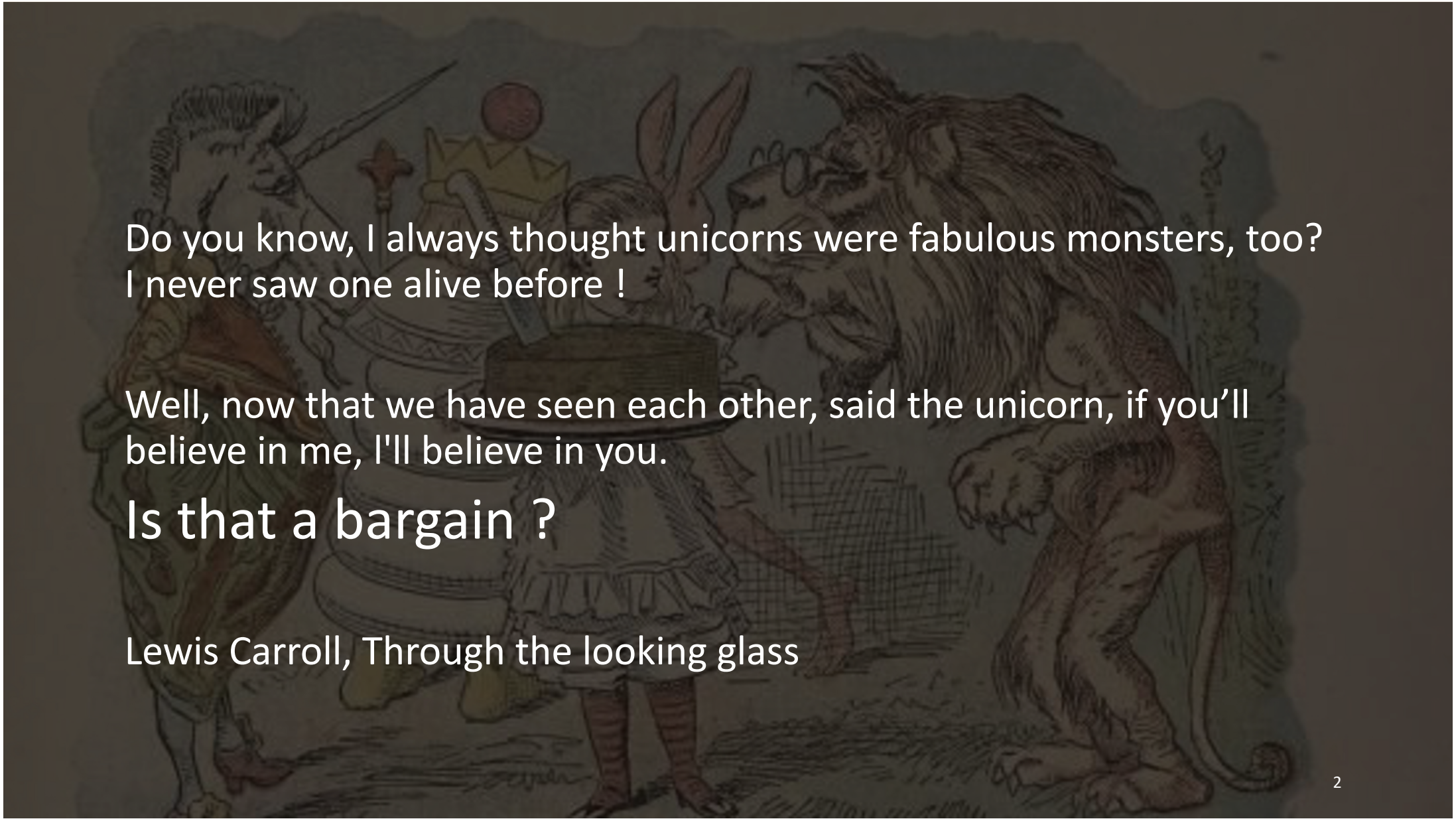


Física da Matéria Condensada

Margarida Telo da Gama

The image is a collage of various physics concepts and equations in condensed matter physics. It includes:

- A diagram of a hexagonal lattice with dashed lines and arrows, possibly representing a topological phase or a specific symmetry.
- A diagram of a honeycomb lattice with a central site and arrows, representing a spin system or a topological phase.
- The equation $G(k, \omega) = \frac{1}{G_0^{-1}(k, \omega) - \Sigma(k, \omega)}$.
- The equation $C = \frac{1}{2\pi} \iint \mathcal{F}(k) \cdot d^2k$.
- A diagram of a honeycomb lattice with a central site and arrows, representing a spin system or a topological phase.
- The equation $\mathcal{H}_{SO} = \Delta_{SO} \psi^\dagger \sigma_z \tau_z s_z \psi$.
- A diagram of a honeycomb lattice with a central site and arrows, representing a spin system or a topological phase.
- The equation $q_s = \text{Tr}_B (|\varphi\rangle\langle\varphi|)$.
- A diagram of a honeycomb lattice with a central site and arrows, representing a spin system or a topological phase.
- The equation $\dot{q} = \gamma \mathcal{L}[q]$.
- A diagram of a honeycomb lattice with a central site and arrows, representing a spin system or a topological phase.



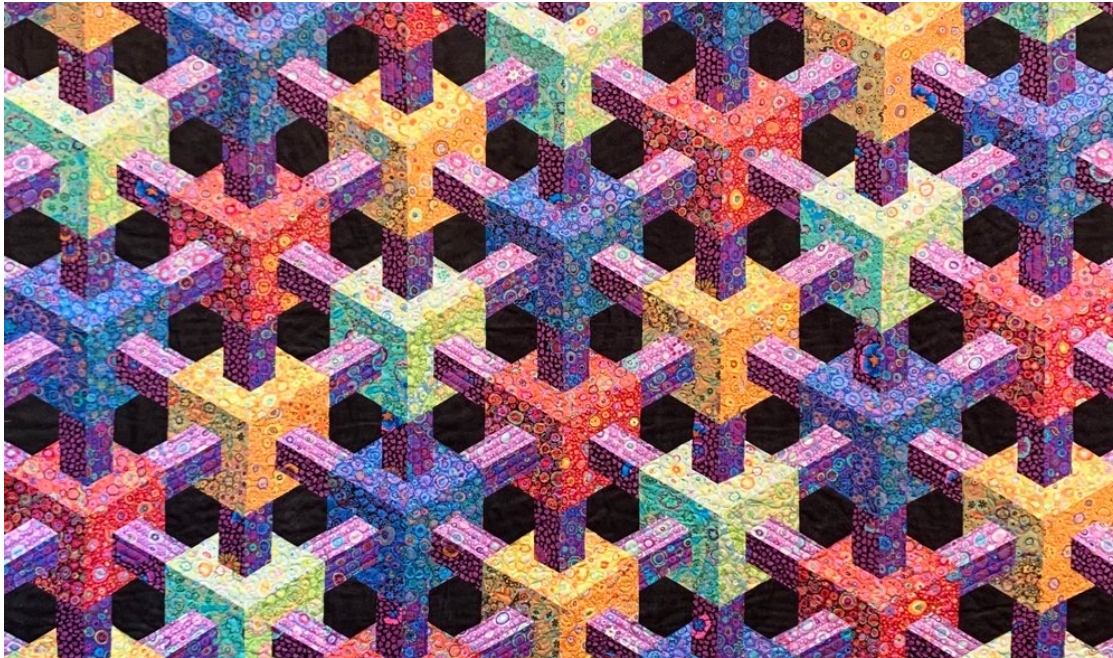
Do you know, I always thought unicorns were fabulous monsters, too?
I never saw one alive before !

Well, now that we have seen each other, said the unicorn, if you'll
believe in me, I'll believe in you.

Is that a bargain ?

Lewis Carroll, Through the looking glass

PROGRAMA (OUTLINE)



1. INTRODUÇÃO
2. ESTRUTURA CRISTALINA
3. ESTRUTURAS DOS SÓLIDOS
4. DIFRAÇÃO E DIFUSÃO ELÁSTICA DE ONDAS
5. LIGAÇÕES QUÍMICAS
6. VIBRAÇÕES ATÓMICAS
7. TERMODINÂMICA DE FONÕES
8. ESTADOS ELECTRÓNICOS
9. TERMODINÂMICA DE ELECTRÕES EM METAIS
10. CONDUTIVIDADE ELÉCTRICA E TÉRMICA
11. ELECTRÕES EM SEMICONDUTORES



BIBLIOGRAFIA

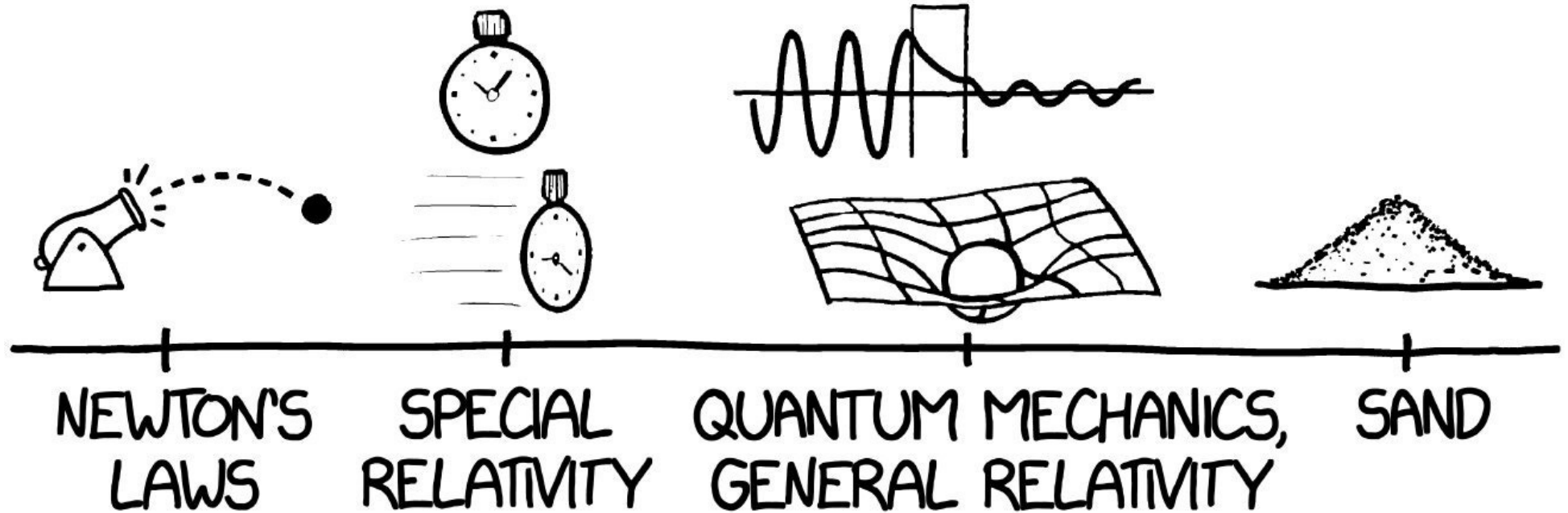
1. Fundamentals of Solid State Physics, J. R. Christman, Wiley, 1988.
2. Solid State Physics, N. W. Ashcroft and N. D. Mermin, Holt, 1976.
3. Introduction to Solid State Physics, 5th ed., C. Kittel, Wiley, 1976.
4. Solid State Physics, An Introduction to Principles of Materials Science, H. Ibach and H. Luth, Springer, 1995.
5. Solid State Physics, H. E. Hall, Wiley, 1974

AValiação

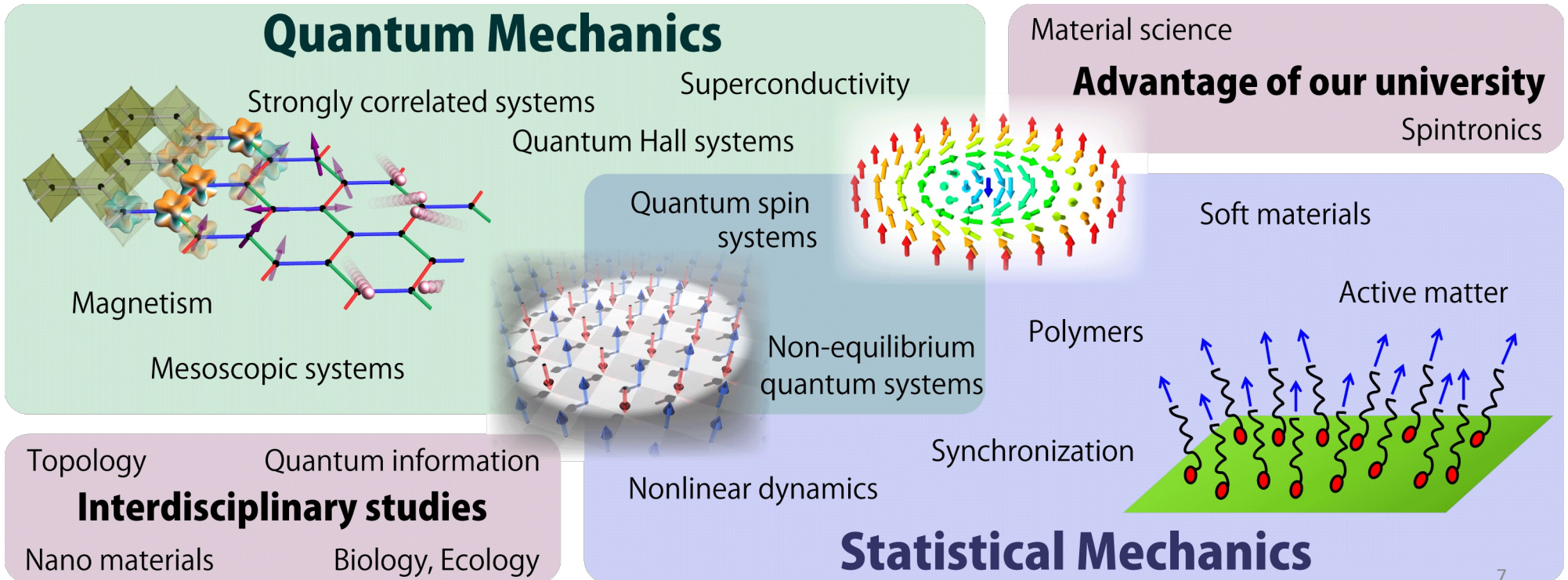
- Contínua: entrega da resolução escrita de 1-3 problemas das séries (grupos de 2 alunos) seguida da resolução no quadro durante as TPs (25%) e exame final (75%). Válida para a primeira data em que os alunos se apresentarem a exame.
- Exame
- O default é a modalidade de avaliação contínua. Para realizarem apenas o exame os alunos devem comunicar ao docente por escrito, justificando esta escolha.

AREAS OF PHYSICS BY DIFFICULTY

HARDER →



1. Introduction



What is condensed matter ?

Collective properties that emerge from the interactions of many particles:

- Quantum or classical Dynamics to calculate the energy spectrum (states) - E_N
- Statistical Mechanics to calculate the occupation probability of each state - $P(E_N)$

What is condensed matter physics ?

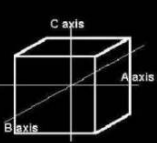
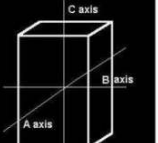
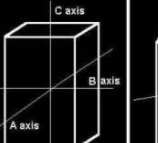

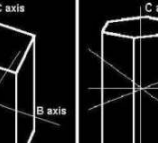
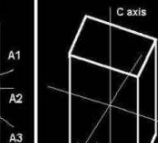





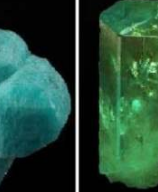


Properties of materials in terms of the interacting building blocks:

- Hard condensed matter: electrons & nuclei
- Soft condensed matter: polymers, colloids ...

