Parity is violated by the weak interactions

The discovery set the stage for the development of the Standard Model, as the model relied on the idea of symmetry of particles and forces and how particles can sometimes break that symmetry.

We know how to build the fermionic currents. We can ask if they invariant under P and C transformations.

	C	P	CP
$\bar{\psi}_i\,\psi_j$	$\bar{\psi}_j\,\psi_i$	$\bar{\psi}_i\,\psi_j$	$\bar{\psi}_j\,\psi_i$
$\bar{\psi}_i\,\gamma_5\,\psi_j$	$\bar{\psi}_j\,\gamma_5\,\psi_i$	$-\bar{\psi}_i\,\gamma_5\,\psi_j$	$-\bar{\psi}_j\,\gamma_5\,\psi_i$
$\bar{\psi}_i\,\not{\partial}\,\psi_j$	$\bar{\psi}_j\,\not{\partial}\,\psi_i$	$\bar{\psi}_i\,\not{\partial}\,\psi_j$	$\bar{\psi}_j\,\not{\partial}\,\psi_i$
$\bar{\psi}_i\,\not{\partial}\,\gamma_5\psi_j$	$-\bar{\psi}_j\,\not{\partial}\,\gamma_5\,\psi_i$	$-\bar{\psi}_i\,\not{\partial}\,\gamma_5\,\psi_j$	$\bar{\psi}_j\,\not{\partial}\,\gamma_5\,\psi_i$

The way to deal with the weak interactions is to treat left and right components of the fermions as independent. What is left and right?

$$\psi_L = \gamma_L\psi = \frac{1-\gamma_5}{2}\psi; \qquad \psi_R = \gamma_R\psi = \frac{1+\gamma_5}{2}\psi;$$

P: Let us build a theory invariant under C with two scalars with $C(\phi_1)$=1 and $C(\phi_2)$=-1 and one fermion. Assuming that the scalar fields are even under P, build one P invariant and another one CP invariant. Add a photon and redo the exercise.

Particles and their quantum numbers

Fermions

$$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix};\ L_L^i = \begin{pmatrix} \nu_L^i \\ l_L^i \end{pmatrix};\ u_R^i;\ d_R^i;\ l_R^i;\ \ldots$$

Hypercharge is chosen to give the fermions the right electric charge.

Gauge bosons

$$(W_\mu^1,\ W_\mu^2,\ W_\mu^3);\ B_\mu$$

Hypercharge is chosen such that the charged particle is on the top.

Scalars

$$\Phi_i = \begin{pmatrix} a^- \\ b_i + i\, c_i \end{pmatrix} \qquad \langle \Phi_i \rangle = \begin{pmatrix} v_1^i + i v_2^i \\ v_3^i + i v_4^i \end{pmatrix} \qquad x_i = \Phi_i^\dagger \Phi_i$$

$$\omega_i = \alpha_i + i\beta_i \qquad \langle \omega_i \rangle = \sigma_1^i + i\sigma_2^i \qquad y_i = \omega_i^* \omega_i$$

If Y ≠ 0 the fields have electric charge and couple to the gauge bosons Z and photon.

The Yukawa Lagrangian - scalars quantum numbers

$$\Phi \quad \Longrightarrow \quad Y=1 \qquad I_3 \rightarrow \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix} \qquad Q \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\Phi^* \quad \Longrightarrow \quad Y=-1 \qquad I_3 \rightarrow \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix} \qquad Q \rightarrow \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} a^* \\ b^* \end{bmatrix}$$

$$i\sigma_2\Phi \quad \Longrightarrow \quad Y=1 \qquad I_3 \rightarrow \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix} \qquad Q \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} b \\ -a \end{bmatrix}$$

$$\tilde{\Phi} = i\sigma_2\Phi^* \quad \Longrightarrow \quad Y=-1 \qquad I_3 \rightarrow \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix} \quad Q \rightarrow \begin{bmatrix} 0 \\ -1 \end{bmatrix} \begin{bmatrix} b^* \\ -a^* \end{bmatrix}$$

$$Q = I_3 + \frac{Y}{2} \quad \Longleftarrow \quad \text{Remember}$$

Note: note the difference in isospin in the up and down components is what gives a unit difference of charge. Hipercharge then fixes the actual value of the charge.

The Yukawa Lagrangian - Yukawa invariants

	T	T_3	$Y/2$	Q
ν_{eL}	1/2	1/2	-1/2	0
e_L	1/2	-1/2	-1/2	-1
u_L	1/2	1/2	1/6	2/3
d_L	1/2	-1/2	1/6	-1/3
e_R	0	0	-1	-1
u_R	0	0	2/3	2/3
d_R	0	0	-1/3	-1/3

$$\mathscr{L}_{lep} = -\,Y_e L_L \phi e_R = -\,Y_e \left(\bar{\nu} \quad \bar{e}_L\right)\begin{pmatrix} a \\ b \end{pmatrix} e_R$$

$$Y = 1 + 1 - 2$$

$$(T_3)_{up} = -\,1/2 + 1/2 + 0$$

$$\mathscr{L}_{qd} = -\,Y_d Q_L \phi d_R = -\,Y_d \left(\bar{u} \quad \bar{d}\right)\begin{pmatrix} a \\ b \end{pmatrix} d_R$$

$$Y = -\,1/3 + 2 - 2/3$$

$$\mathscr{L}_{qu} = -\,Y_u Q_L \tilde{\phi} u_R = -\,Y_u \left(\bar{u} \quad \bar{d}\right)\begin{pmatrix} b^* \\ -a^* \end{pmatrix} u_R$$

$$Y = -\,1/3 - 1 + 4/3$$

$$\tilde{\Phi} = i\sigma_2 \Phi^*$$

Add the hermitian conjugate to all terms. This is a version of the Lagrangian where the neutrinos are massless. As we now know this is not true. More later.

The Yukawa Lagrangian - mass eigenstates

$$-\mathrm{L}_{\mathrm{Y}} = \begin{bmatrix} \bar{\mathrm{U}} & \bar{\mathrm{D}} \end{bmatrix}_{\mathrm{L}} \Phi \mathrm{Y}_{\mathrm{d}}\, \mathrm{D}_{\mathrm{R}} + \begin{bmatrix} \bar{\mathrm{U}} & \bar{\mathrm{D}} \end{bmatrix}_{\mathrm{L}} \tilde{\Phi}\, \mathrm{Y}_{\mathrm{u}} \mathrm{U}_{\mathrm{R}} + \begin{bmatrix} \bar{\mathrm{N}} & \bar{\mathrm{E}} \end{bmatrix}_{\mathrm{L}} \Phi\, \mathrm{Y}_{\mathrm{e}} \mathrm{E}_{\mathrm{R}} + \mathrm{h.c}$$

where the gauge eigenstates are

$$\mathrm{U} = \begin{bmatrix} \mathrm{u}_{\mathrm{g}} & \mathrm{c}_{\mathrm{g}} & \mathrm{t}_{\mathrm{g}} \end{bmatrix};\ \mathrm{D} = \begin{bmatrix} \mathrm{d}_{\mathrm{g}} & \mathrm{s}_{\mathrm{g}} & \mathrm{b}_{\mathrm{g}} \end{bmatrix};\ \mathrm{N} = \begin{bmatrix} \nu_{\mathrm{e}} & \nu_{\mu} & \nu_{\tau} \end{bmatrix};\ \mathrm{E} = \begin{bmatrix} \mathrm{e} & \mu & \tau \end{bmatrix}$$

and Y are matrices in flavour space. To get the mass terms we just need the vacuum expectation values of the scalar fields

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix}; \quad \langle \tilde{\Phi} \rangle = i\tau_2 \Phi^* = \begin{pmatrix} \mathrm{v}/\sqrt{2} \\ 0 \end{pmatrix}$$

The mass terms are

$$-\mathrm{L}_{\mathrm{Y}}^{\mathrm{mass}} = \frac{\mathrm{v}}{\sqrt{2}} \bar{\mathrm{U}}_{\mathrm{L}} \mathrm{Y}_{\mathrm{u}} \mathrm{U}_{\mathrm{R}} + \frac{\mathrm{v}}{\sqrt{2}} \bar{\mathrm{D}}_{\mathrm{L}} \mathrm{Y}_{\mathrm{d}} \mathrm{D}_{\mathrm{R}} + \frac{\mathrm{v}}{\sqrt{2}} \bar{\mathrm{E}}_{\mathrm{L}} \mathrm{Y}_{\mathrm{e}} \mathrm{E}_{\mathrm{R}} + \mathrm{h.c.}$$

which have to be diagonalised. We are still dealing with the group eigenstates.

