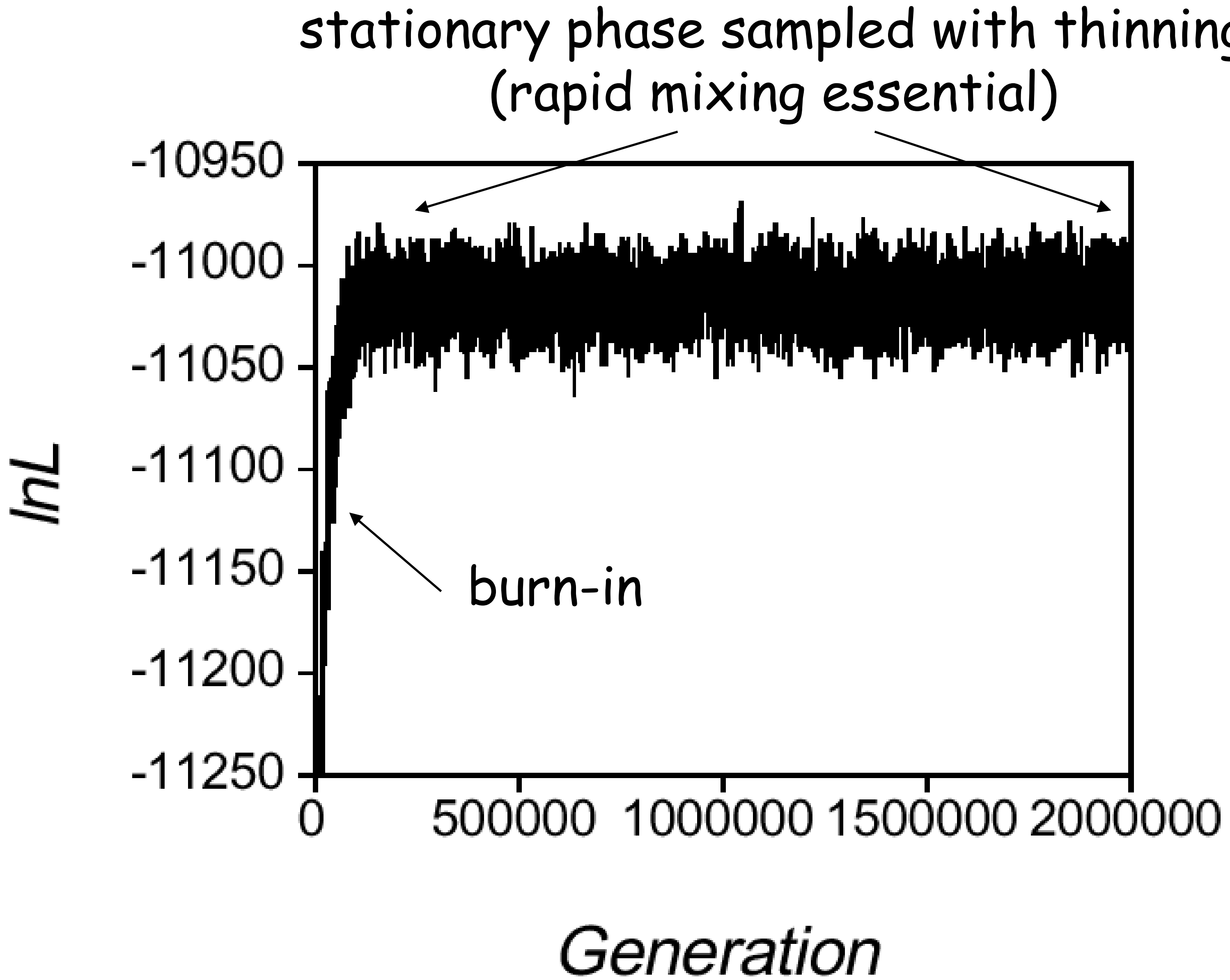

cold chain
hot chain


cold chain
hot chain


cold chain
successful swap
hot chain


cold chain
hot chain

Burn-in and stationarity



Bayesian Inference