Response to external fields:

- Linear
- Non-linear

2. Crystal structure: Lattices

Crystal Systems						
Isometric	Tetragonal	Orthorhombic	Monoclinic	Triclinic	Hexagonal	Trigonal
						
						
Fluorite	Wulfenite	Tanzanite	Azurite	Amazonite	Emerald	Rhodochrosite

GeologyIn.com

$$\log_c\left(\frac{a}{b}\right) = \log_c a - \log_c b$$

$$\sum_{k=1}^n k = \frac{1}{2}n(n+1)$$

$$\log_a 1 = 0$$



$$\begin{aligned} 100 &= c^2 \\ \sqrt{100} &= \sqrt{c^2} \\ \pm 10 &= c \end{aligned}$$

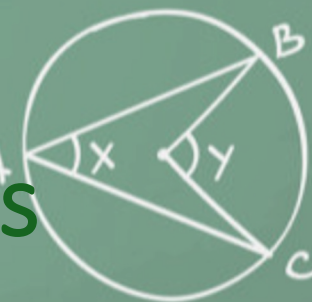
B.14

$$f(-x) = a(-x) + b = -(ax - b)$$

$$(x+y)^n = \sum_{k=0}^n {}^n C_k x^{n-k} y^k$$



$$3^0 = 1$$



$$\begin{aligned} \frac{x}{x+2} - \frac{8}{x+6} &= \\ &= \frac{16}{x^2 + 8x + 6} \end{aligned}$$



$$a^b a^c = a^{b+c}$$

$$\sin^2 y + \cos^2 y = 1 \quad y = \frac{k}{x} \quad \sqrt[n]{x} = x^{\frac{1}{n}}$$

Geometry of crystals

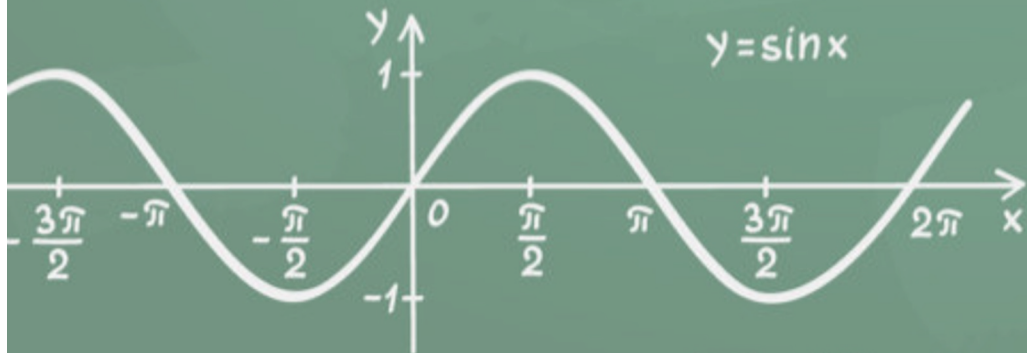
$$c^2 = a^2 + b^2$$

$$+2lh + 2wh$$

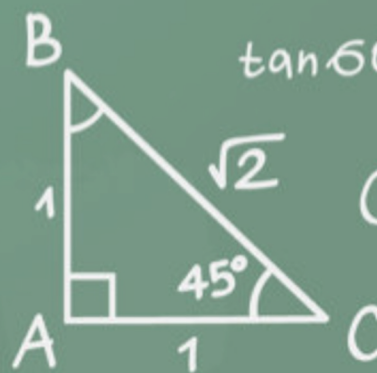
$$(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$$

$$y = ax^2 + bx + c$$

$$\tan 60^\circ = \sqrt{3}$$



$$A = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr$$



$$C = 2\pi r$$

$$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 \quad S = \frac{a+b+c}{2}$$

$$A = sr \quad r = \frac{A}{s} \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$(8^2)^3 = 8^{2 \times 3} = 8^6$$

Ideal solid

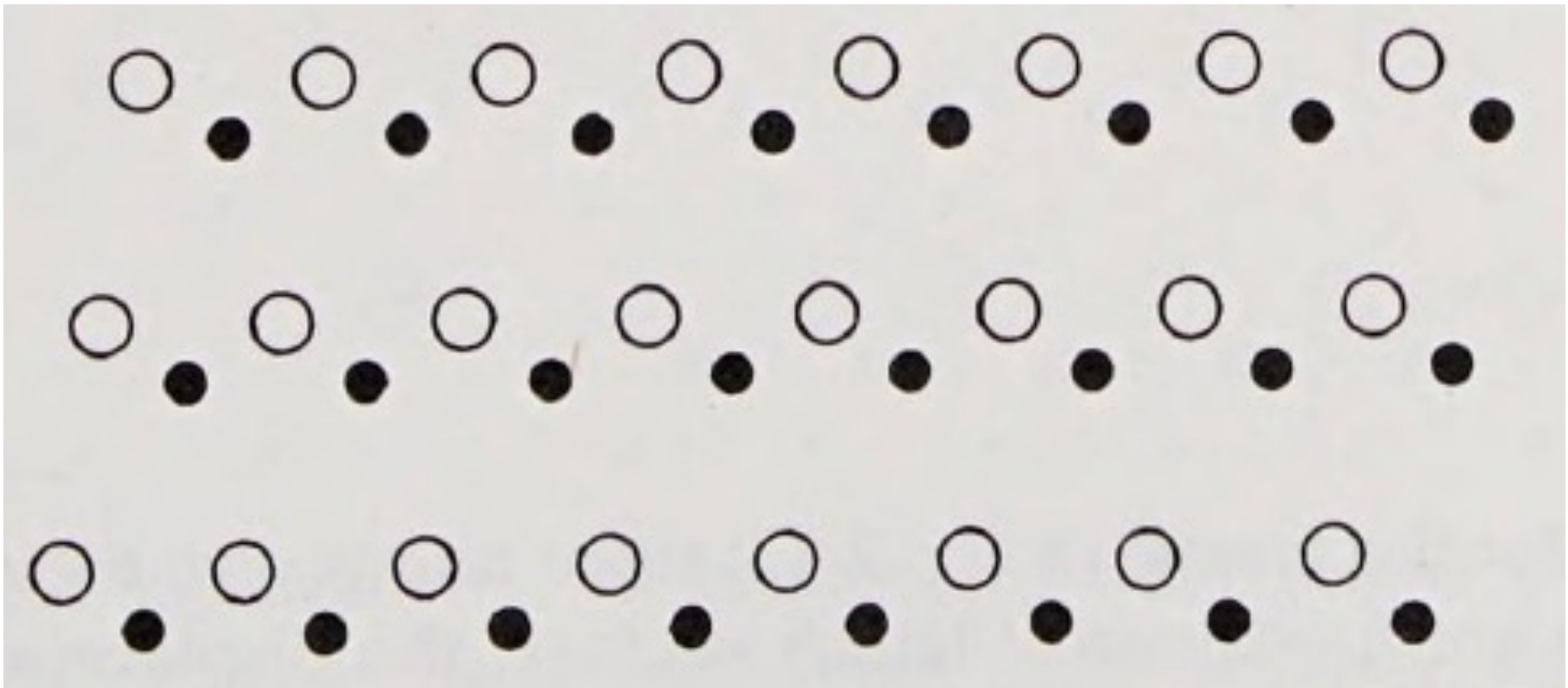
Periodic structure where the atoms are placed regularly within the medium exhibiting symmetry of translation.

Mathematically, there is symmetry of translation, in 3d, when there are, 3 no coplanar, vectors such that the medium is invariant for a translation **T**:

$$\mathbf{T} = n_1\mathbf{a} + n_2\mathbf{b} + n_3\mathbf{c}$$

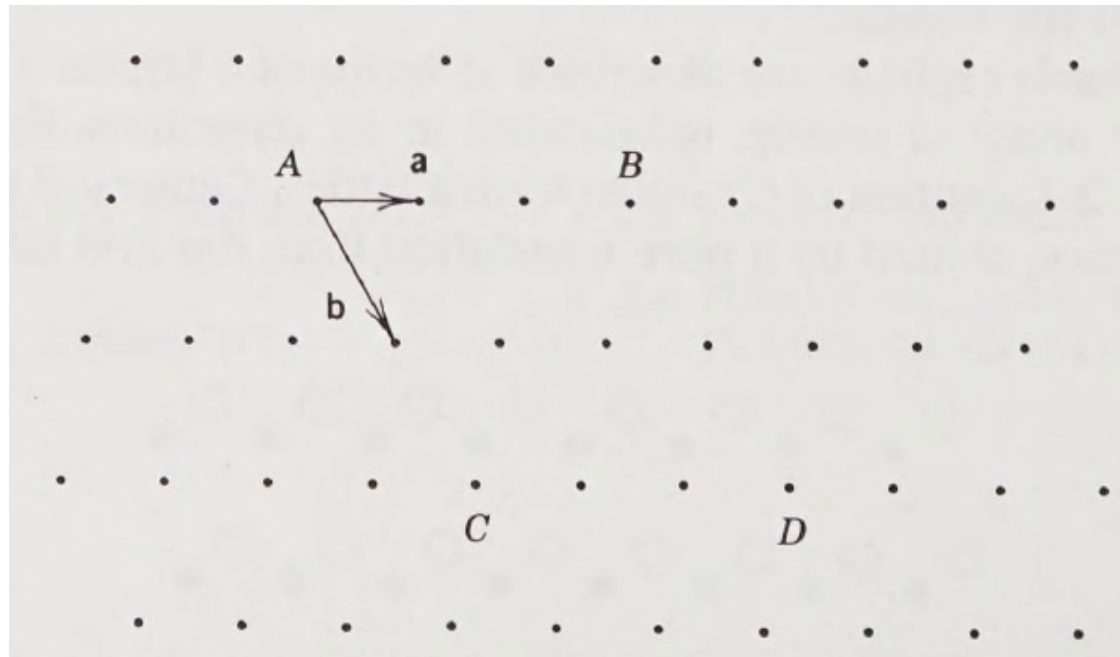
for all integers n_i .

2D crystalline solid: the basis of two atoms is repeated periodically

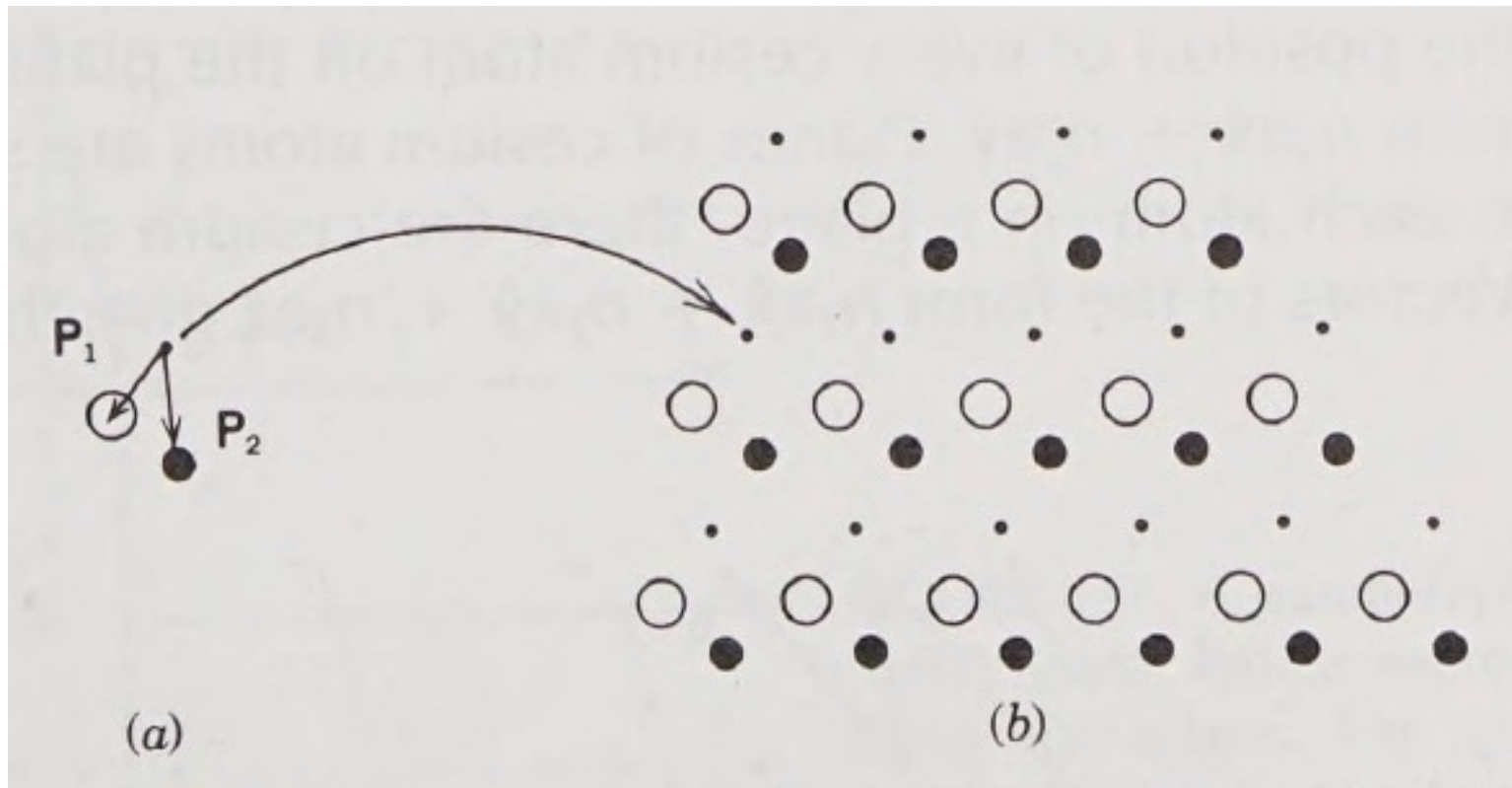


Lattice points give the positions of the basis:
a and **b** are the fundamental lattice vectors

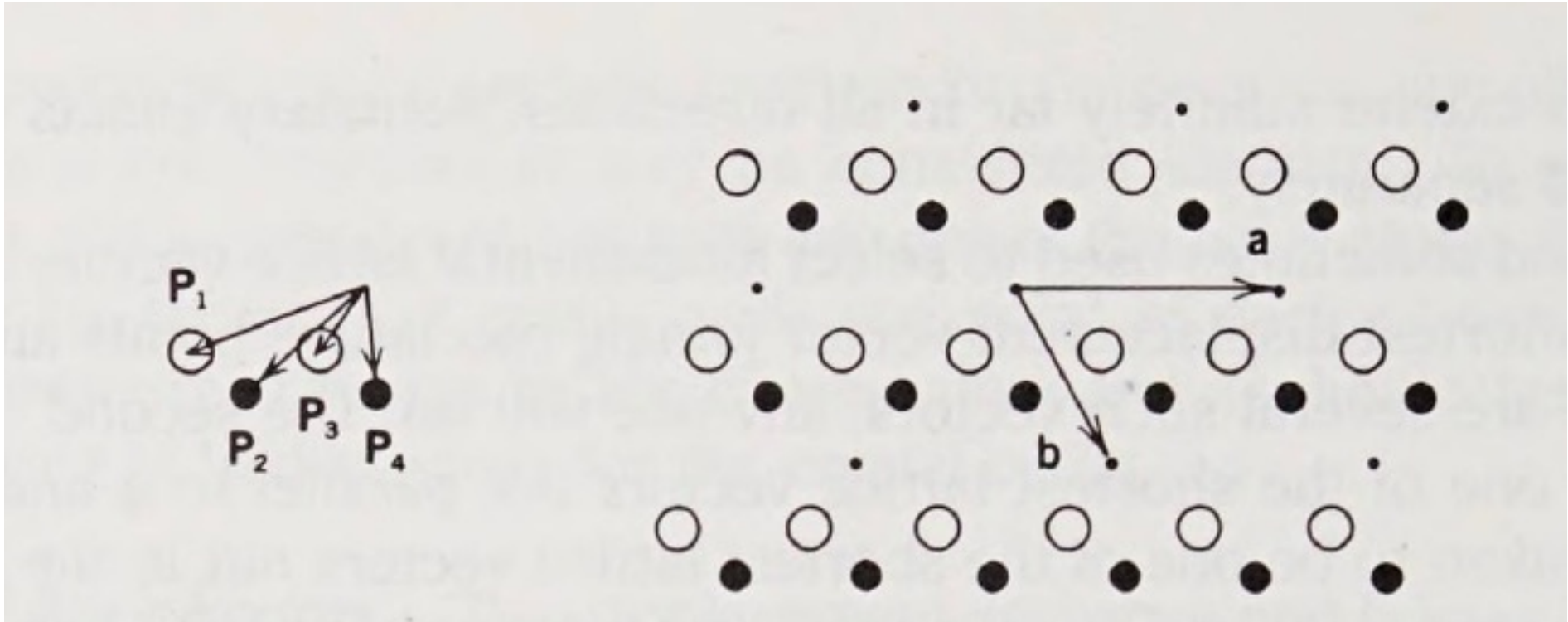
Displacement of any lattice point is $n_1\mathbf{a}+n_2\mathbf{b}$



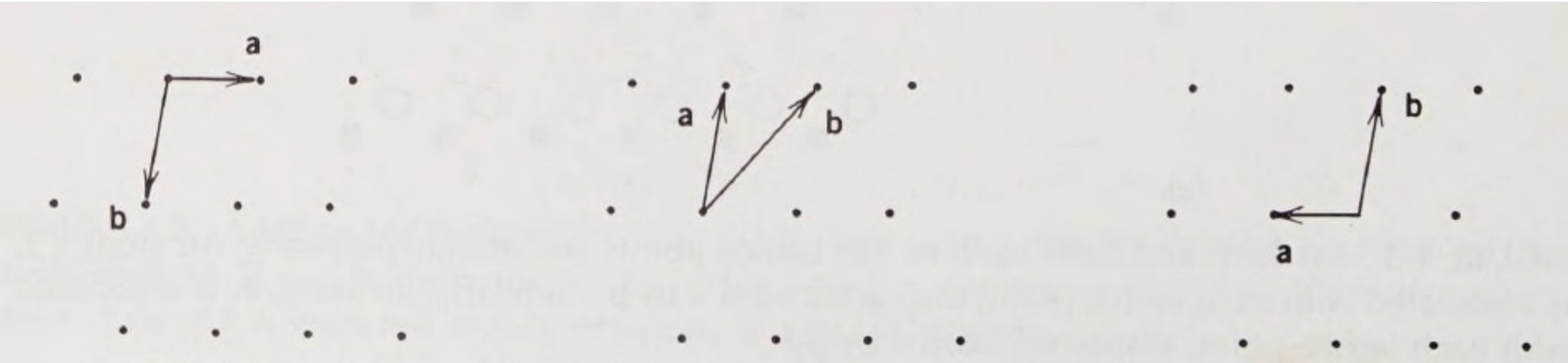
Basis and basis vectors (a) lattice points and atomic positions (b)