The Yukawa Lagrangian - mass eigenstates

So we define

$$D_R \to N_R^{-1} D_R ; D_L \to N_L^{-1} D_L ; U_R \to K_R^{-1} U_R ; U_L \to K_L^{-1} U_L$$

and the mass matrices are

$$-\frac{v}{\sqrt{2}} N_L^\dagger Y_d N_R = M_d ; \qquad -\frac{v}{\sqrt{2}} K_L^\dagger Y_u K_R = M_u$$

and finally the CKM matrix revealed

$$\frac{v}{\sqrt{2}} \bar{U}_L K_L^\dagger Y_d N_R D_R \ldots + \frac{v}{\sqrt{2}} \bar{D}_L N_L^\dagger Y_u K_R U_R \ldots =$$

$$\frac{v}{\sqrt{2}} \bar{U}_L K_L^\dagger N_L M_D D_R \ldots + \frac{v}{\sqrt{2}} \bar{D}_L N_L^\dagger K_L M_U U_R \ldots =$$

$$\frac{v}{\sqrt{2}} \bar{U}_L V_{CKM} M_D D_R \ldots + \frac{v}{\sqrt{2}} \bar{D}_L V_{CKM}^\dagger M_U U_R \ldots$$

and flavour changing neutral currents (FCNC)?

$$\Phi = \begin{pmatrix} 0 \\ (h+v)/\sqrt{2} \end{pmatrix}$$

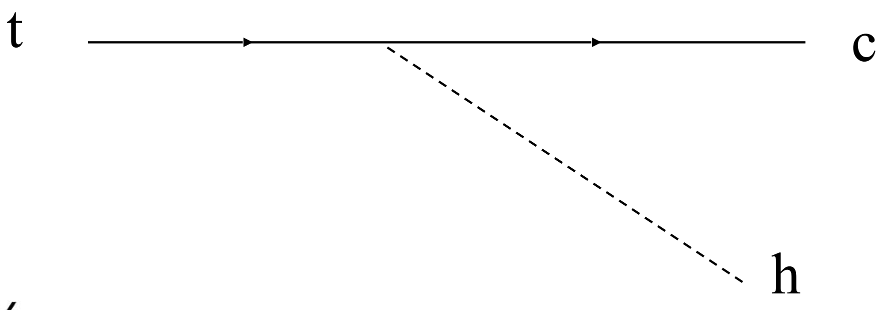
$$-L_Y^{\text{interactions}} = \frac{h}{\sqrt{2}} \bar{D}_L Y_d D_R \propto \frac{v}{\sqrt{2}} \bar{D}_L Y_d D_R$$

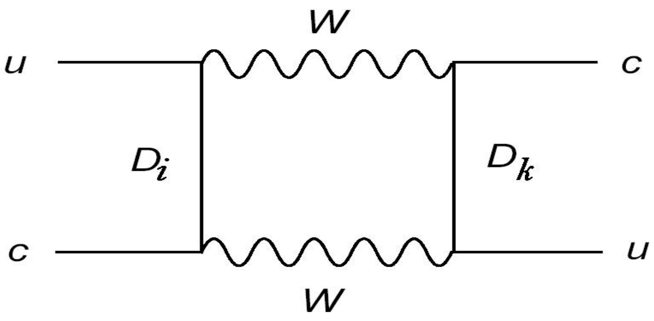
The Yukawa Lagrangian - FCNC

$$-\mathrm{L}_{\mathrm{Y}}^{\mathrm{interactions}} = \frac{\mathrm{h}}{\sqrt{2}}\,\bar{\mathrm{D}}_{\mathrm{L}}\mathrm{Y}_{\mathrm{d}}\mathrm{D}_{\mathrm{R}} \propto \frac{\mathrm{v}}{\sqrt{2}}\,\bar{\mathrm{D}}_{\mathrm{L}}\mathrm{Y}_{\mathrm{d}}\mathrm{D}_{\mathrm{R}} \qquad -\frac{\mathrm{v}}{\sqrt{2}}\,\mathrm{N}_{\mathrm{L}}^{\dagger}\mathrm{Y}_{\mathrm{d}}\mathrm{N}_{\mathrm{R}} = \mathrm{M}_{\mathrm{d}}$$

Higgs couplings to the fermions are proportional to the fermions mass! No tree-level FCNC.

$\bar{t}ch$



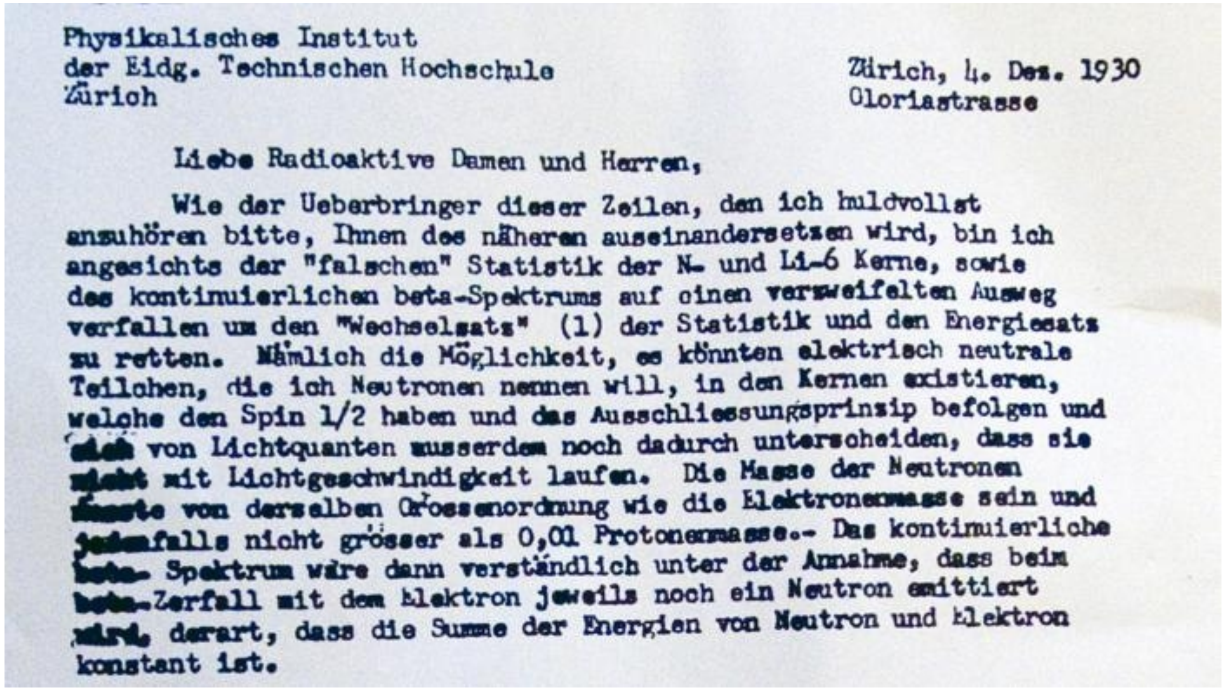


D-Mixing in the Standard Model is very small

Tree level couplings fine tuned to be unusually small

Neutrino masses

Dear radioactive Ladies and Gentlemen,



Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Des. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst ansuhören bitte, Ihnen des näheren auseinandersetsen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsats" (1) der Statistik und den Energiesats su retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinsip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

December 4th, 1930
Letter to his colleagues in Tübingen

1930
Pauli's neutrino hypothesis

Neutrino masses

Zürich, Dec. 4, 1930

Physics Institute of the ETH

Gloriastrasse

Zürich

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, because of the "wrong" statistics of the N- and Li-6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that in the nuclei there could exist electrically neutral particles, which I will call neutrons, that have spin 1/2 and obey the exclusion principle and that further differ from light quanta in that they do not travel with the velocity of light.

(.../...)

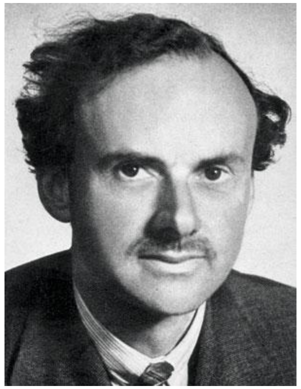
But so far I do not dare to publish anything about this idea, and trustfully turn first to you, dear radioactive people, with the question of how likely it is to find experimental evidence for such a neutron if it would have the same or perhaps a 10 times larger ability to get through [material] than a gamma-ray.

I admit that my remedy may seem almost improbable because one probably would have seen those neutrons, if they exist, for a long time. (.../...) Thus, dear radioactive people, scrutinize and judge. - Unfortunately, I cannot personally appear in Tübingen since I amindispensable here in Zürich because of a ball on the night from December 6 to 7. With my best regards to you, and also to Mr. Back, your humble servant

signed W. Pauli

Neutrino masses

Paul Dirac



$$m_D \overline{f}_L f_R + \text{h.c.} = m_D \overline{f} f$$

$$\boxed{f \neq f^c}$$

Ettore Majorana

$$\frac{1}{2} m_M \overline{f^c_X} f_X + \text{h.c.}$$

$$\boxed{f = f^c}$$

Only neutral fermions can be Majorana

Note:

$$f^c \equiv C\overline{f}^T$$

Pontecorvo, the “father” of neutrino oscillations, recalls the origin of Majorana neutrinos in the following way: Dirac discovers his famous equation describing the evolution of the electron; Majorana goes to Fermi to point out a fundamental detail: ” I have found a representation where all Dirac γ matrices are real. In this representation it is possible to have a real spinor that describes a particle identical to its antiparticle.”