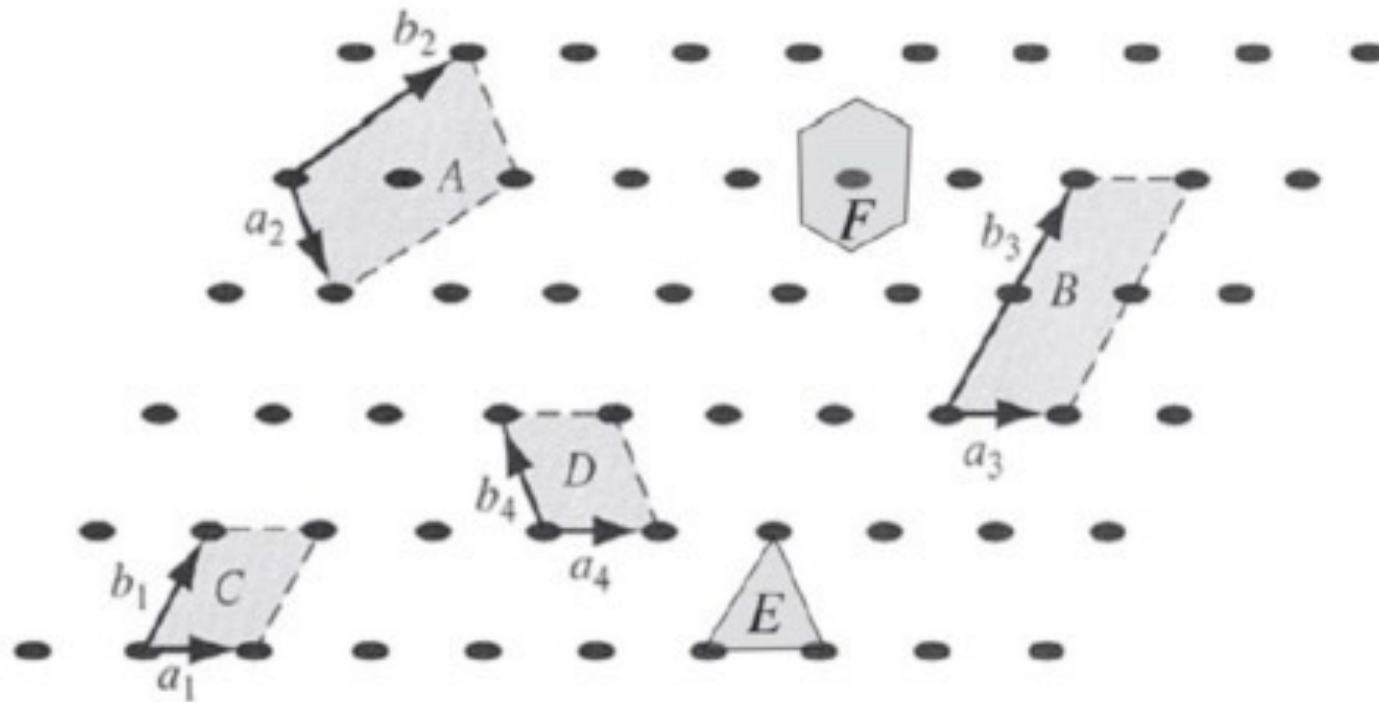
Another basis and the same lattice



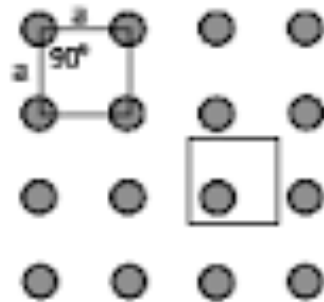
Primitive lattice vectors correspond to the smallest possible basis