It would be interesting though to know if neutrino are Dirac or Majorana particles. However, there is no problem with the neutrino masses. Just extend the SM with right-handed neutrinos.

The Gauge groups

Here the important conventions are for the field strengths and the covariant derivatives. We have

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu, \quad a = 1, \dots, 8 \tag{D.1}$$

where f^{abc} are the group structure constants, satisfying

$$\left[T^a, T^b\right] = i f^{abc} T^c \tag{D.2}$$

and T^a are the generators of the group. The covariant derivative of a (quark) field q in some representation T^a of the gauge group is given by

$$D_\mu q = \left(\partial_\mu - i\, g\, G^a_\mu T^a\right) q \tag{D.3}$$

In QCD the quarks are in the fundamental representation and $T^a = \lambda^a/2$ where λ^a are the Gell-Mann matrices. A gauge transformation is given by a matrix

$$U = e^{-iT^a \alpha^a} \tag{D.4}$$

and the fields transform as

$$\begin{aligned} & q \to e^{-iT^a\alpha^a} q & & \delta q = -iT^a \alpha^a q \\ & G^a_\mu T^a \to U G^a_\mu T^a U^{-1} - \frac{i}{g} \partial_\mu U U^{-1} & & \delta G^a_\mu = -\frac{1}{g} \partial_\mu \alpha^a + f^{abc} \alpha^b G^c_\mu \end{aligned} \tag{D.5}$$

where the second column is for infinitesimal transformations. With these definitions one can verify that the covariant derivative transforms like the field itself,

$$\delta(D_\mu q) = -i\, T^a \alpha^a (D_\mu q) \tag{D.6}$$

ensuring the gauge invariance of the Lagrangian.

Feynman Rules -
Prof. Jorge Romão.

Feynman Rules and signs - Romão e Silva https://arxiv.org/pdf/1209.6213.pdf.

D.2.2 Gauge Group $SU(2)_L$

This is similar to the previous case. We have

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu, \quad a = 1, \ldots, 3 \tag{D.7}$$

where, for the fundamental representation of $SU(2)_L$ we have $T^a = \sigma^a/2$ and ϵ^{abc} is the completely anti-symmetric tensor in 3 dimensions. The covariant derivative for any field ψ_L transforming non-trivially under this group is,

$$D_\mu \psi_L = \left(\partial_\mu - i\, g\, W^a_\mu T^a\right) \psi_L \tag{D.8}$$

D.2.3 Gauge Group $U(1)_Y$

In this case the group is abelian and we have

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{D.9}$$

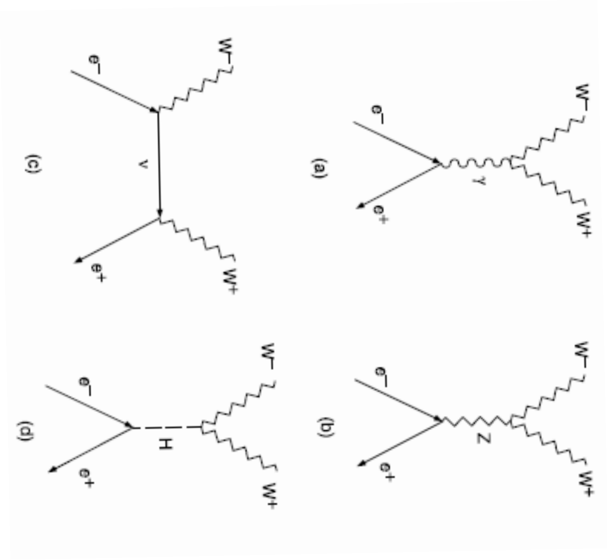
with the covariant derivative given by

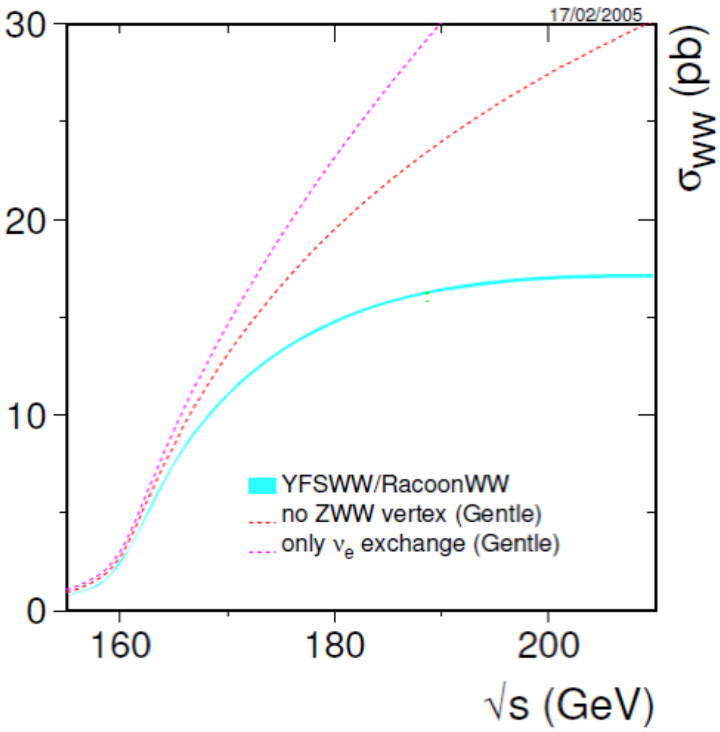
$$D_\mu \psi_R = \left(\partial_\mu + i\, g'\, Y\, B_\mu\right) \psi_R \tag{D.10}$$

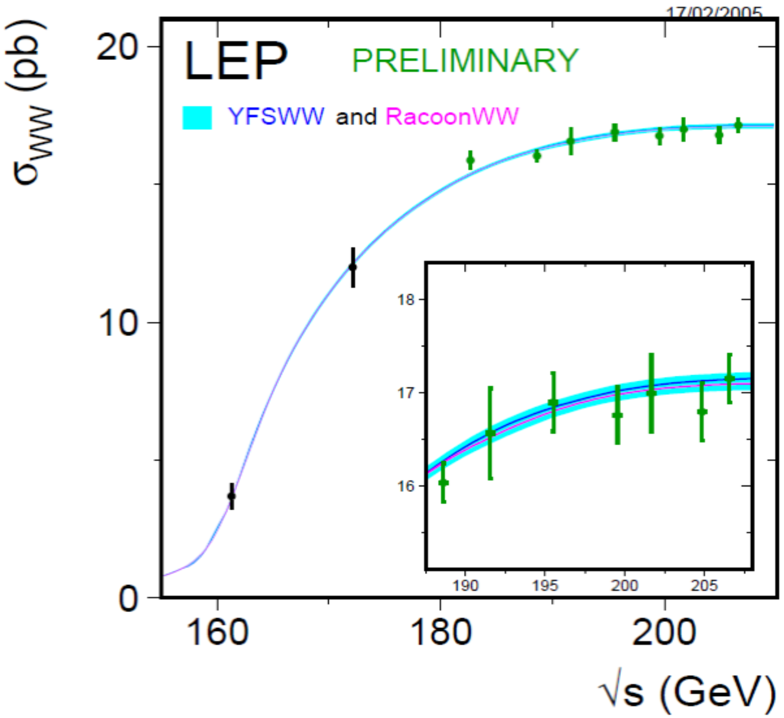
where Y is the hypercharge of the field. Notice the different sign convention between Eq. (D.8) and Eq. (D.9). This is to have the usual definition[1]

$$Q = T_3 + Y\ . \tag{D.12}$$

Where is the gauge structure?







The gauge fixing Lagrangian in QED

In order to quantise the theory we still need to fix the gauge and therefore to introduce the gauge fixing Lagrangian

$$\mathscr{L} = \mathscr{L}_{free} + \mathscr{L}_{GF} = -\frac{1}{4}F^2 - \frac{1}{2\xi}(\partial \cdot A)^2$$

and the equations of motion are now

$$\Box A_\mu + (\frac{1}{\xi} - 1)\partial_\mu(\partial \cdot A) = 0$$

Choosing $\xi = 1$ the components of the photon field obey a Klein-Gordon equation and can be treated as a scalar field. Using the parameter a Lagrange multiplier we get

$$\partial_\mu A^\mu = 0 \qquad\qquad k_\mu \epsilon^\mu = 0$$

Note: note the gauge fixing Lagrangian is not unique and (underline twice now) the physical observables cannot depend on the gauge fixing parameter.