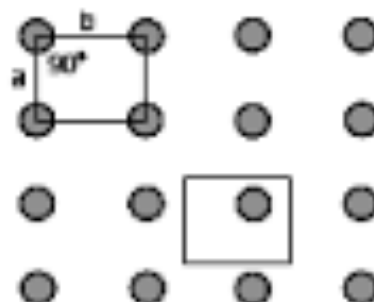
Lattice vectors and unit cells



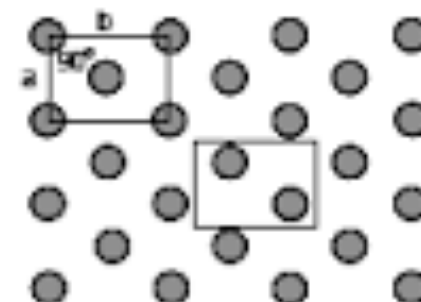
Unit cells



square lattice
square unit cell

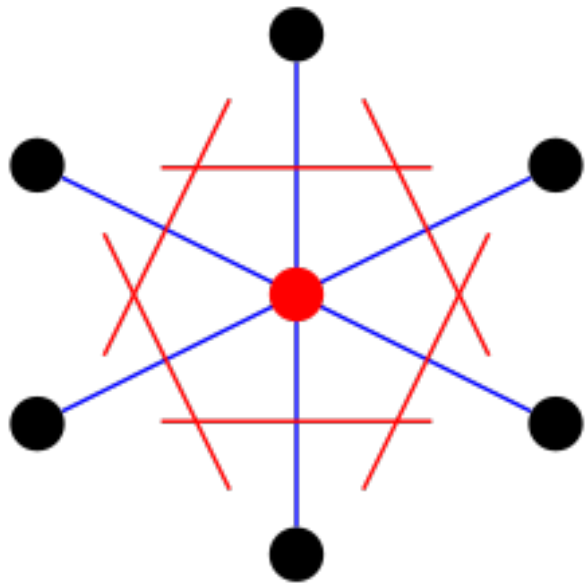


rectangular lattice
rectangular unit cell

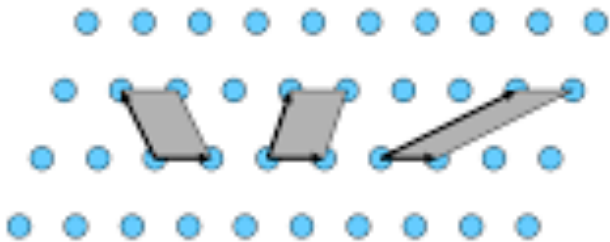


rectangular lattice
centered rectangular unit cell

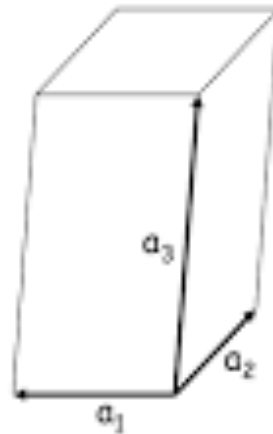
Wigner-Seitz cell



Volume of a unit cell



There is more than one choice for a primitive unit cell



Primitive unit cell

Volume of a unit cell
 $|\mathbf{c} \cdot \mathbf{a} \times \mathbf{b}|$

Rigid symmetry operations: Point & spatial



Reflection



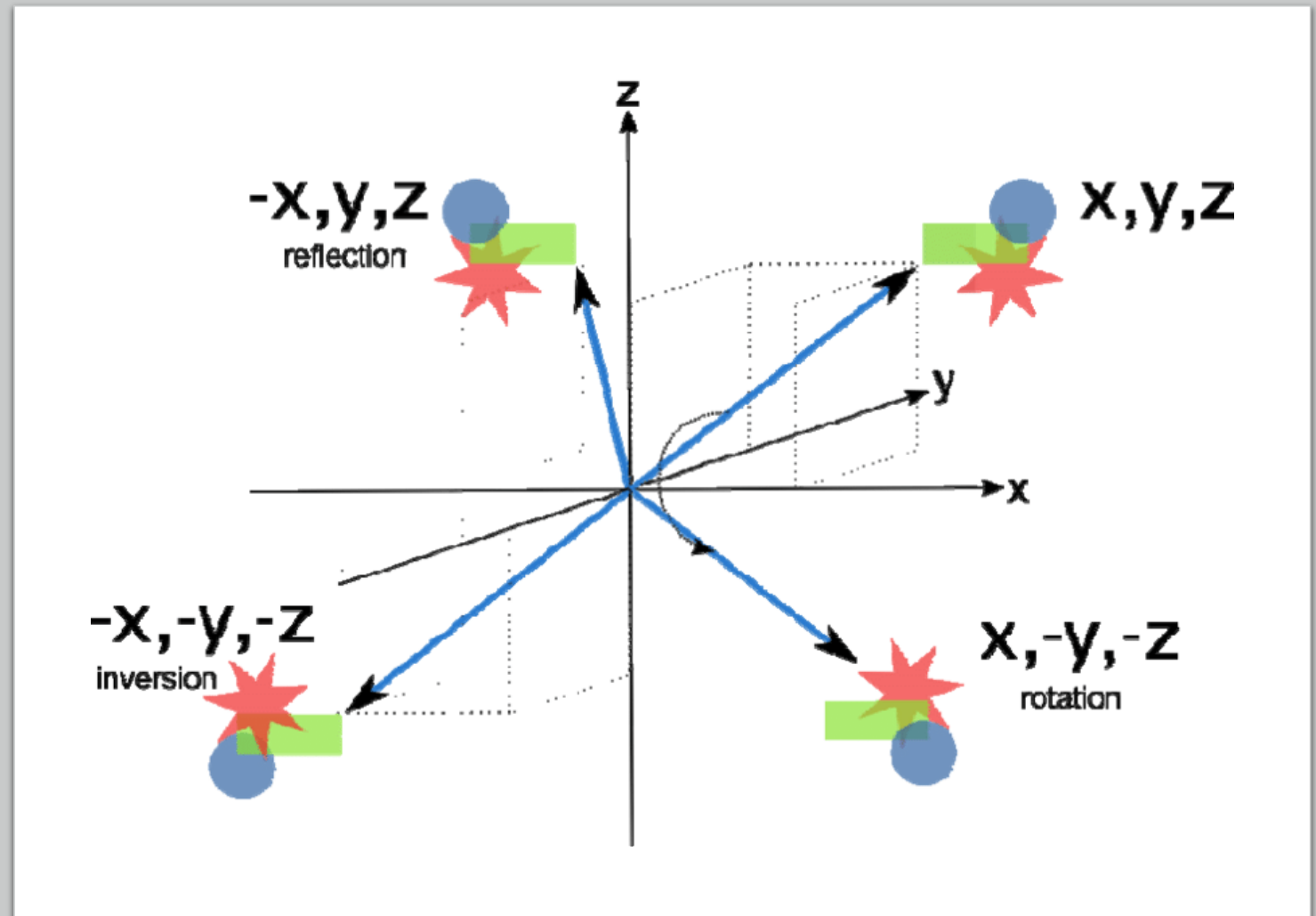
Rotation



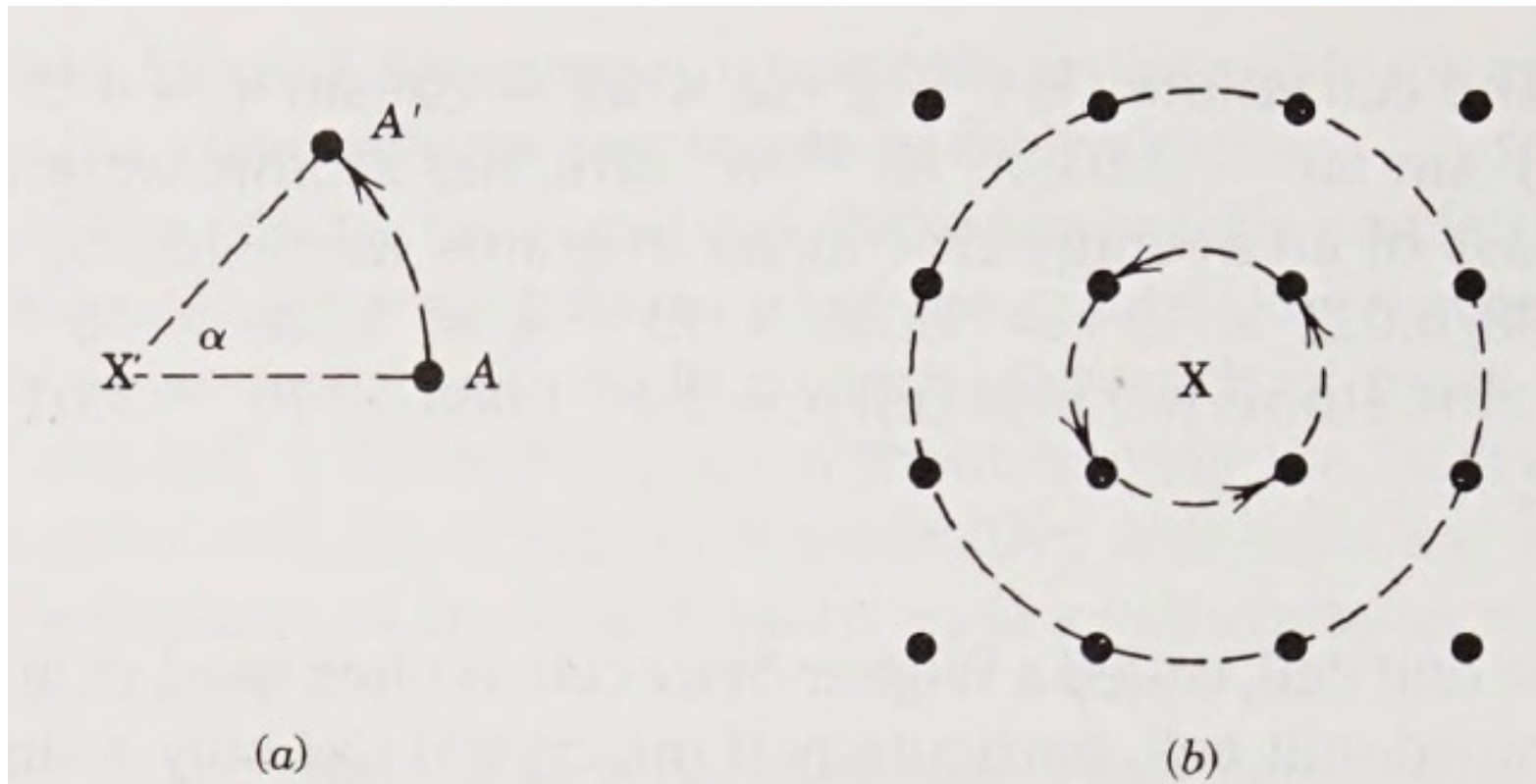
Translation

Point symmetries

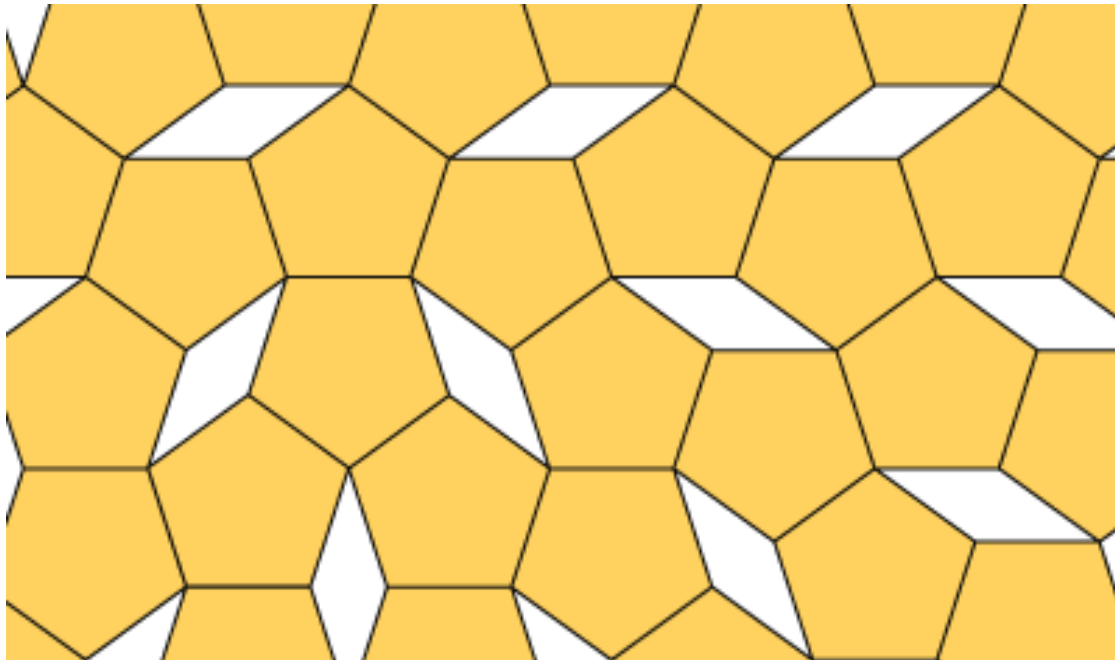
Mirror,
rotation and
inversion

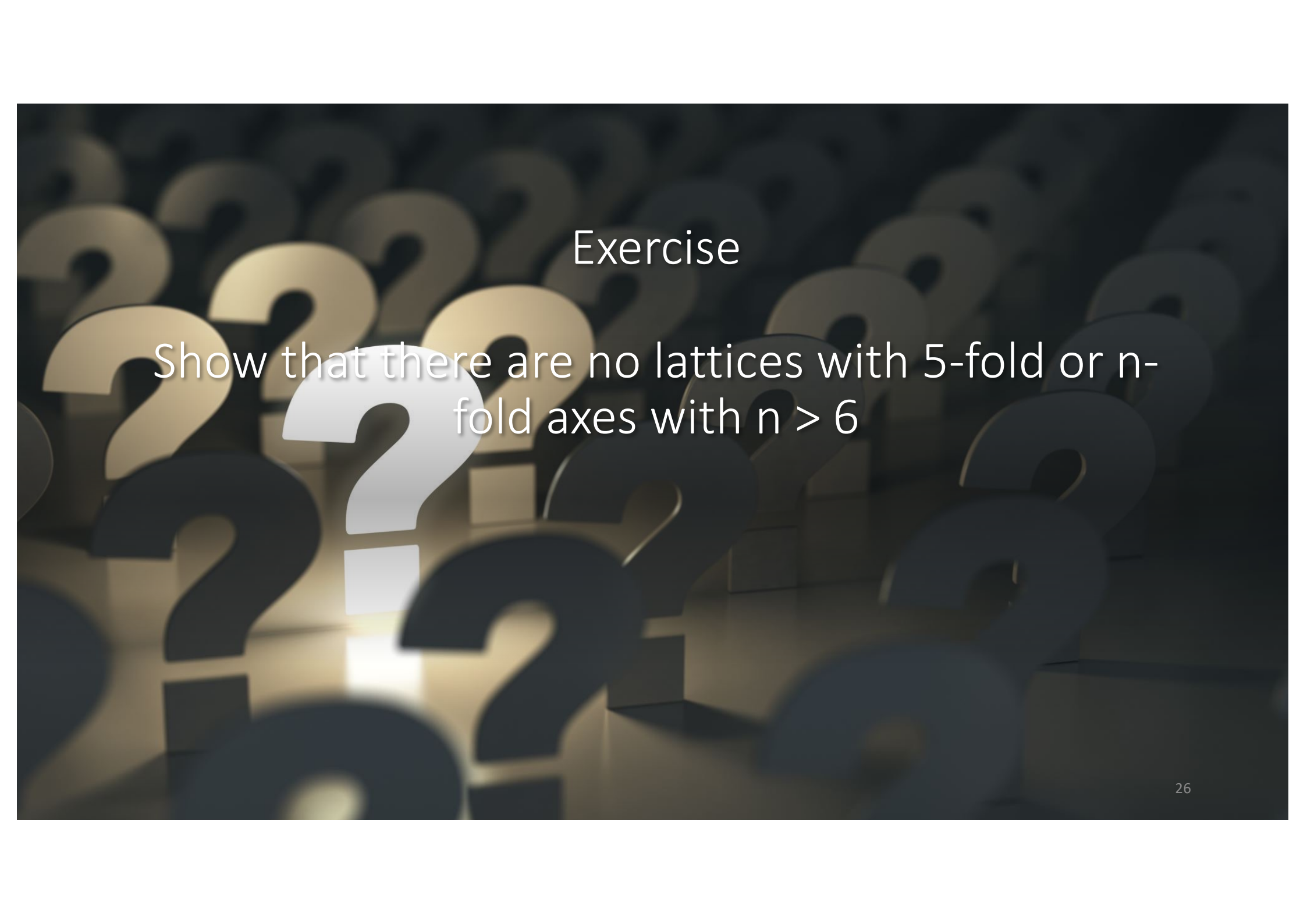


Rotational symmetry



Crystals do not have 5-fold rotational axes

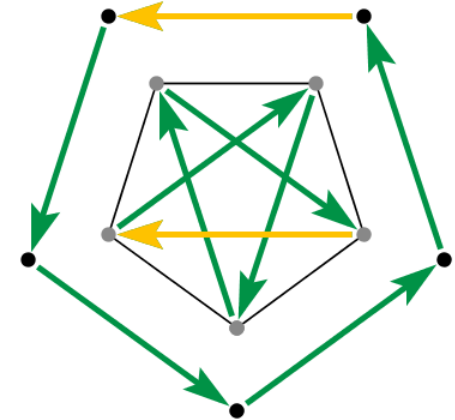
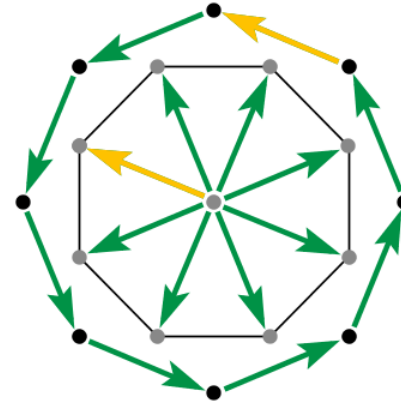
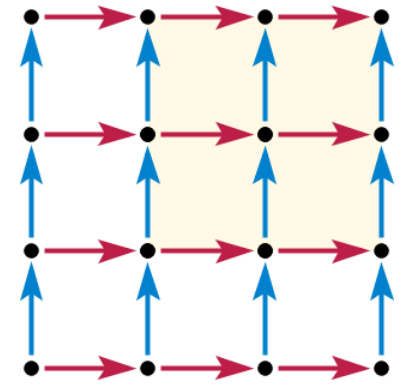
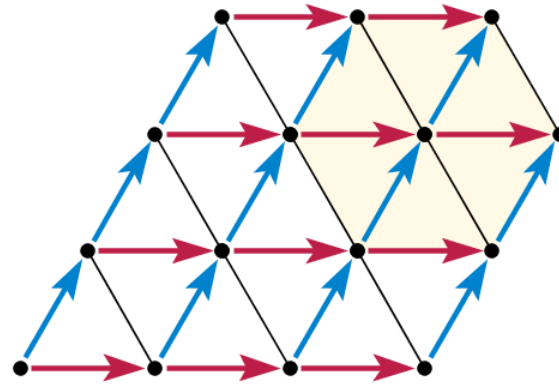


The background of the slide is a dark, textured surface covered with numerous question marks of varying sizes and colors, including shades of gold, brown, and dark grey. The question marks are scattered across the entire frame, creating a sense of mystery and inquiry.

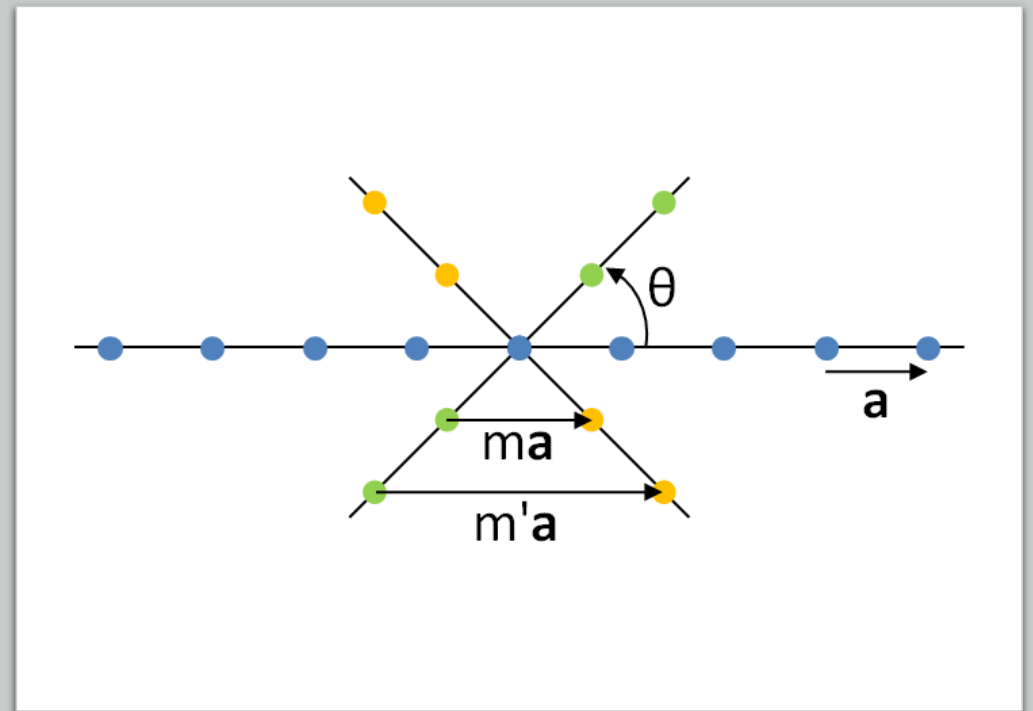
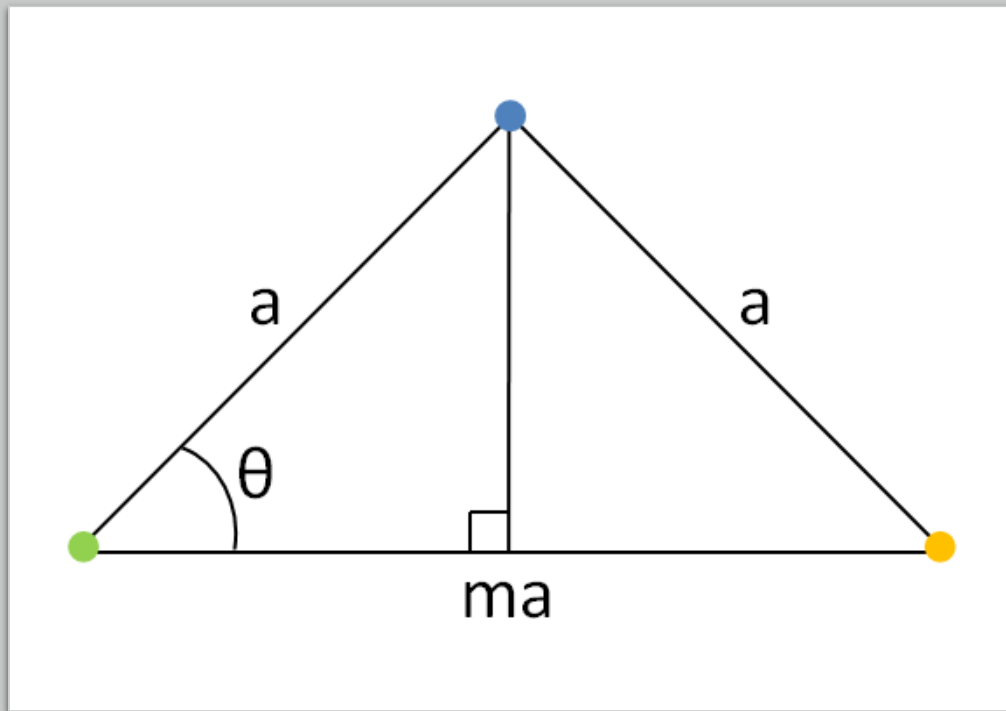
Exercise

Show that there are no lattices with 5-fold or n -fold axes with $n > 6$

Lattice proof



Geometric proof



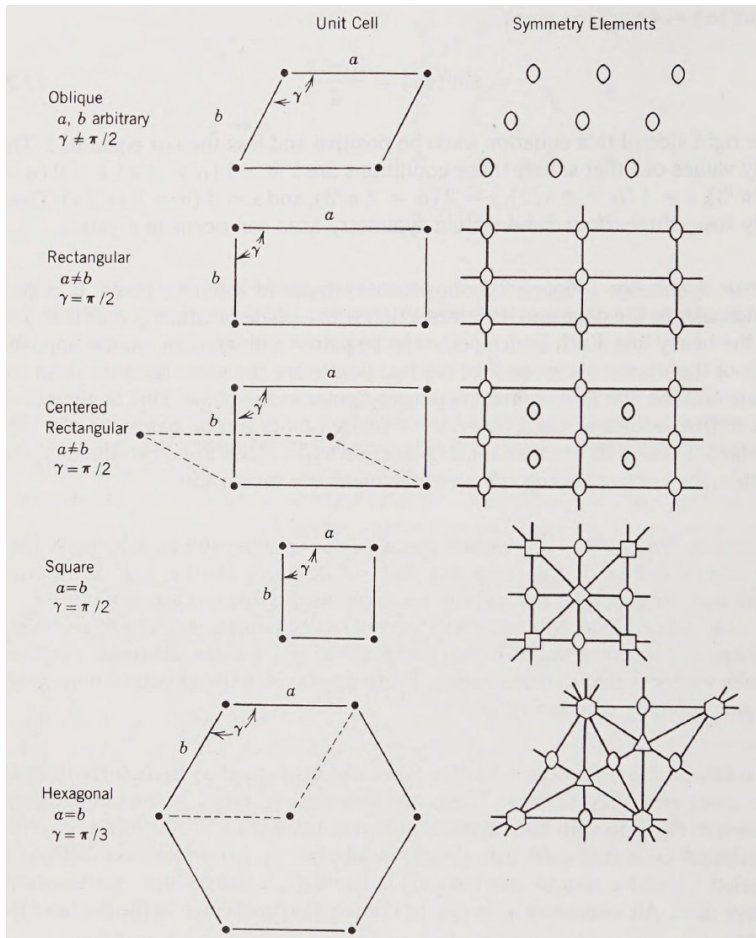
Rigid symmetries are not independent

For example, a 2-fold axis perpendicular to a mirror plane implies inversion symmetry (prove this).

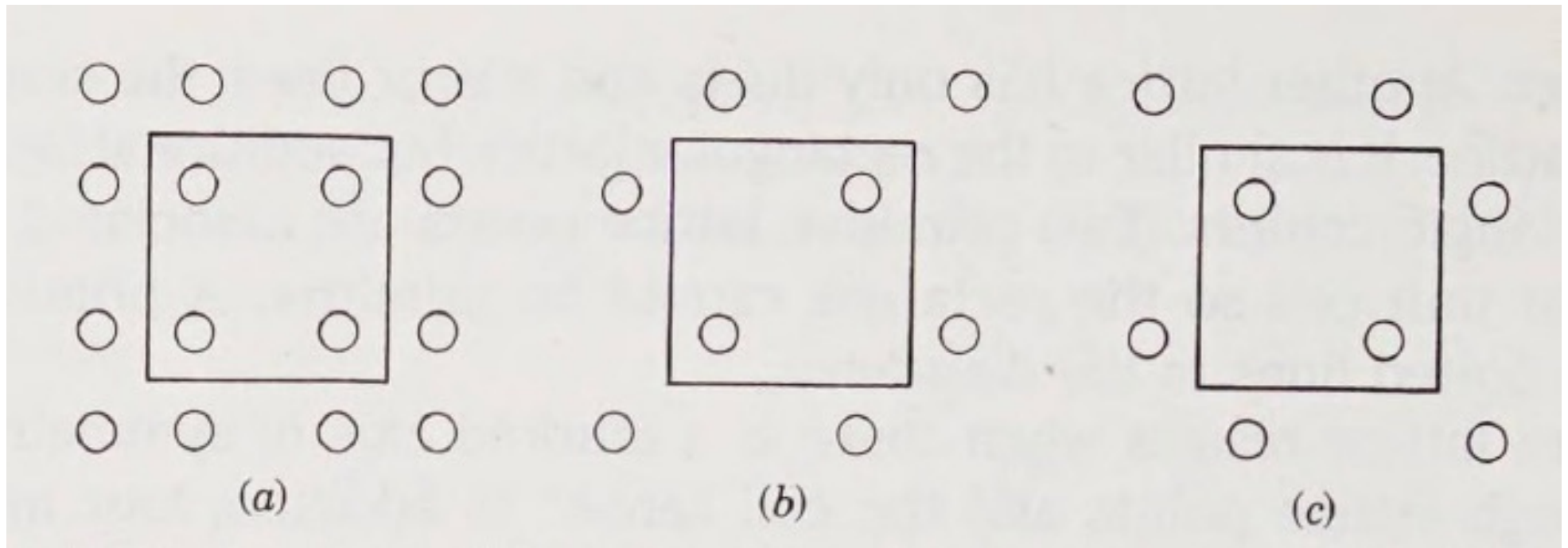
Small number of symmetry groups in 2 and 3 dimensions.

Point symmetry groups: Crystallographic systems

Spatial symmetry groups: Bravais lattices



2D
 Unit cells and symmetry groups
 5 Bravais lattices
 4 crystallographic systems