The gauge fixing Lagrangian in the SM

In the case of the SM the gauge fixing Lagrangian has the form

$$\mathscr{L}_{GF} = -\frac{1}{2\beta_A}(\partial \cdot A - \gamma_A M_Z G_0) - \frac{1}{2\beta_Z}(\partial \cdot Z - \gamma_Z M_Z G_0) - \frac{1}{\beta_W} | \partial \cdot W^+ - \gamma_W M_W G^+ |$$

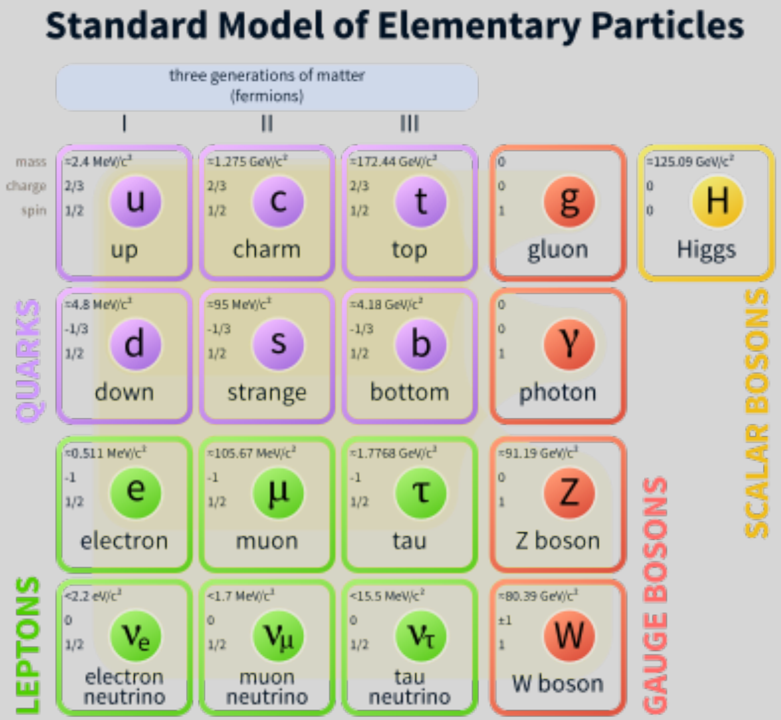
one fixes the gauge with the added bonus of cancelling the bilinear term below allowing us to define the propagator.

$$\mathscr{L}_S = (D_\mu \phi)^\dagger (D^\mu \phi) = -\tfrac{1}{2} h \partial^2 h - \tfrac{1}{2} G_0 \partial^2 G_0 - G^+ h \partial^2 G^-$$
$$+ M_W^2 W^+ W^- + M_W (i W_m u^- \partial^\mu G^+ + h\,.\,c.) + \tfrac{1}{2} M_Z^2 Z^2 - M_Z (Z_m u \partial^\mu G_0)$$

The Fadeev-Popov Lagrangian

Go home and study the FP Lagrangian.

The SM (and beyond) for people in a hurry



$$G = SU(3)_c \times SU(2)_L \times U(1)_Y$$

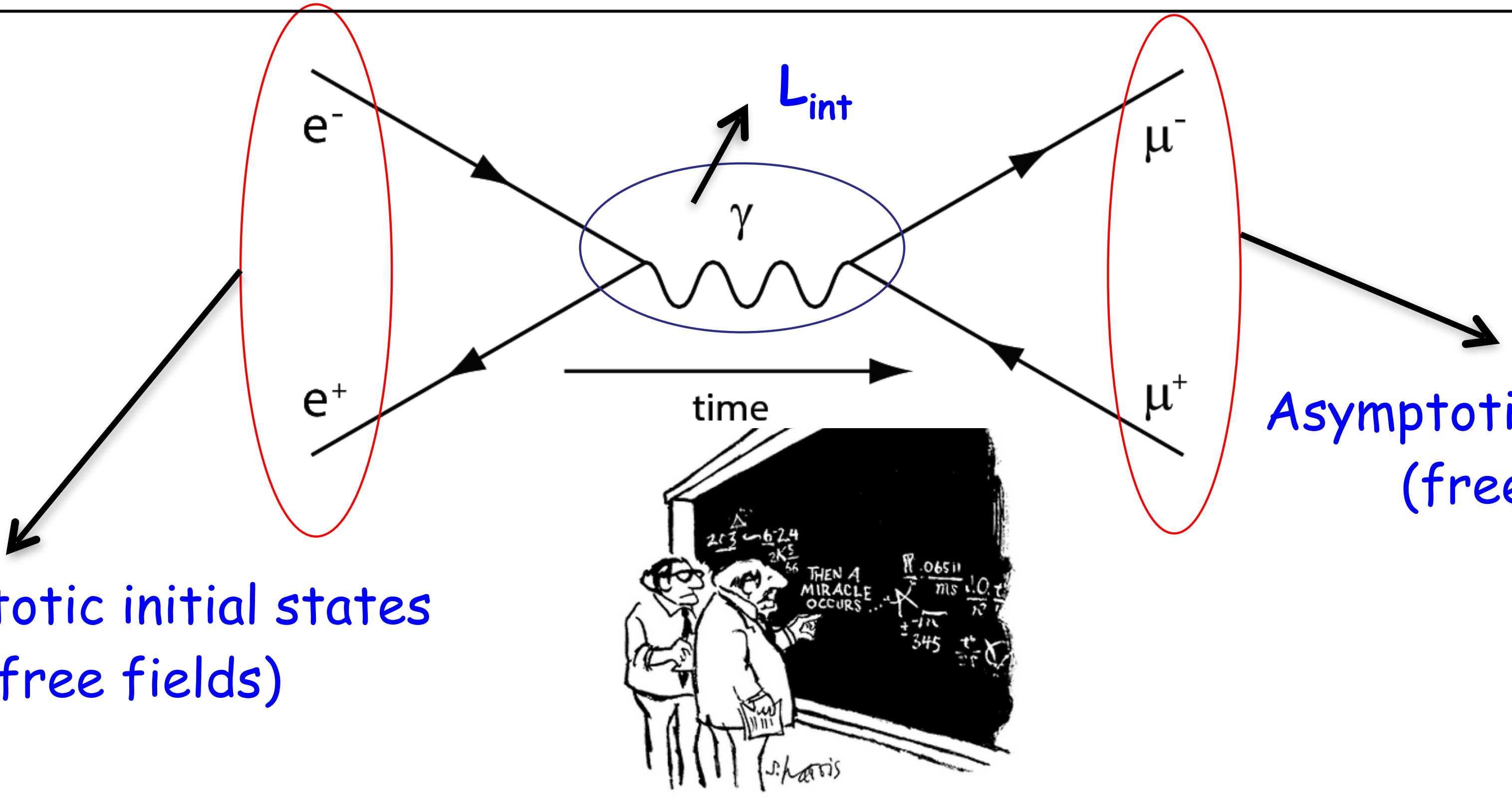
$$\Big\downarrow$$

$$G' = SU(3)_c \times U(1)_Q$$

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{Ghost}}$$

Cross Sections and Branching ratios - circumventing QFT

Feynman diagrams



Asymptotic final states (free fields)

Asymptotic initial states (free fields)

"I think you should be more explicit here in step two."

An electron and a positron collide, exchange a virtual photon, and create a pair of muon and anti-muon (there are more diagrams).

The time arrow tells us that the diagram has to be read from left to right. This is the most common definition now.

The initial and final states are free states obeying the Dirac equation.

Asymptotic states

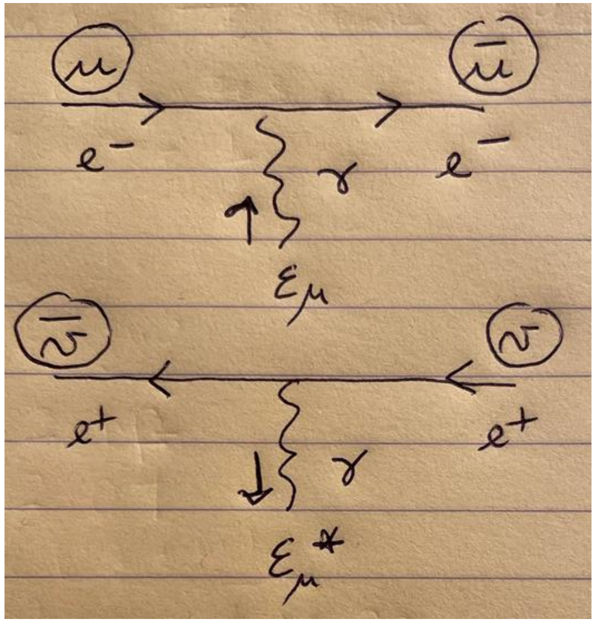
The way calculations in QFT work are as follows. Particles colide in free states and move away from the interaction in free states as well (asymptotic). For the type of particles

1 Scalar

ϵ_μ Vector -polarisation vector

$u, v, \bar{u}, \bar{v}$ Spinor

All solutions are multiplied by the corresponding solution of the equation of motion - a plane wave solution.



Processes in QED with electrons, positrons and photons.