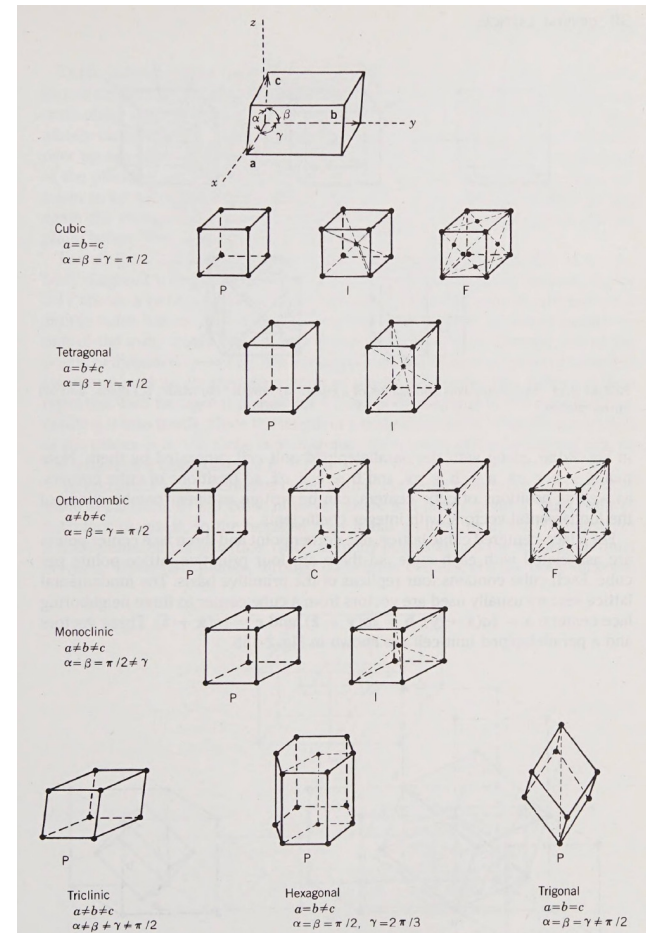


3D

Unit cells and symmetry groups

14 Bravais lattices

7 crystallographic systems

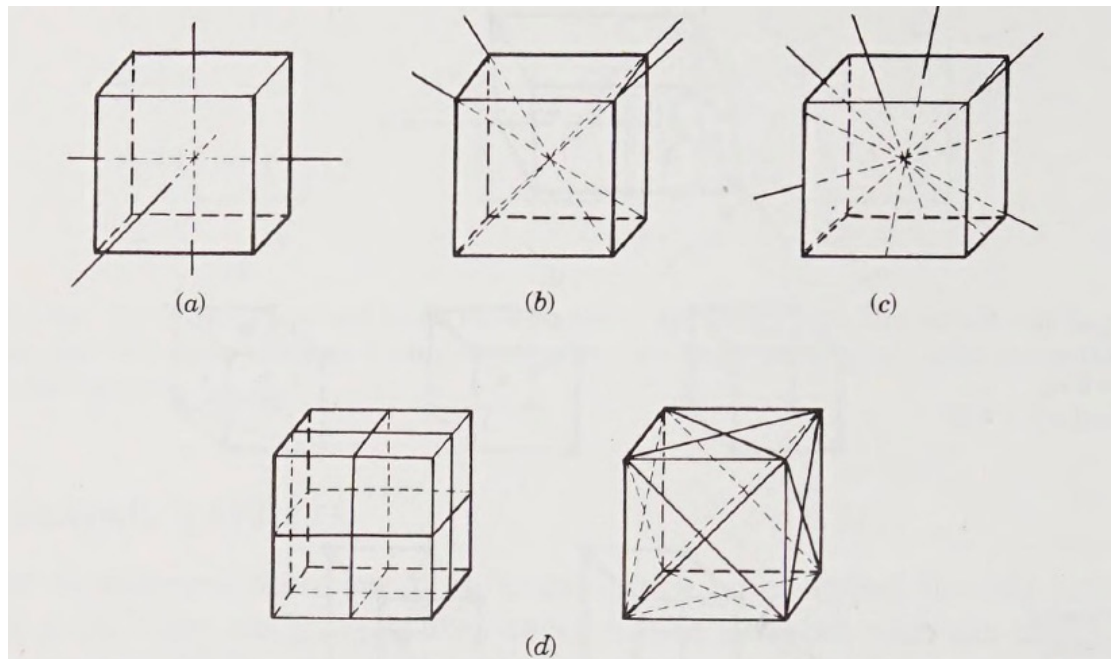




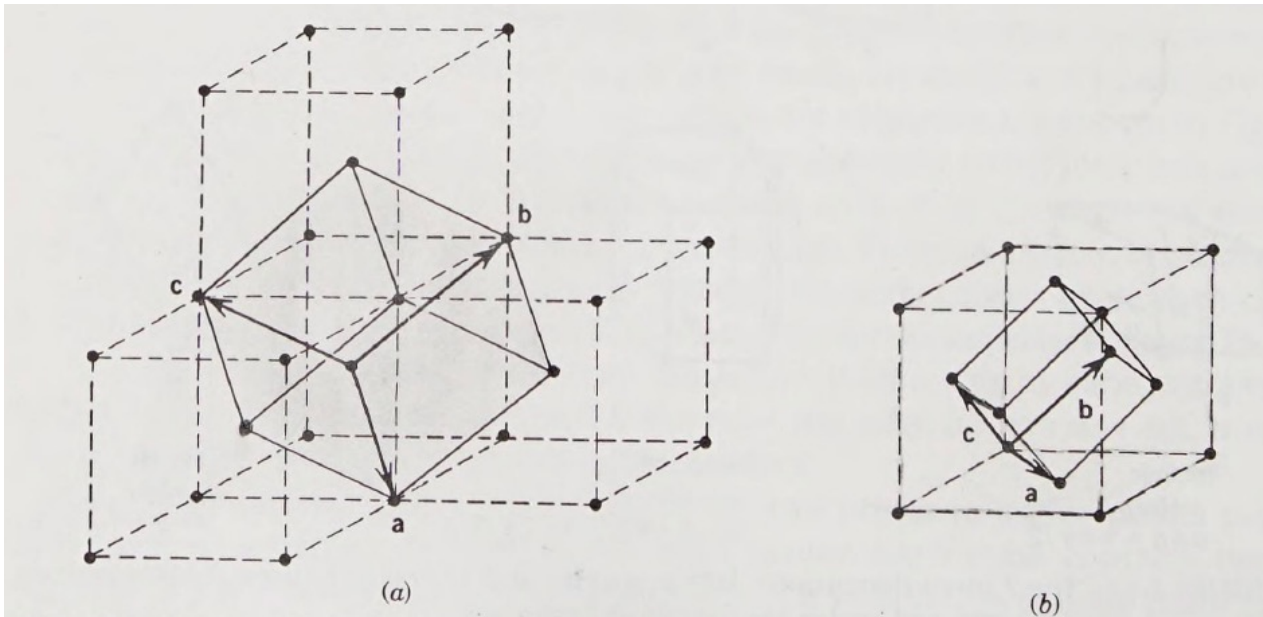
Questions

Why is there no cubic lattice of type C ?
And tetragonal of type F ?

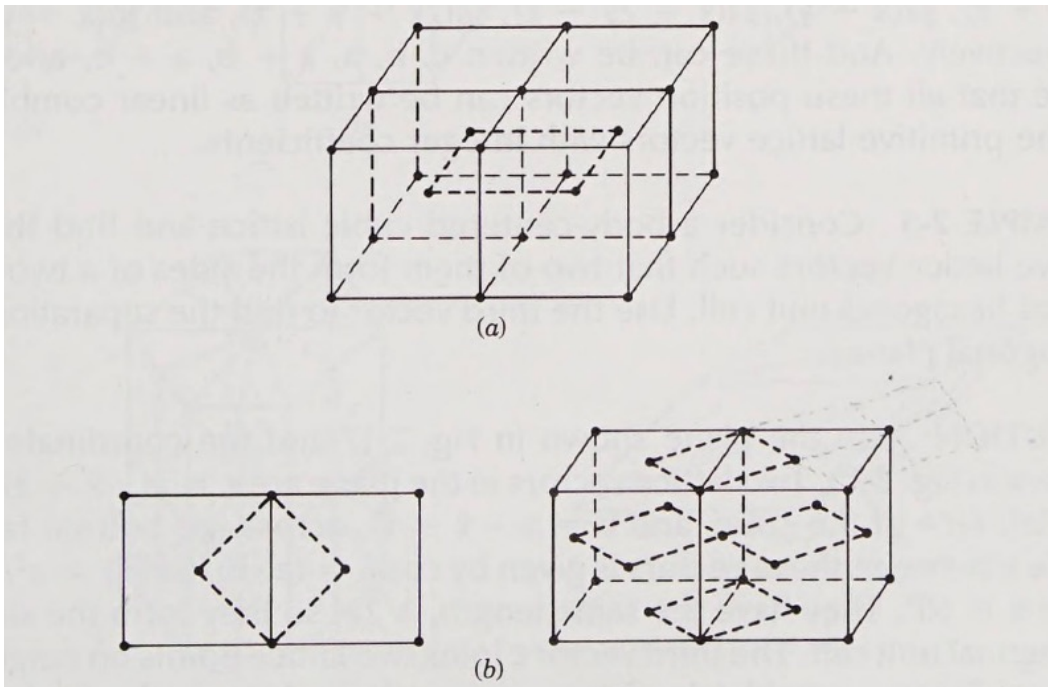
Symmetry axes and planes of a cube



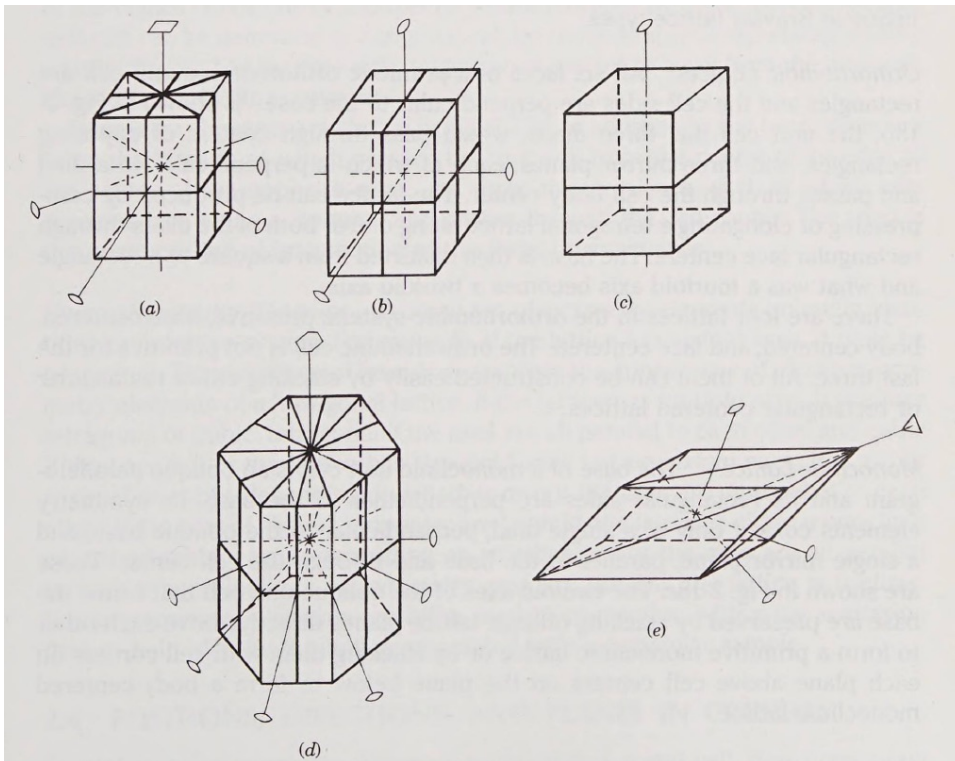
Primitive translation vectors and primitive cells for bcc and fcc



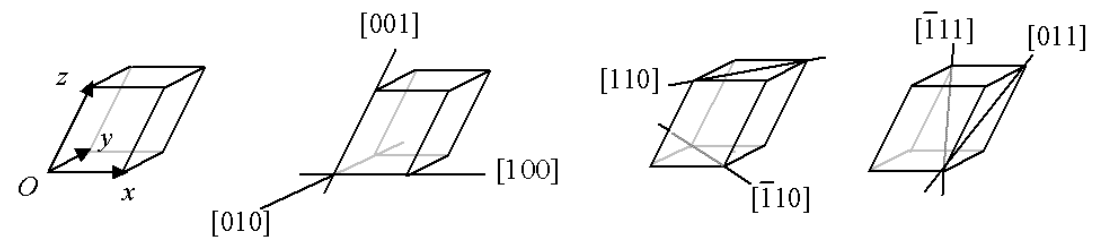
Stacking of square lattices to form bcc and fcc



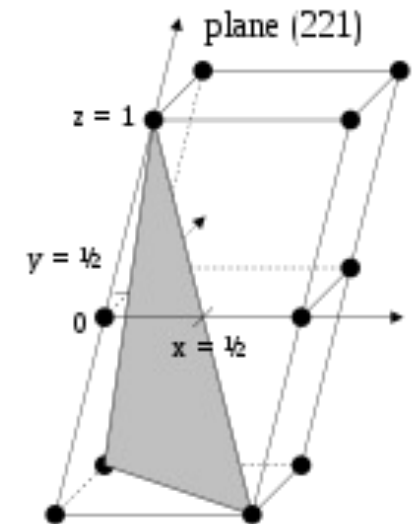
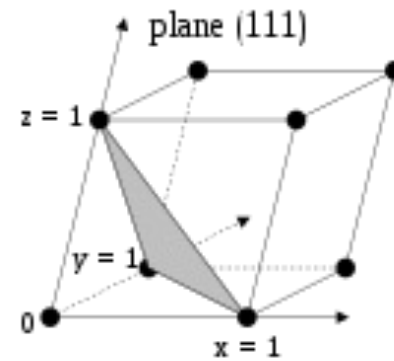
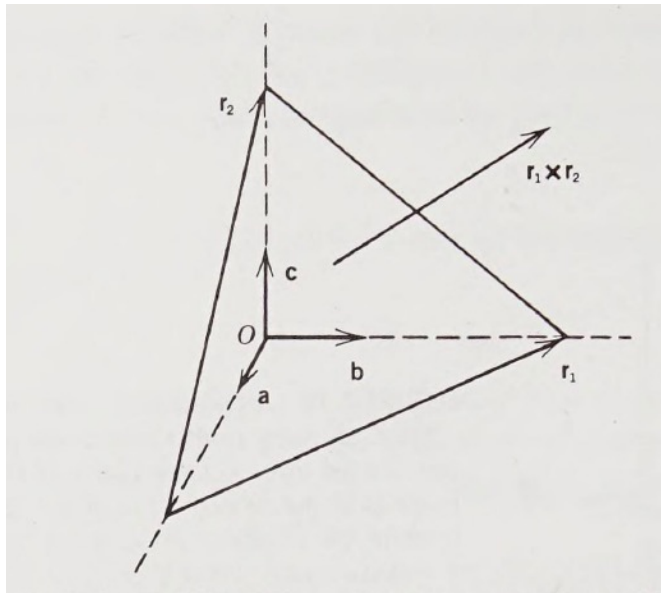
Symmetry elements of unit cells



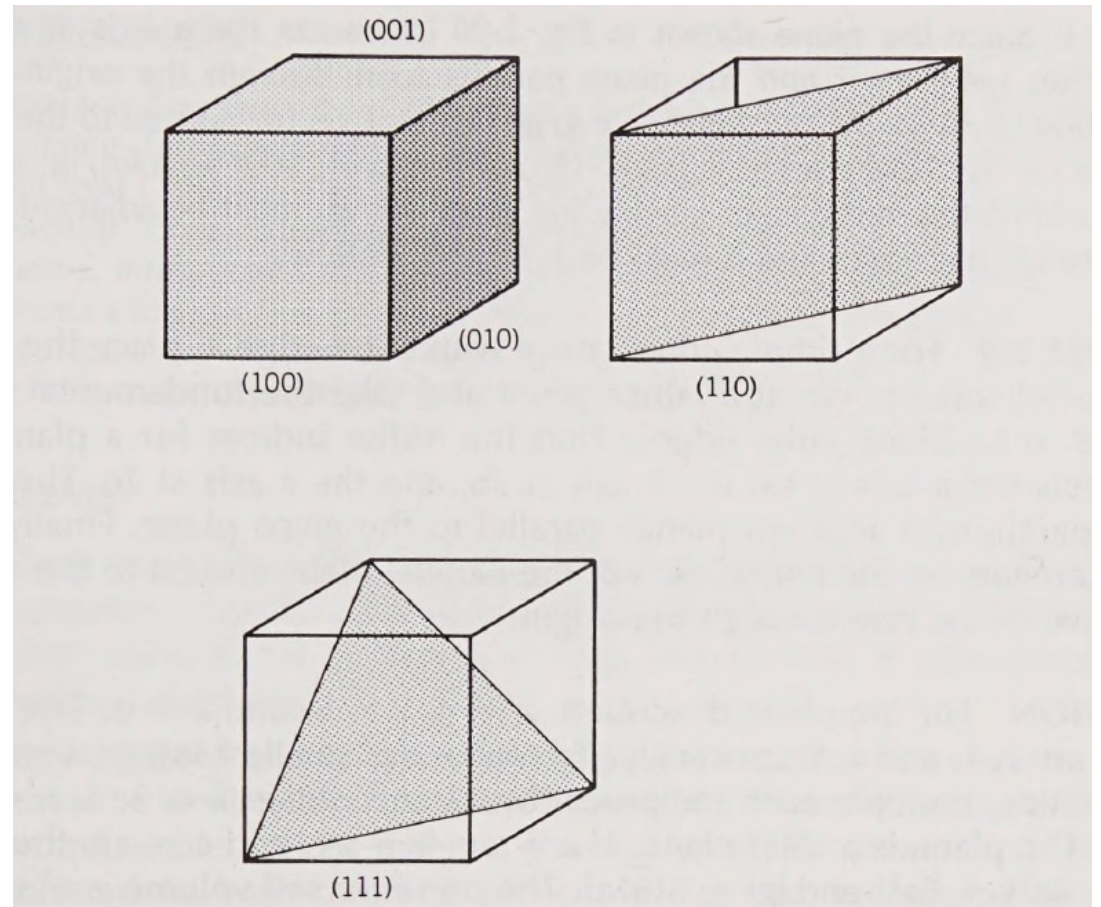
Directions



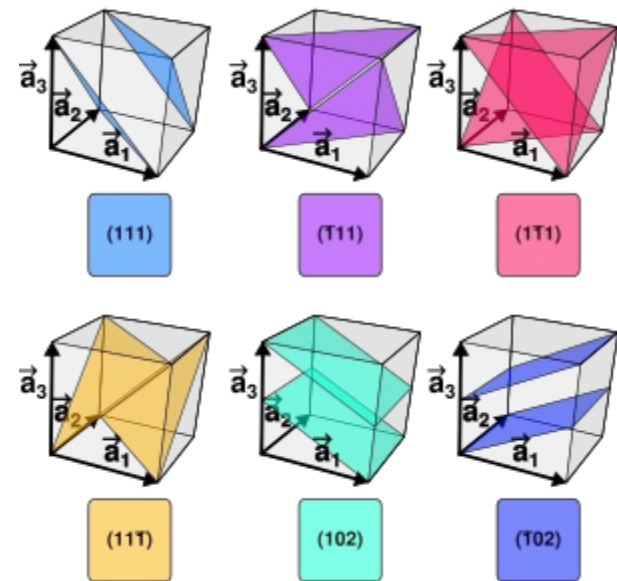
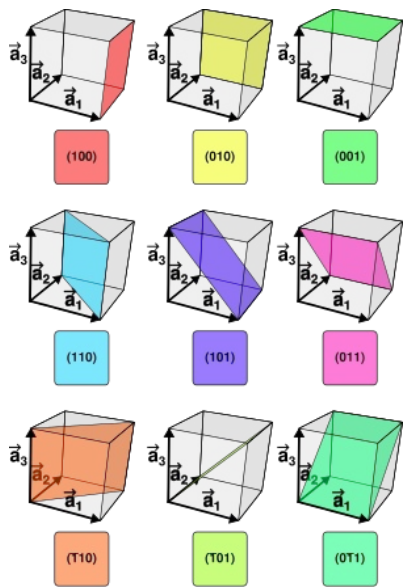
Crystallographic planes: Miller indices



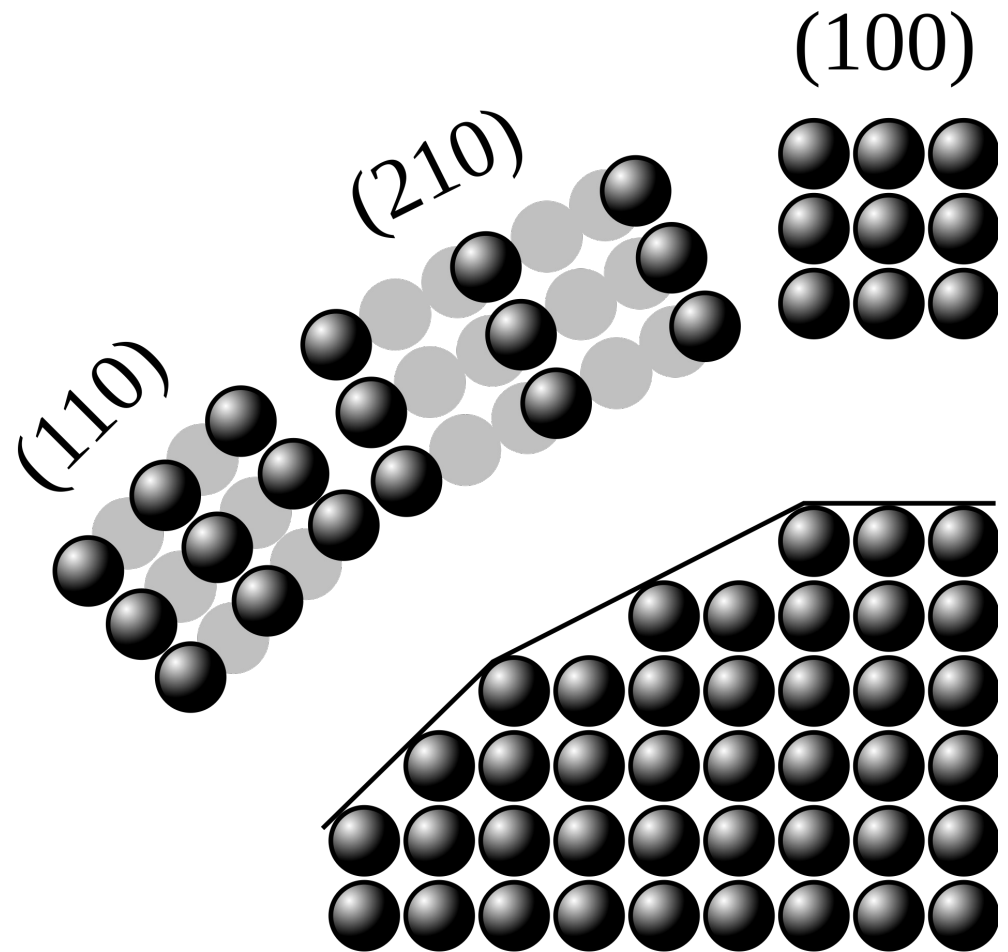
Planes of cubic lattices



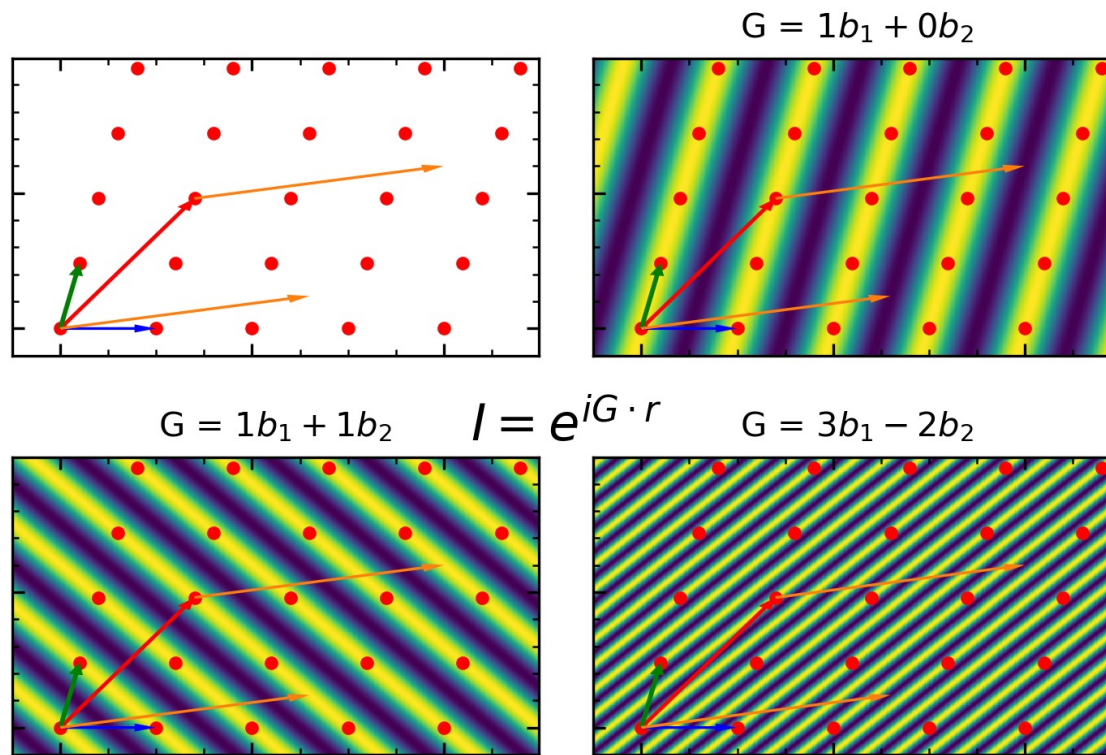
Planes of cubic lattices



Dense
crystallographic
planes



Reciprocal lattice



Reciprocal lattice vectors

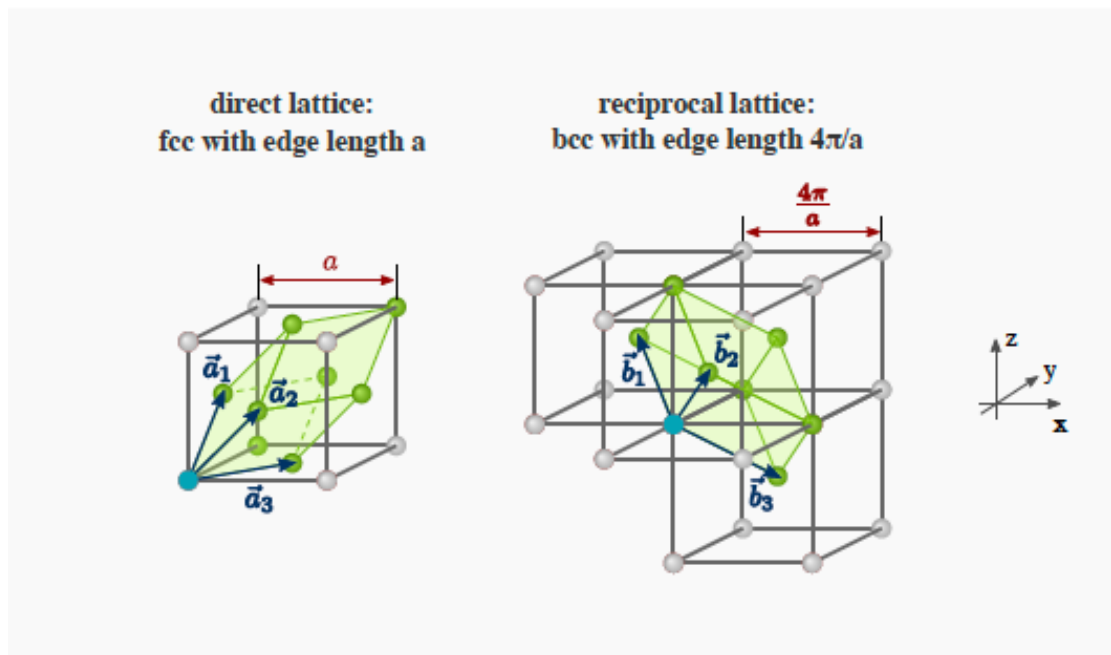
$$\vec{b}_1 = 2\pi \cdot \frac{\vec{a}_2 \times \vec{a}_3}{V}$$

$$\vec{b}_2 = 2\pi \cdot \frac{\vec{a}_3 \times \vec{a}_1}{V}$$

$$\vec{b}_3 = 2\pi \cdot \frac{\vec{a}_1 \times \vec{a}_2}{V}$$

- As we have seen above, the reciprocal lattice of a Bravais lattice is again a Bravais lattice.
- The reciprocal lattice of a reciprocal lattice is the (original) direct lattice.
- The length of the reciprocal lattice vectors is proportional to the reciprocal of the length of the direct lattice vectors. This is where the term reciprocal lattice arises from.

Reciprocal lattice of an fcc lattice



$$\vec{b}_1 = \frac{8\pi}{a^3} \cdot \vec{a}_2 \times \vec{a}_3 = \frac{4\pi}{a} \cdot \left(-\frac{\hat{x}}{2} + \frac{\hat{y}}{2} + \frac{\hat{z}}{2} \right)$$

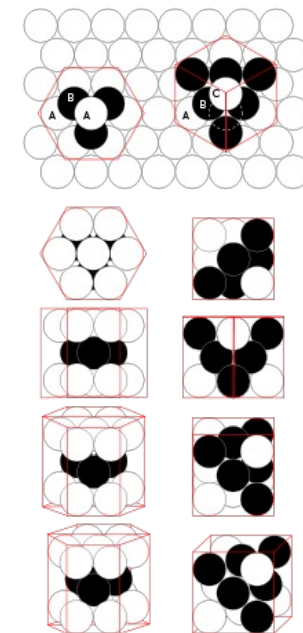
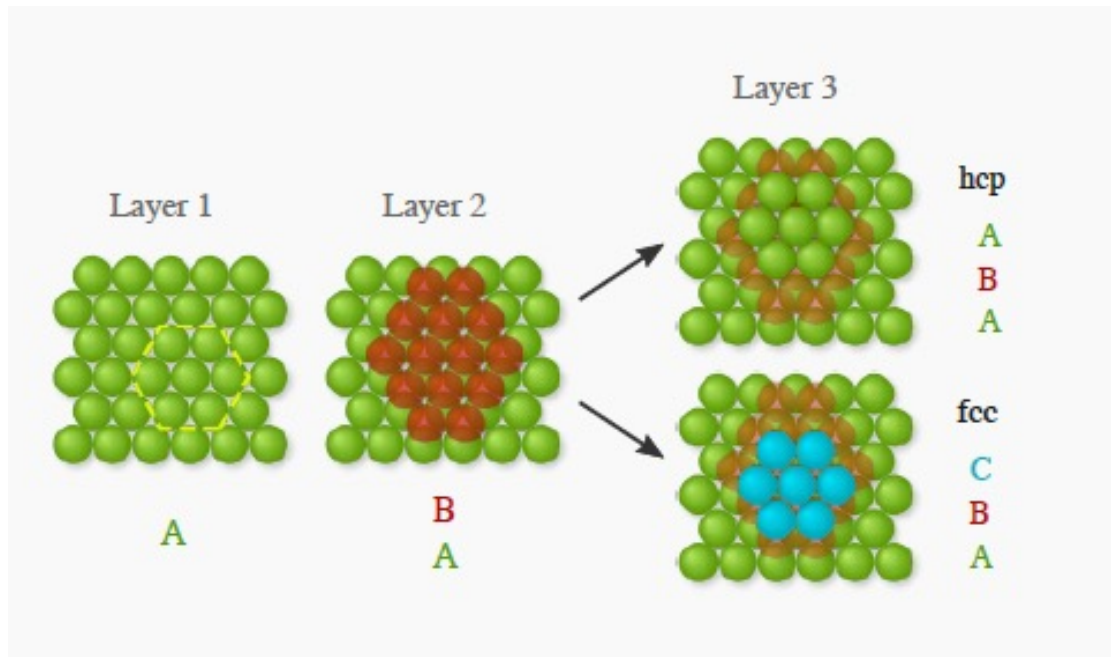
$$\vec{b}_2 = \frac{8\pi}{a^3} \cdot \vec{a}_3 \times \vec{a}_1 = \frac{4\pi}{a} \cdot \left(\frac{\hat{x}}{2} - \frac{\hat{y}}{2} + \frac{\hat{z}}{2} \right)$$

$$\vec{b}_3 = \frac{8\pi}{a^3} \cdot \vec{a}_1 \times \vec{a}_2 = \frac{4\pi}{a} \cdot \left(\frac{\hat{x}}{2} + \frac{\hat{y}}{2} - \frac{\hat{z}}{2} \right)$$

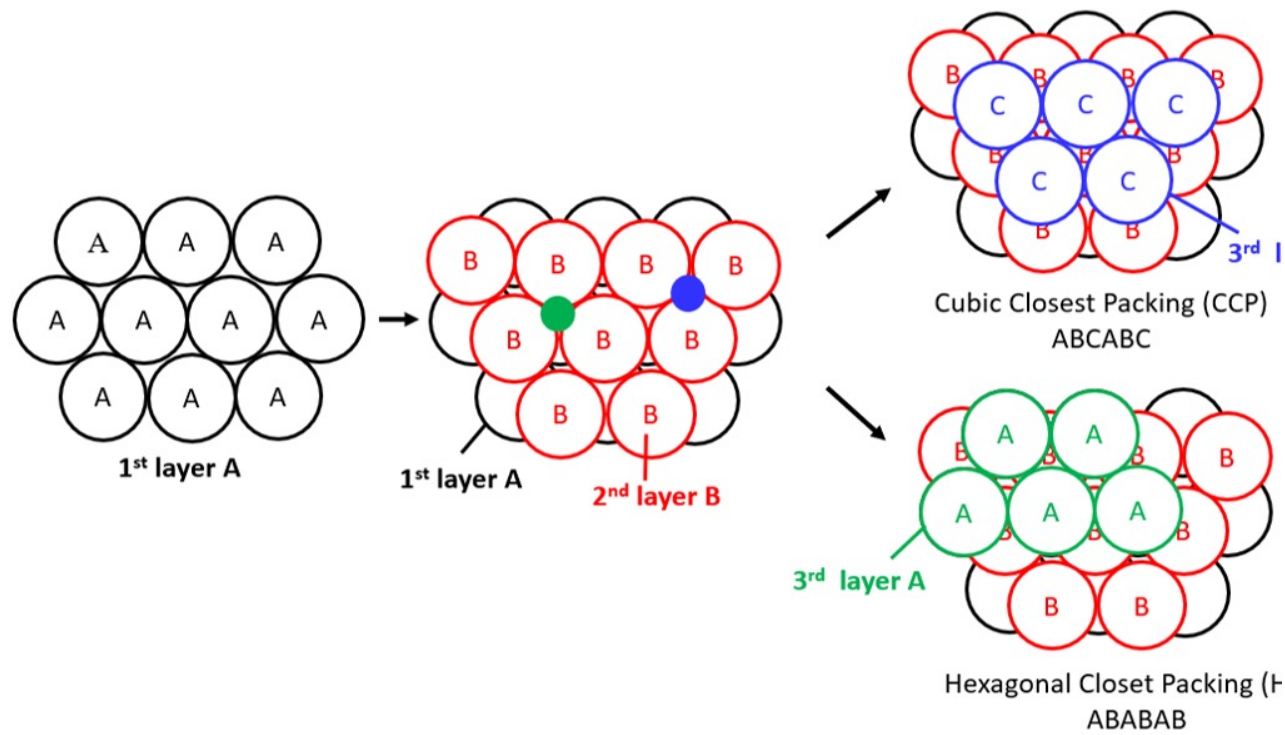
3. Structures of solids



Close packed structures



Close packed structures

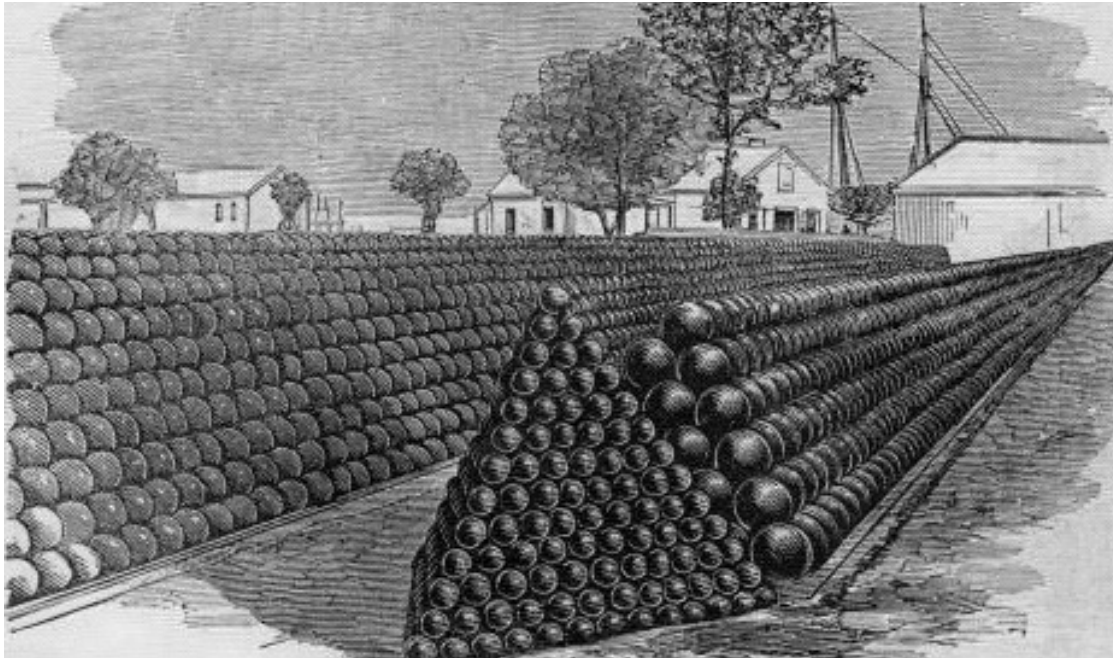


Close packed structures



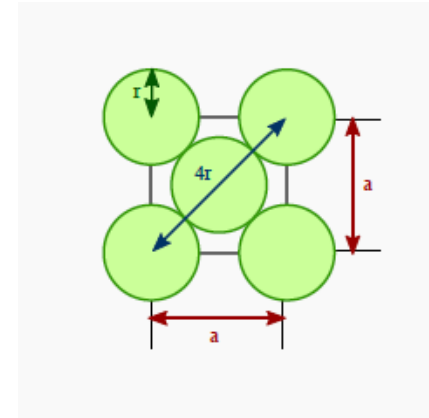
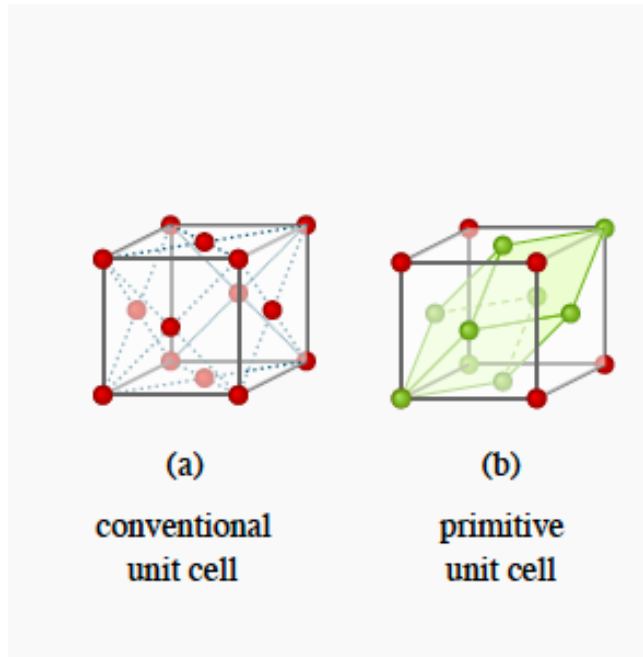
Snowballs stacked in preparation for a snowball fight. The front pyramid is hexagonal close-packed and rear is face-centered cubic.