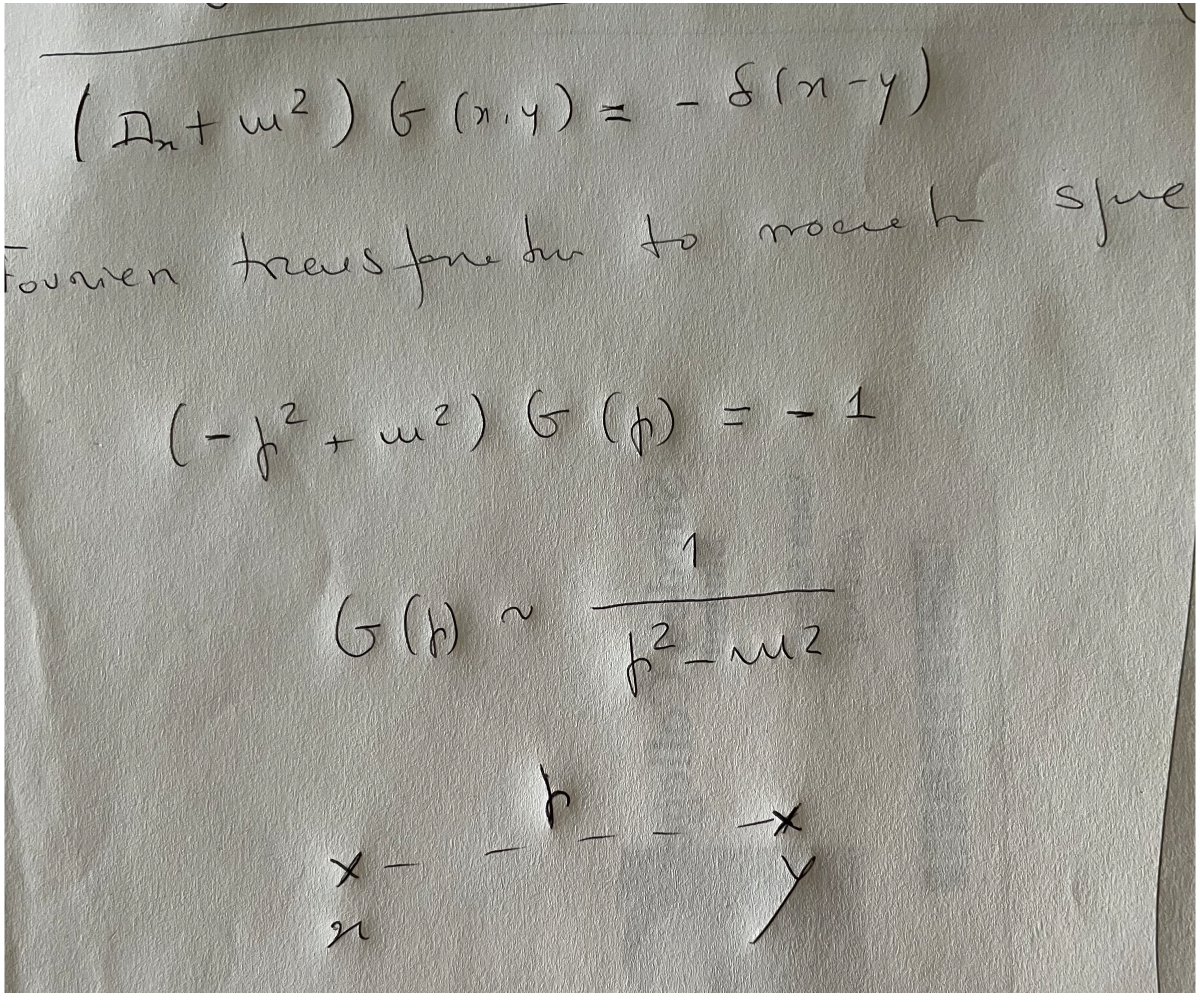
Always go against the arrow.

The arrows represent particle flow. An arrow with time is a particle and an arrow against time is an anti-particle.

Propagators

$$\mathscr{L}_{free}^{KG} = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right) \;\Rightarrow\; \frac{i}{p^2 - m^2}$$

Scalar field



$$(\Box_x + m^2)\, G(x,y) = -\delta(x-y)$$

Fourier transformation to momentum space

$$(-p^2 + m^2)\, G(p) = -1$$

$$G(p) \sim \frac{1}{p^2 - m^2}$$

Propagators

$$\mathscr{L}^{KG}_{free} = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right) \Rightarrow \frac{i}{p^2 - m^2}$$

Scalar field

The photon is described by a 4-component vector. Yet it has only two degrees of freedom. In classical electromagnetism we choose a gauge. The Lorenz gauge is

$$\partial_\mu A^\mu = 0 \to p_\mu \epsilon^\mu = 0$$

What is done in QFT is to add a gauge fixing Lagrangian

$$\mathscr{L}_{GF} = -\frac{1}{4}F^2 + \frac{1}{2}\lambda(\partial\cdot A)^2 \quad \Rightarrow \quad \Box A^\mu + (\lambda - 1)\partial^\mu(\partial_\nu A^\nu) = 0$$

$$-\frac{ig_{\mu\nu}}{p^2 - m^2}$$

Vector field in the 't Hooft-Feynman Gauge

$$i\frac{p_\mu\gamma^\mu + m}{p^2 - m^2}$$

Spinor 4 by 4 matrix

Interactions

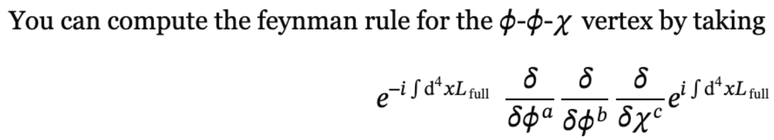
Interactions can be founds using functional derivatives of the Lagrangian

$\mathscr{L}_{int} = \frac{\lambda}{4!}\phi^4 \;\Rightarrow\; i\lambda$ (Real) Scalar theory with self-interactions

$\mathscr{L}_{int} = \frac{\lambda}{4}(\phi^\dagger\phi)^2 \;\Rightarrow\; i\lambda$ (Complex) Scalar theory with self-interactions

$\mathscr{L}_{int} = e\overline{\psi}\gamma_\mu A^\mu \psi \;\Rightarrow\; ie\gamma_\mu$ QED

$\mathscr{L}_{int} = -e^2 g_{\mu\nu}A^\mu A^\nu \phi^\dagger\phi \;\Rightarrow\; -2ie^2 g_{\mu\nu}$ Scalar QED



You can compute the feynman rule for the ϕ-ϕ-χ vertex by taking

$$e^{-i\int \mathrm{d}^4x L_{\text{full}}}\frac{\delta}{\delta\phi^a}\frac{\delta}{\delta\phi^b}\frac{\delta}{\delta\chi^c}e^{i\int \mathrm{d}^4x L_{\text{full}}}$$

Cross Sections

The S-matrix (scattering matrix) is the unitary operator S that determines the evolution of the initial state |i> at t=-∞ to state |f> at t =+∞.

The probability amplitude for a transition between initial state |i> and state |f> is

$$S_{fi} = \langle f | S | i \rangle$$

and S is the scattering matrix. S is expanded at each order and it depends on the interaction Lagrangian

$$S \Leftarrow \mathscr{L}_{int}$$

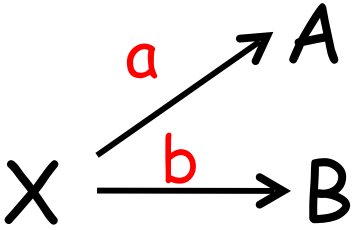
The cross section is proportional to the transition amplitude

$$\sigma \propto |\langle f | S | i \rangle|^2$$

and is the probability of a given process to occur.

Decay width and branching ratios

Here again the interaction Lagrangian appears in the calculation of the decay width just like with a cross section. When a particle decays it may decay to different sets of particles. The decay width is like the decay constant in Nuclear Physics



Nucleus X decays to A with decay constant λ_a and to B with decay constant λ_b.

$$\begin{cases} \left(\dfrac{dN}{dt}\right)_a = -\lambda_a N & \text{to A} \\ \left(\dfrac{dN}{dt}\right)_b = -\lambda_b N & \text{to B} \end{cases} \Rightarrow \quad \frac{dN}{dt} = \left(\frac{dN}{dt}\right)_a + \left(\frac{dN}{dt}\right)_b = -(\lambda_a + \lambda_b)N$$

$$N(t) = N_0 \; e^{-\lambda_T \, t} \quad \text{com} \quad \lambda_T = \lambda_a + \lambda_b$$

In particle physics the notation is

$$\lambda \rightarrow \Gamma$$

Particle lifetime is the time taken for the sample to reduce to 1/e of original sample.

Decay width and branching ratios

In natural units (more later) the decay width is the decay constant which is the inverse of the lifetime of the particle. For instance for the Z boson the total width is

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\nu_i\nu_i} + \Gamma_{q_iq_i}$$

with the measured value of

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

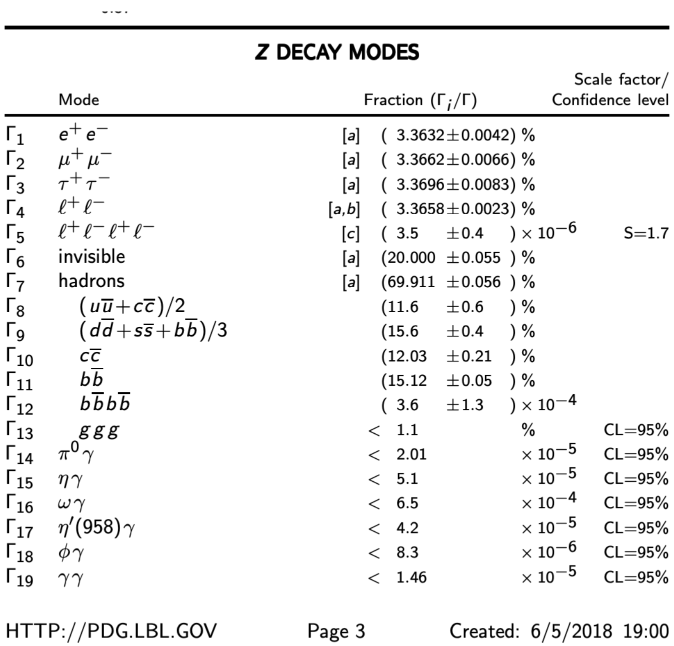
and a lifetime of about 2.7 × 10^{-25} s. Each of the terms are called partial width for the corresponding channel.