The cannon ball mathematical problem (1587)



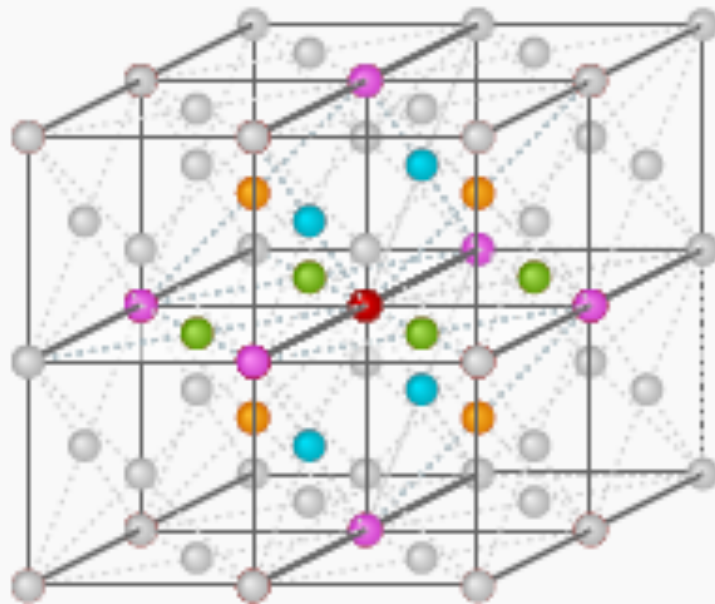
Cannonballs piled on a triangular (*front*) and rectangular (*back*) base, both fcc lattices.

Close packed density: fcc lattice



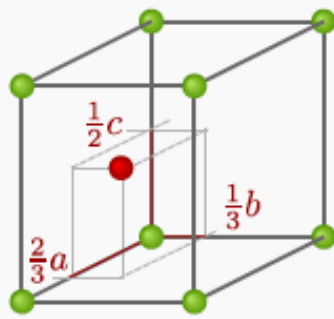
$$\rho = \frac{n \cdot V_{\text{sph}}}{V_{\text{uc}}} = \frac{4 \cdot \frac{4}{3} \pi \cdot \left(\frac{\sqrt{2}}{4}\right)^3 a^3}{a^3}$$
$$= \frac{\sqrt{2}\pi}{6} \approx 74\%$$

Nearest-neighbours

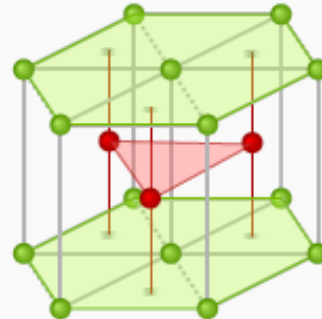


- reference point
- ● ● 12 nearest neighbours
- 6 next-nearest neighbours

Second close packed density: hcp structure

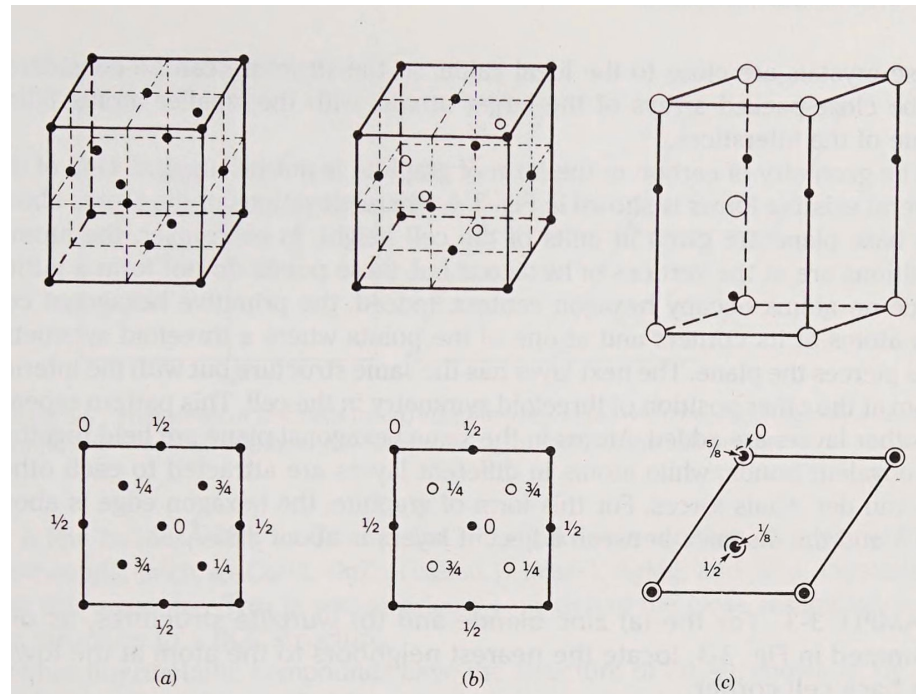


(a)

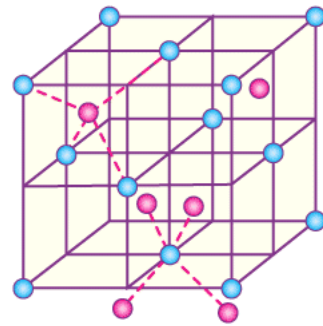
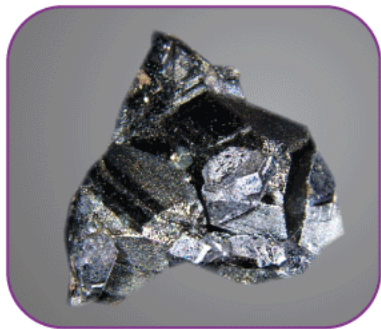


(b)

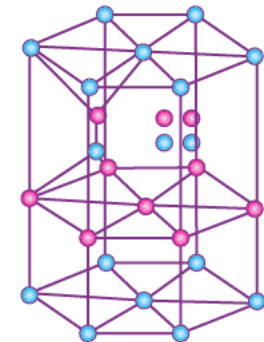
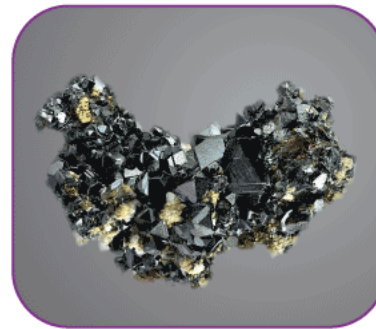
Other crystal structures: diamond, zinc blende and wurzite



ZINC BLENDE STRUCTURE

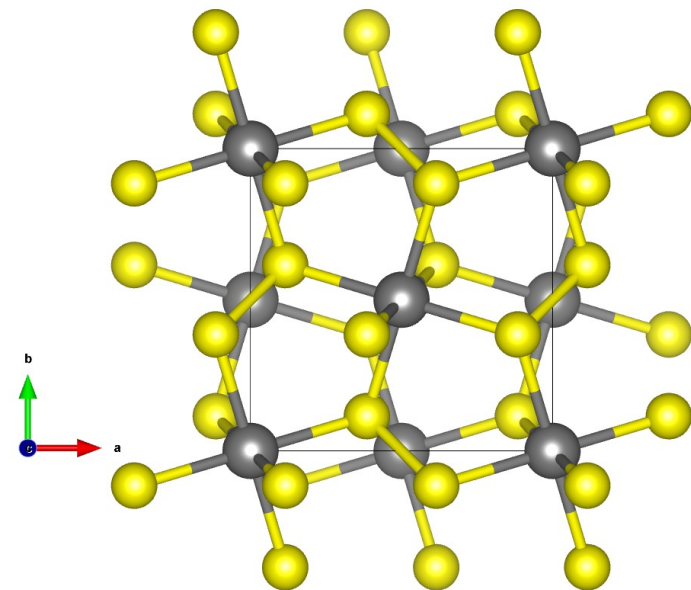
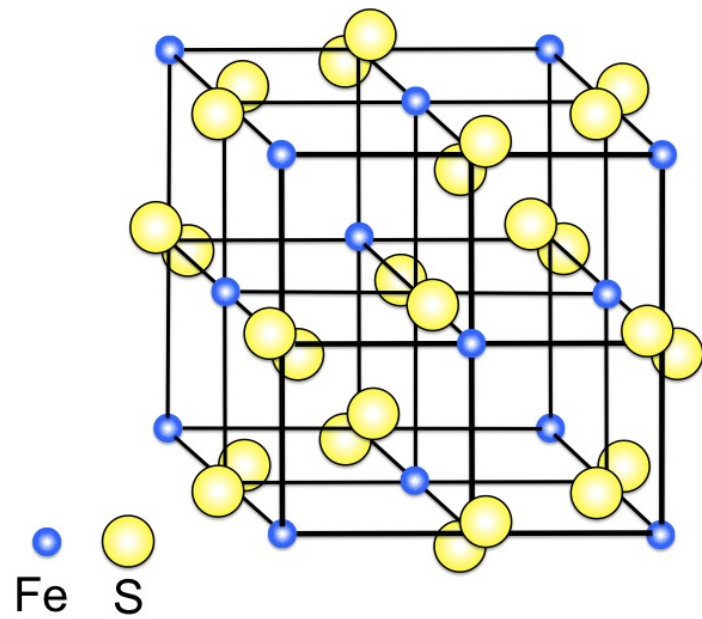


WURTZITE STRUCTURE OF ZINC SULFIDE



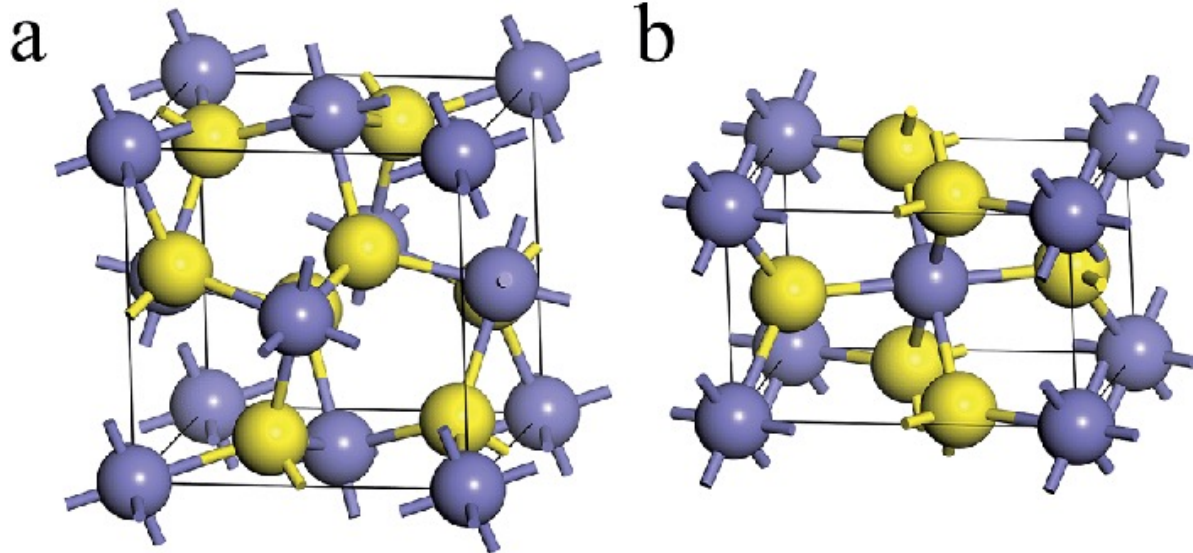
Zinc blende and wurzite
(Zinc sulfide)

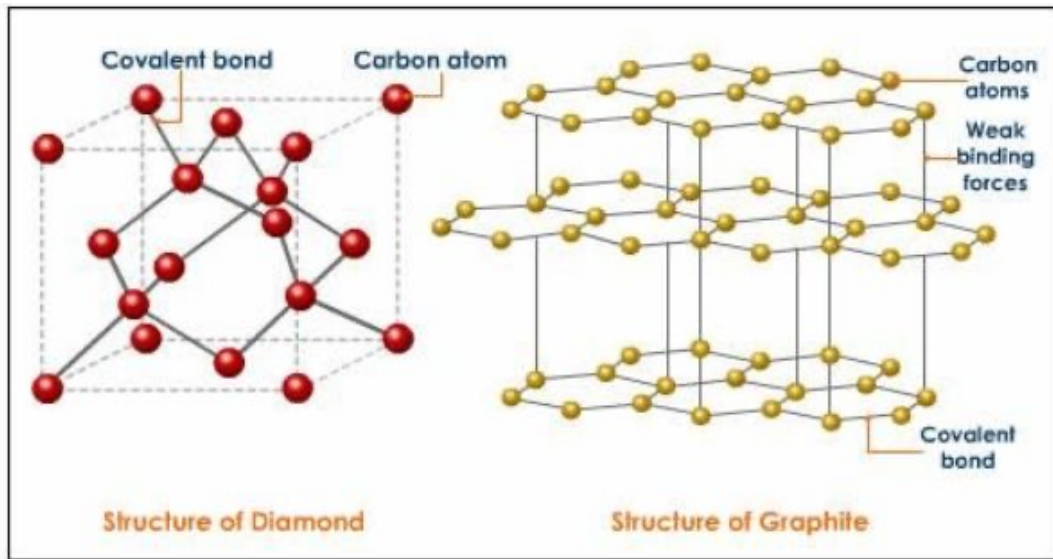
Pyrite (Fools Gold)



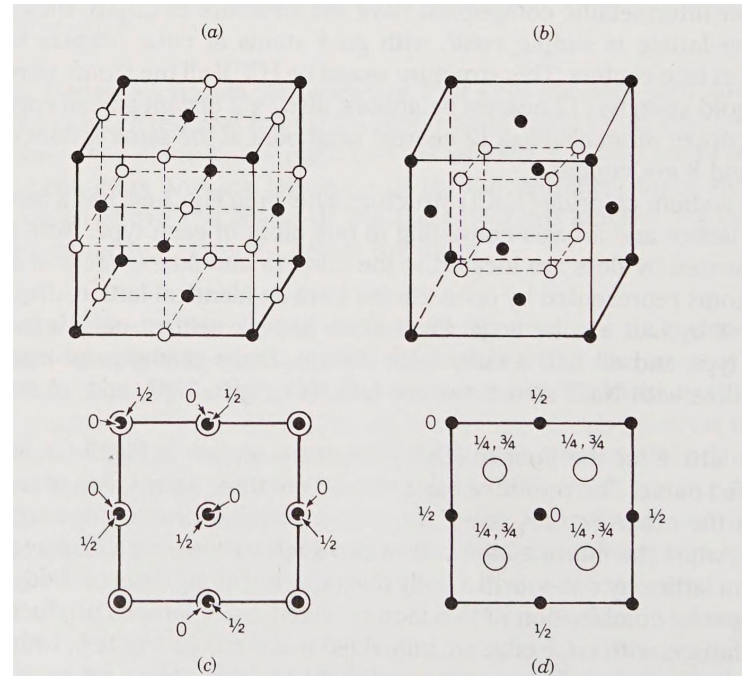
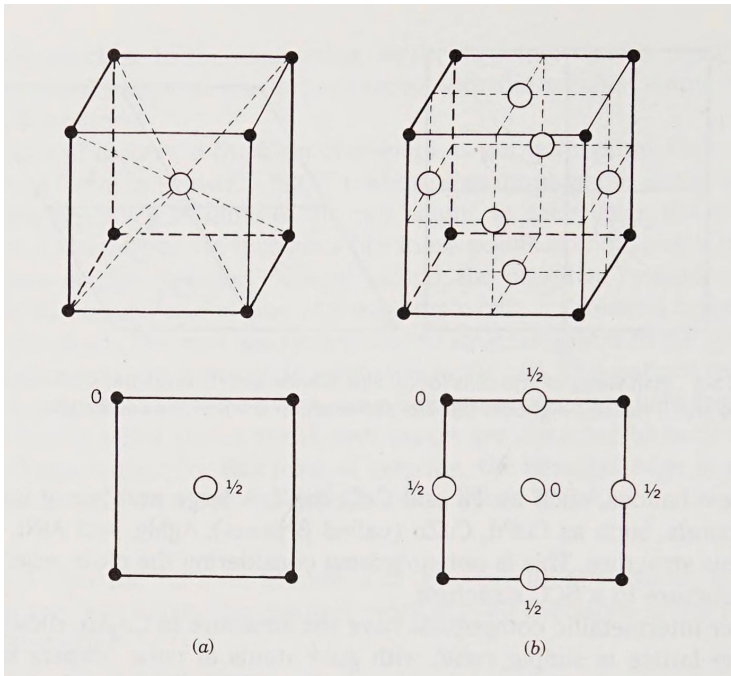