The branching ratio for a specific channel

$$\text{BR}_X = \frac{\Gamma_X}{\Gamma_Z}$$

Fraction of electrons coming from the Z decay

$$\text{BR}_{ee} = \frac{\Gamma_{ee}}{\Gamma_Z}$$



Z DECAY MODES

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	e^+e^-	[a]	(3.3632±0.0042) %	
Γ_2	$\mu^+\mu^-$	[a]	(3.3662±0.0066) %	
Γ_3	$\tau^+\tau^-$	[a]	(3.3696±0.0083) %	
Γ_4	$\ell^+\ell^-$	[a,b]	(3.3658±0.0023) %	
Γ_5	$\ell^+\ell^-\ell^+\ell^-$	[c]	(3.5 ±0.4) × 10^{-6}	S=1.7
Γ_6	invisible	[a]	(20.000 ±0.055) %	
Γ_7	hadrons	[a]	(69.911 ±0.056) %	
Γ_8	$(u\overline{u}+c\overline{c})/2$		(11.6 ±0.6) %	
Γ_9	$(d\overline{d}+s\overline{s}+b\overline{b})/3$		(15.6 ±0.4) %	
Γ_{10}	$c\overline{c}$		(12.03 ±0.21) %	
Γ_{11}	$b\overline{b}$		(15.12 ±0.05) %	
Γ_{12}	$b\overline{b}b\overline{b}$		(3.6 ±1.3) × 10^{-4}	
Γ_{13}	ggg		< 1.1 %	CL=95%
Γ_{14}	$\pi^0\gamma$		< 2.01 × 10^{-5}	CL=95%
Γ_{15}	$\eta\gamma$		< 5.1 × 10^{-5}	CL=95%
Γ_{16}	$\omega\gamma$		< 6.5 × 10^{-4}	CL=95%
Γ_{17}	$\eta'(958)\gamma$		< 4.2 × 10^{-5}	CL=95%
Γ_{18}	$\phi\gamma$		< 8.3 × 10^{-6}	CL=95%
Γ_{19}	$\gamma\gamma$		< 1.46 × 10^{-5}	CL=95%

HTTP://PDG.LBL.GOV Page 3 Created: 6/5/2018 19:00

A generalised concept of luminosity

When particles colide at a lepton collider you have to count the number of electrons and positrons that collide. Suppose we are calculated the process

$$e^+e^- \to \gamma\gamma$$

The number of $\gamma\gamma$ events produced at the collider is

$$N_\gamma = 2\,L_{e^+}\,\sigma_{e^+e^-\to\gamma\gamma}$$

and L_{e^+} is the luminosity. Same principles apply to any lepton collider. Two notes: a) the number of electrons that actually count to the collision is not the number of electrons produced; b) the number of photons actually detected depend on the detectors and on the specific experimental analysis.

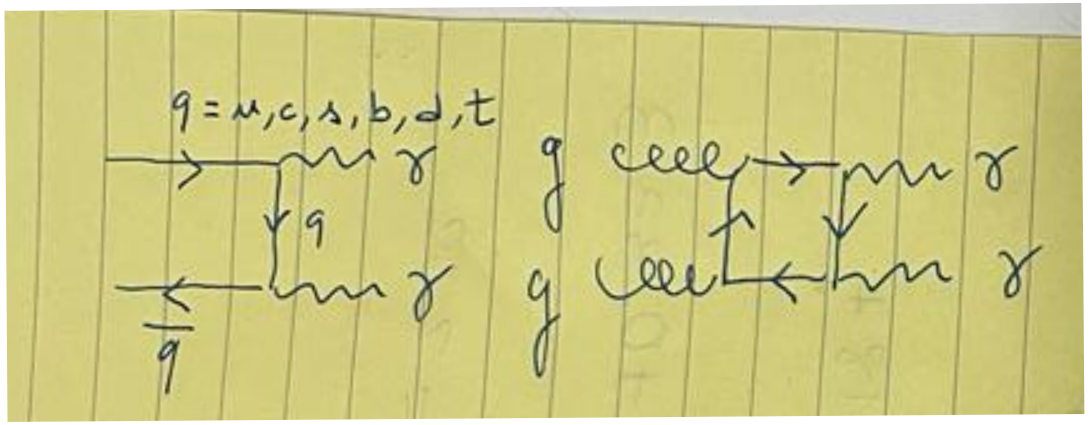
Let us now move to a photon collider. There are various methods of e → γ conversion but the best one is the Compton scattering of laser light on high-energy electrons. In this method a laser photon is scattered backward taking from the high-energy electron a large fraction of its energy. The scattered photon travels along the direction of the initial electron. Here, the CM energy is calculated to be a fraction of the initial leptons.

A generalised concept of luminosity

And finally hadron colliders. Hadron colliders are pp (LHC - Large Hadron Collider) or $p\bar{p}$ (Tevatron) colliders. The protons are made of quarks and gluons. Further, when they collide there is a lot of energy and many elementary particles can be present. Considering again the production of two photons

$$pp \to \gamma\gamma$$

From the point of view of the particles (partons) that constitute the protons we have the following set (just one of each type) of Feynman diagrams



Depending on the energy scale of the problem we may consider only the lightest quarks and the gluon, or the heavier ones. At the LHC there are two schemes: the 4F scheme where the b-quark is not considered and the 5F scheme where it is taken into account. The top quark is too heavy to play a role at the LHC. The u and d are called valence quarks.

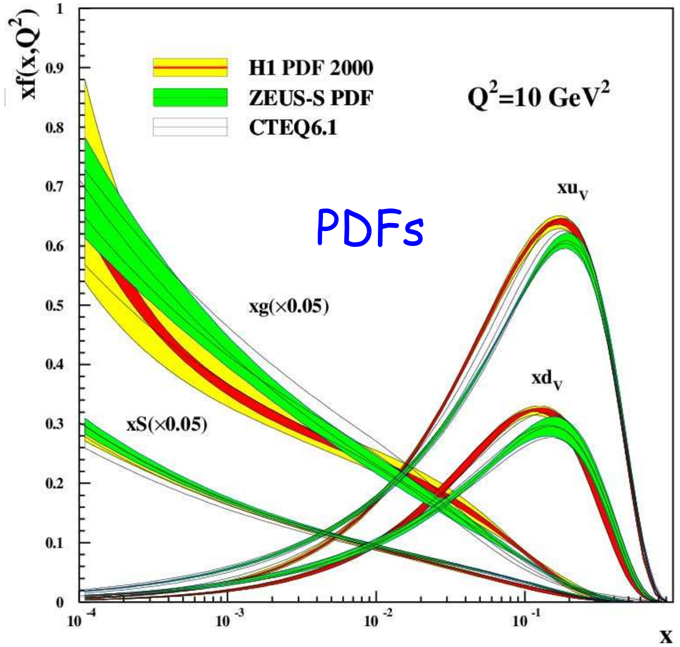
A generalised concept of luminosity

To calculate the cross section $\sigma(pp \to \gamma\gamma)$ we first need to calculate the corresponding Parton level cross section $\sigma(q\bar{q} \to \gamma\gamma)$ for all quarks but the top and $\sigma(gg \to \gamma\gamma)$ for the gluon. The total cross section is calculated using the Parton Distribution Functions (PDF).

The PDF is defined as the probability density for finding a particle with a certain longitudinal momentum fraction x at some energy scale. Because of the non-perturbative nature of partons, which cannot be observed as free particles, parton densities cannot be calculated using perturbative QCD. They are therefore calculated using fits from all collider data available so far. The total cross section for $pp \to \gamma\gamma$ is given by

$$\sigma_{pp\to\gamma\gamma} = \int dx_1\, f_{q/p}(x_1,\mu^2) \int dx_2\, f_{q/p}(x_2,\mu^2)\, \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$$

PDFs combine theoretical knowledge from QCD with experiment. Complicated stuff! Still experiment shows that the proton is indeed a (uud), that the sea quarks play a role and that the gluons carry about 50% of the momentum.



Number of events

To find the number of events in a given process you need: (a) the cross section; (b) the branching ratio; (c) the luminosity; (d) the efficiency. Suppose we are looking for a Higgs in the process

$$pp \to h \to \gamma\gamma$$

The total number of events for a luminosity of 25 fb^{-1} and a center-of-mass energy of 8 TeV is

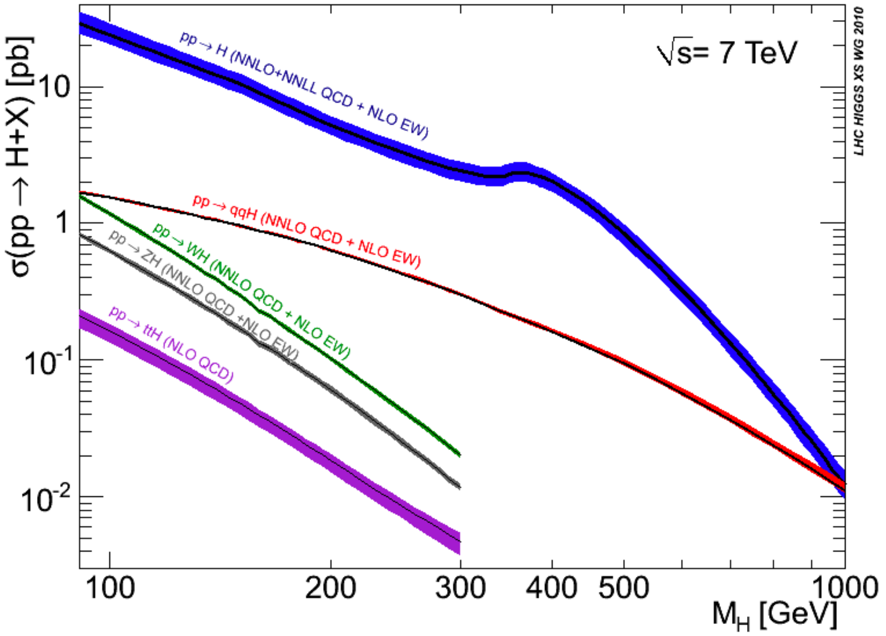
$$N_{Higgs} = \sigma(pp(gg) \to h)\, BR(h \to \gamma\gamma)\, L\, \epsilon$$

$$N_{Higgs} = (21.4 \times 10^{3}) \times (2.28 \times 10^{-3}) \times (25) \times (1) \approx 1220$$

with the efficiency set to 1.

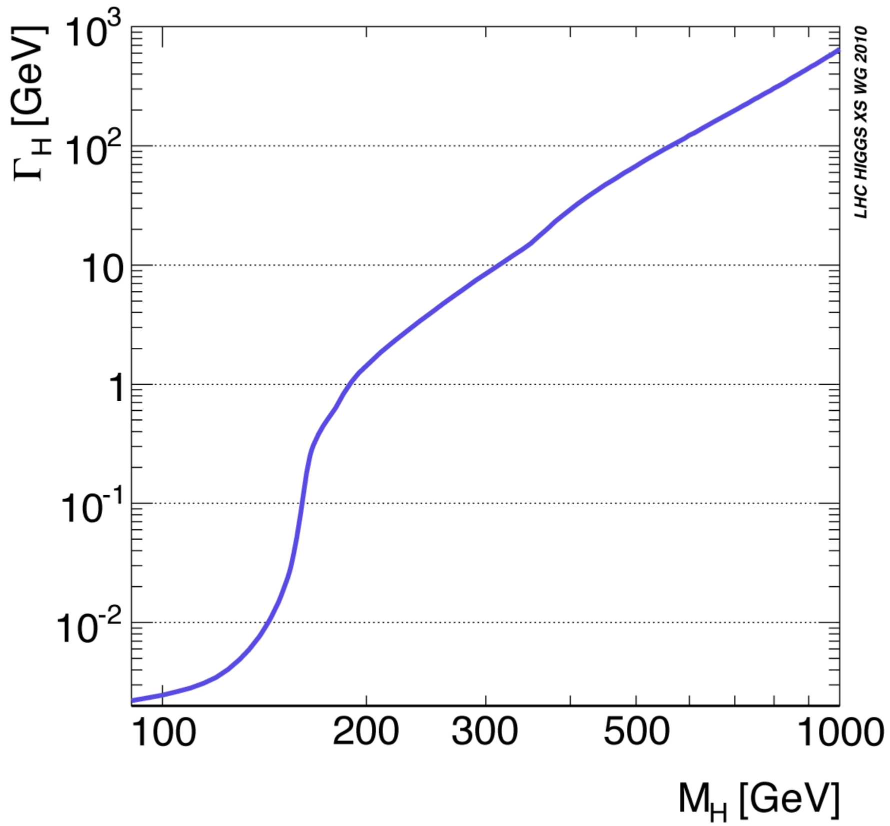
This is the maximum number of events. We then need to take into account the background and the fact that all apparatus and analysis have a specific efficiency.

Profiling the Higgs



Cross section for Higgs production at the LHC
For a center of mass energy of 7 TeV.

Total width of the Higgs as a function of the mass.



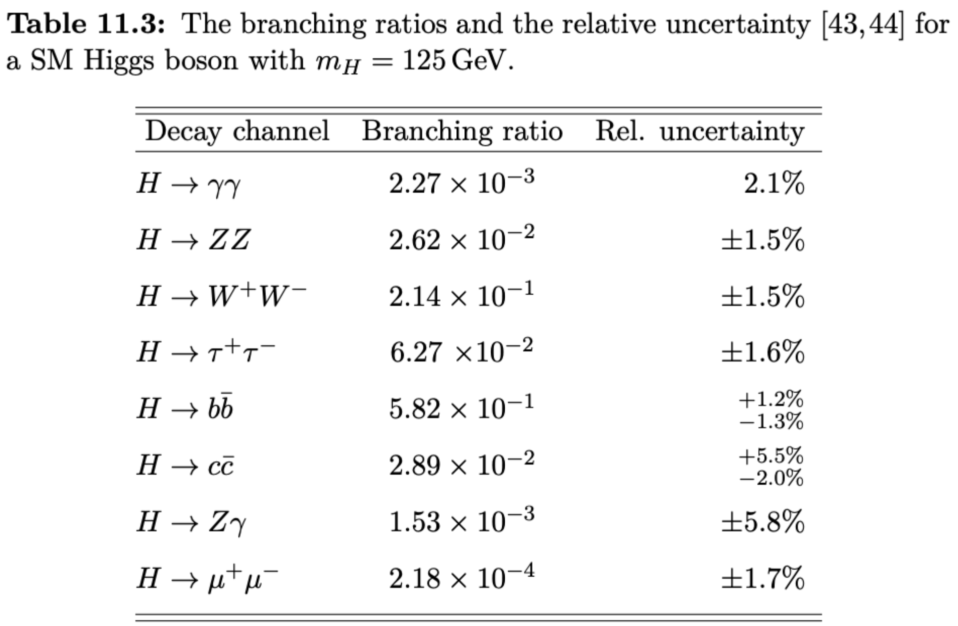
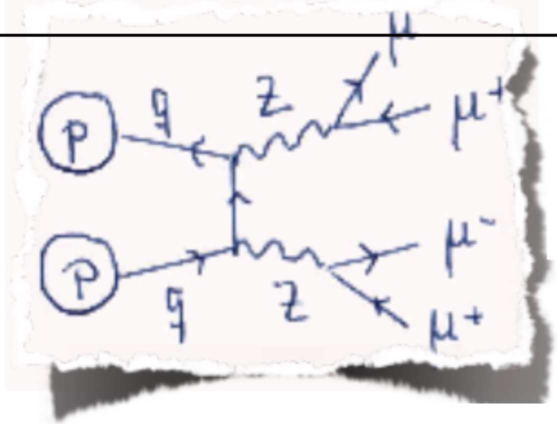


Table 11.3: The branching ratios and the relative uncertainty [43,44] for a SM Higgs boson with $m_H = 125\,\mathrm{GeV}$.

Decay channel	Branching ratio	Rel. uncertainty
$H \to \gamma\gamma$	2.27×10^{-3}	2.1%
$H \to ZZ$	2.62×10^{-2}	±1.5%
$H \to W^+W^-$	2.14×10^{-1}	±1.5%
$H \to \tau^+\tau^-$	6.27×10^{-2}	±1.6%
$H \to b\bar{b}$	5.82×10^{-1}	$^{+1.2\%}_{-1.3\%}$
$H \to c\bar{c}$	2.89×10^{-2}	$^{+5.5\%}_{-2.0\%}$
$H \to Z\gamma$	1.53×10^{-3}	±5.8%
$H \to \mu^+\mu^-$	2.18×10^{-4}	±1.7%

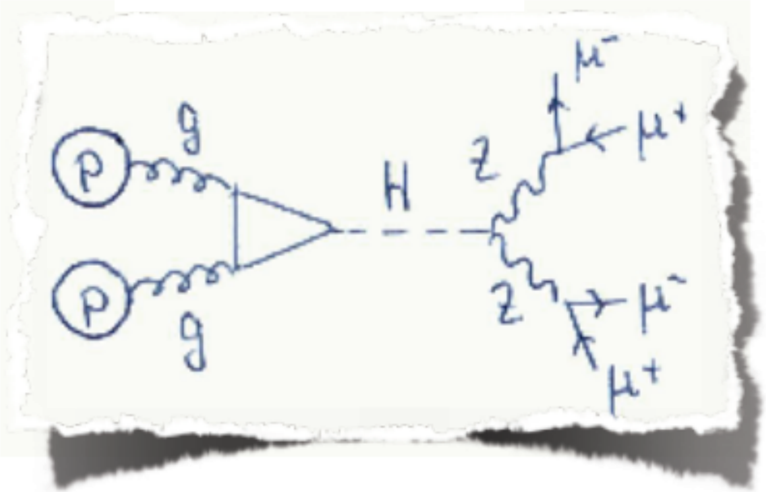
How do we search for the Higgs boson?