Pyrite and marcasite
(Iron sulfide)



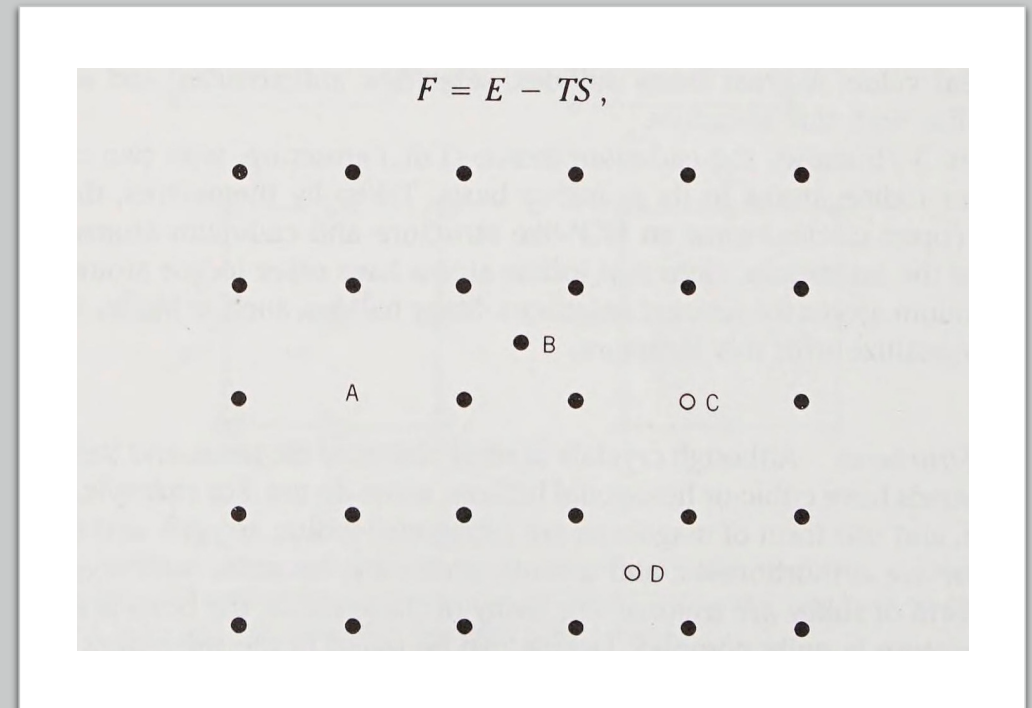
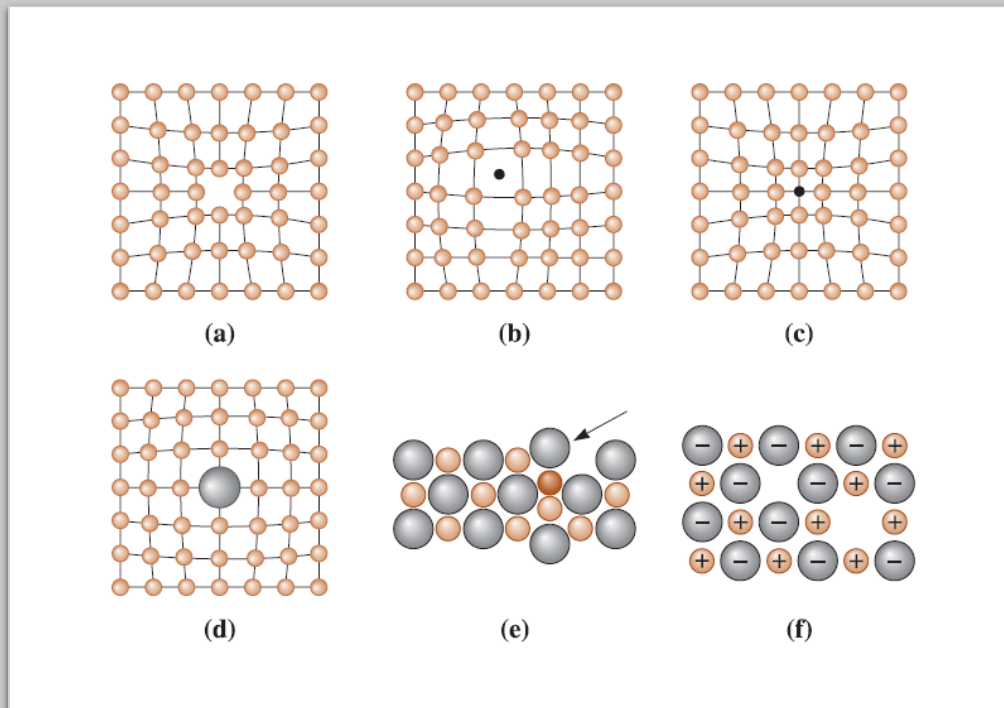


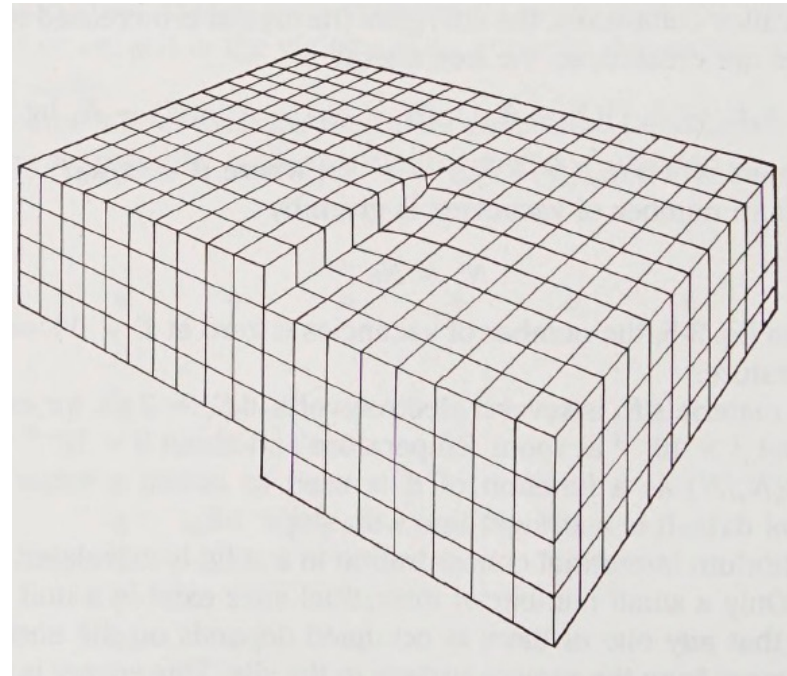
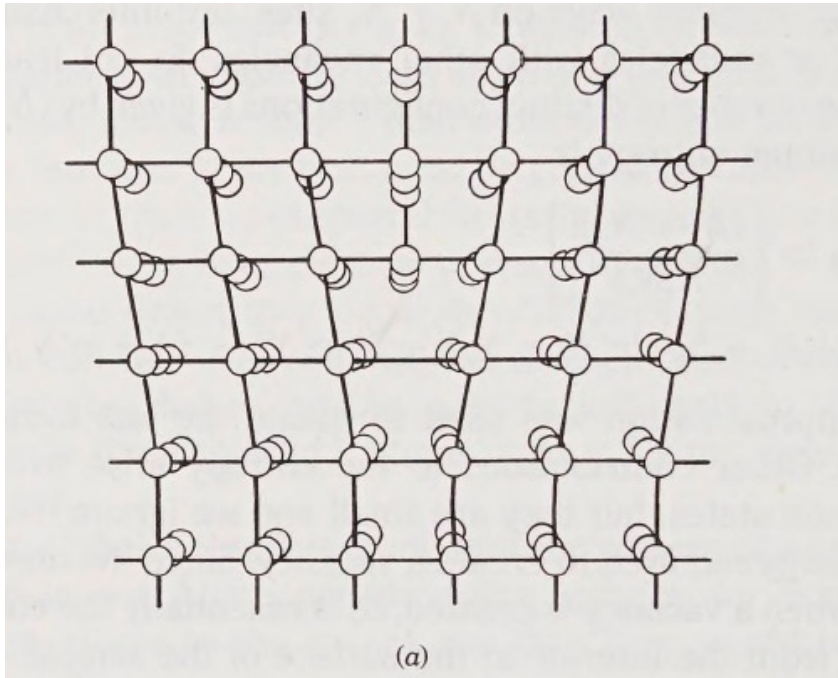
Diamond and graphite



Other cubic structures: CsCl, Cu₃Au, NaCl, CuFe₂

Point defects: A vacancy, B interstitial, C substitutional impurity, D interstitial impurity

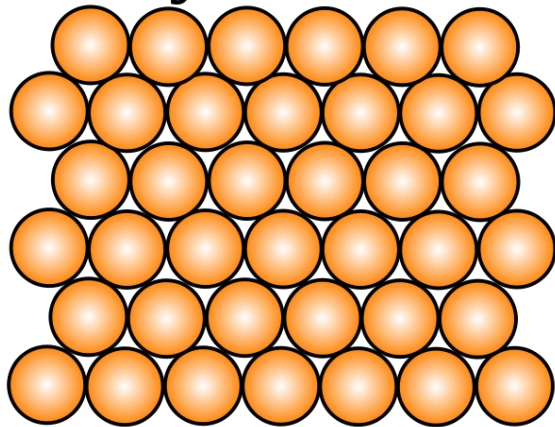




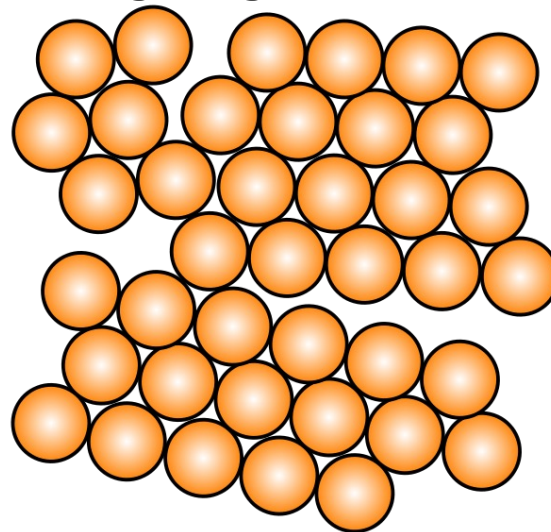
Dislocations: Edge (a) and screw (b)

Amorphous structures

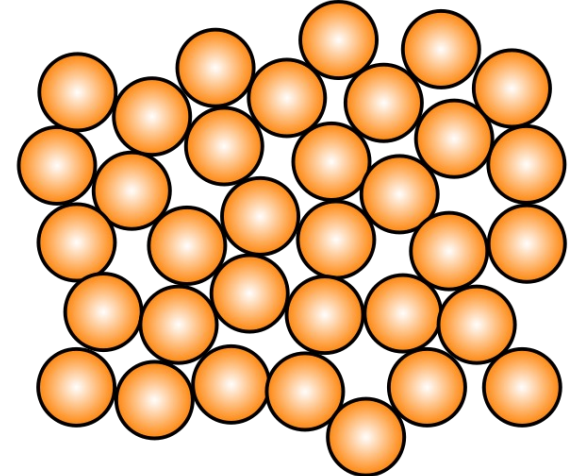
Crystalline



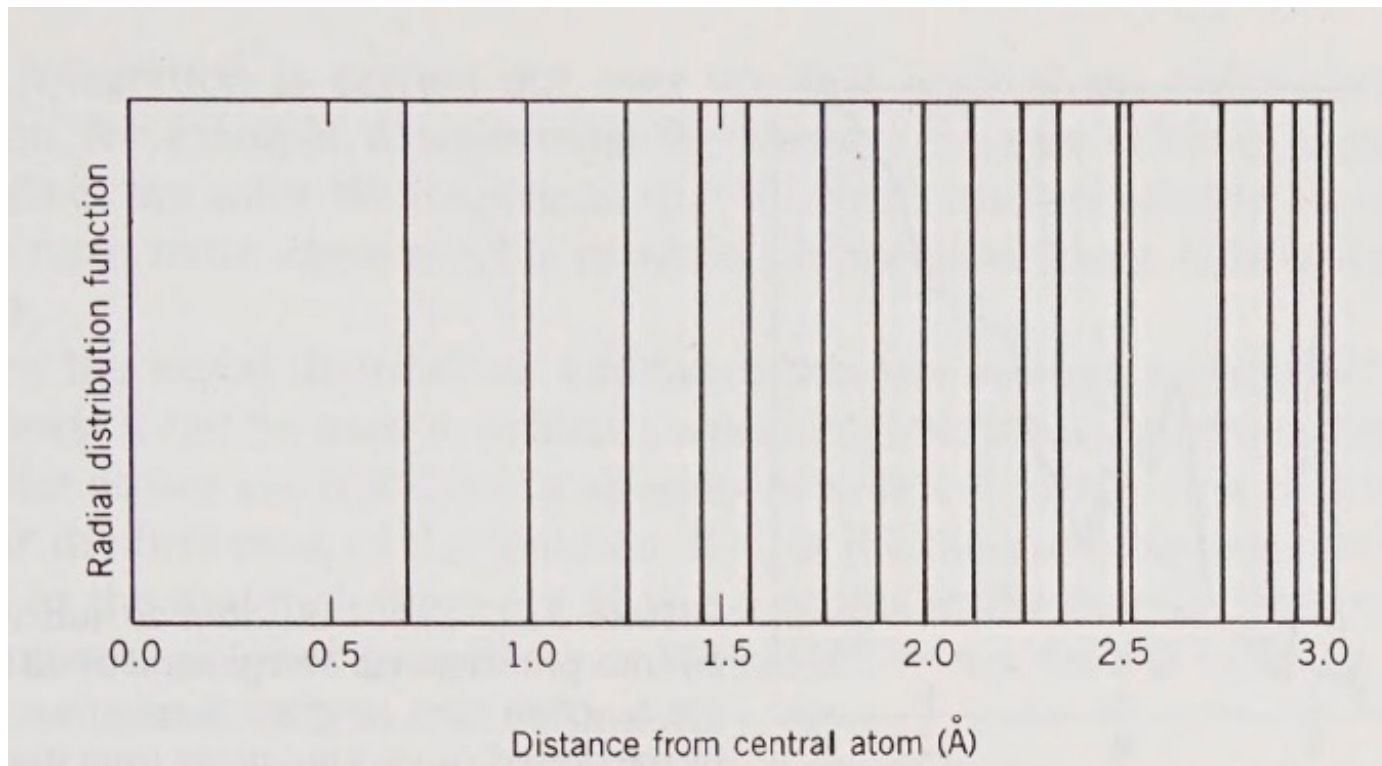
Polycrystalline



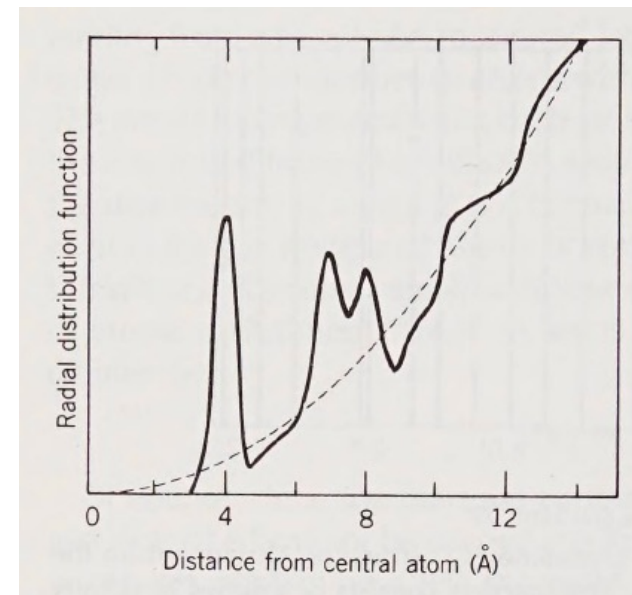
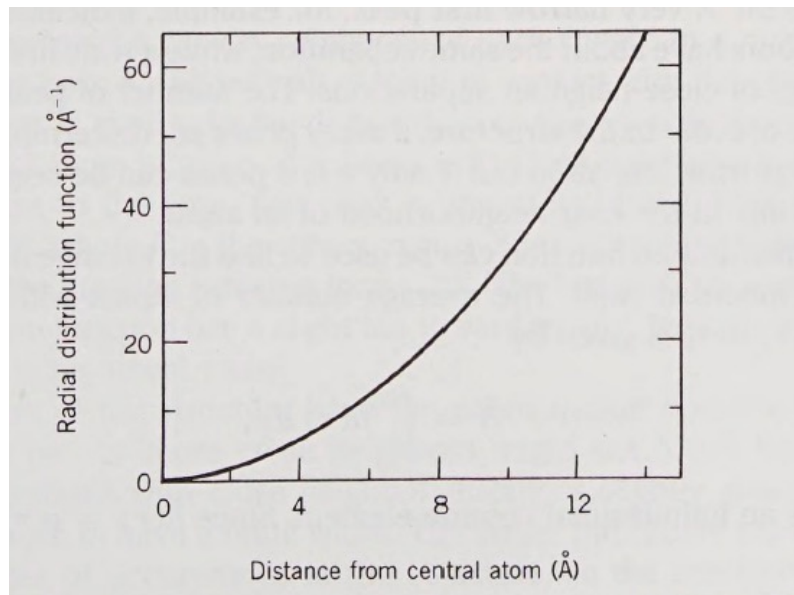
Amorphous



Radial distribution function of crystalline fcc structure



Radial distribution function of amorphous structures



Liquid crystalline order