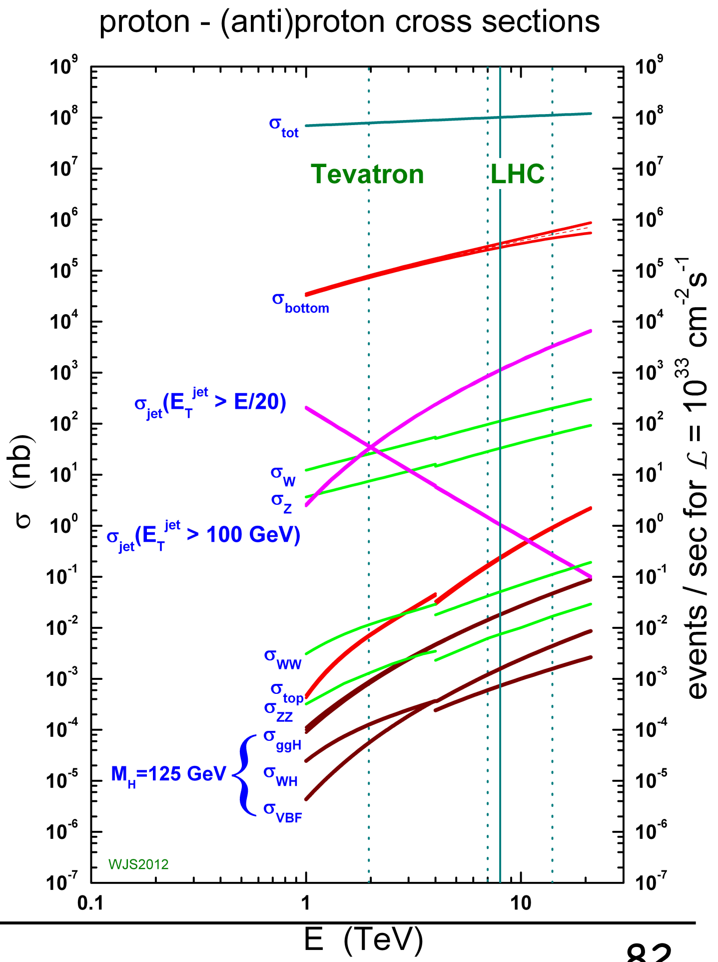
Signal and background



a lot

a few

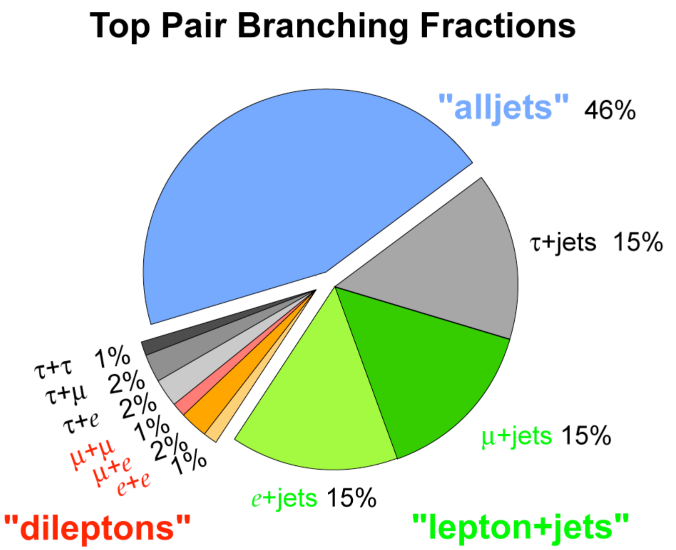


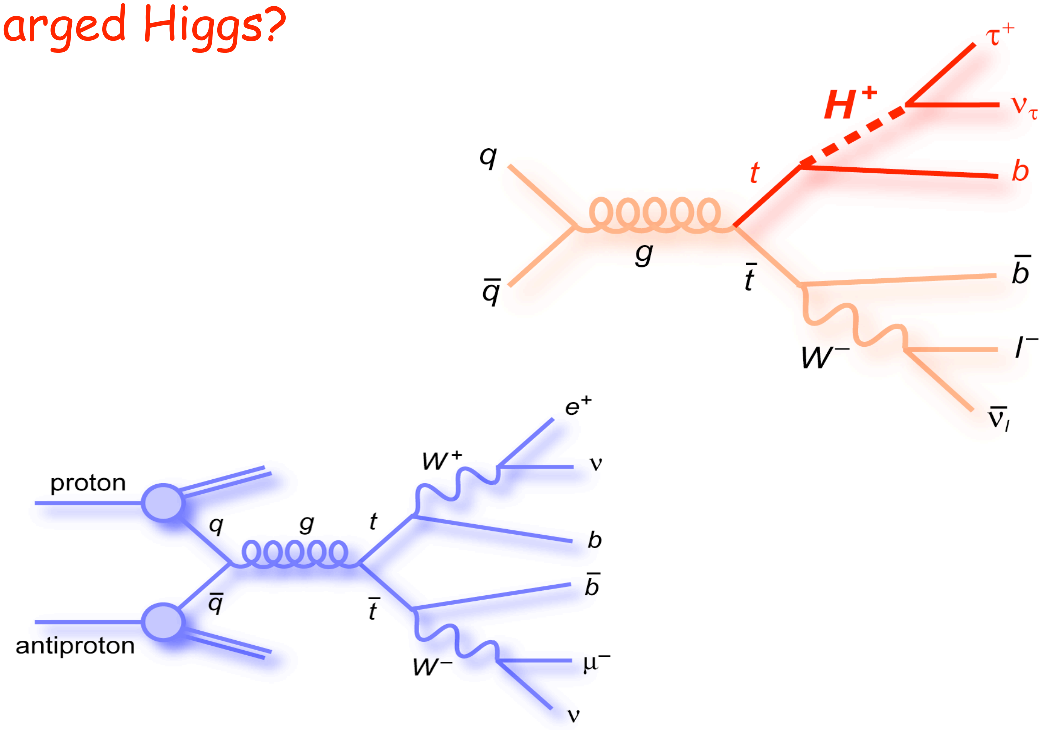


Example of an LHC search

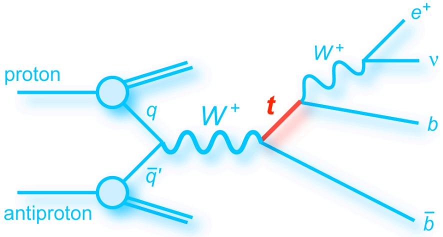
- Suppose we are searching for a charged Higgs at the LHC
- The charged Higgs comes from a top-quark (t -> H^+ b)
- There are two top-quarks in the event
- The charged Higgs will decay to a tau and a neutrino ($H^+ \to \tau^+ \upsilon$)
- The tau will decay to an electron or a muon and neutrinos ($\tau^+ \to e^+ \upsilon\ \upsilon$)
- Experimentalists will look for a an electron and missing energy (from the neutrinos).

- Do they come from a charged Higgs?





Levels of the search - folklore in traditional approach



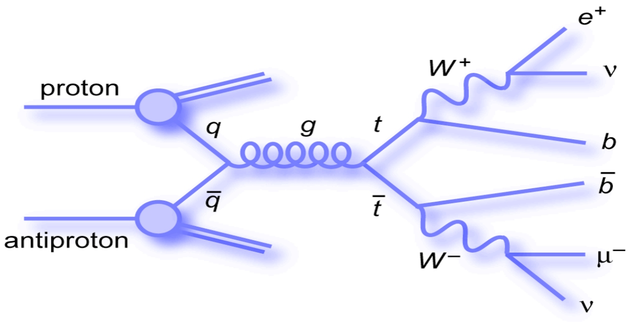
Parton Level pp -> e^+ υ υ υ b + X

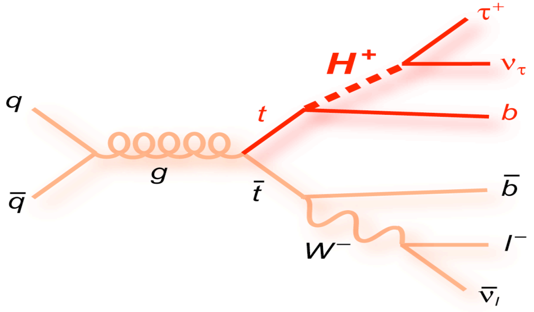
a) Introduce background (again there are several levels)

Irreducible - pp -> t + X -> W^+ b + X-> e^+ υ υ b + X

Reducible - pp -> t t -> W^+ b W^- b

- Number of reducible backgrounds is virtually infinite (a jet has some probability of being misidentified as an electron).





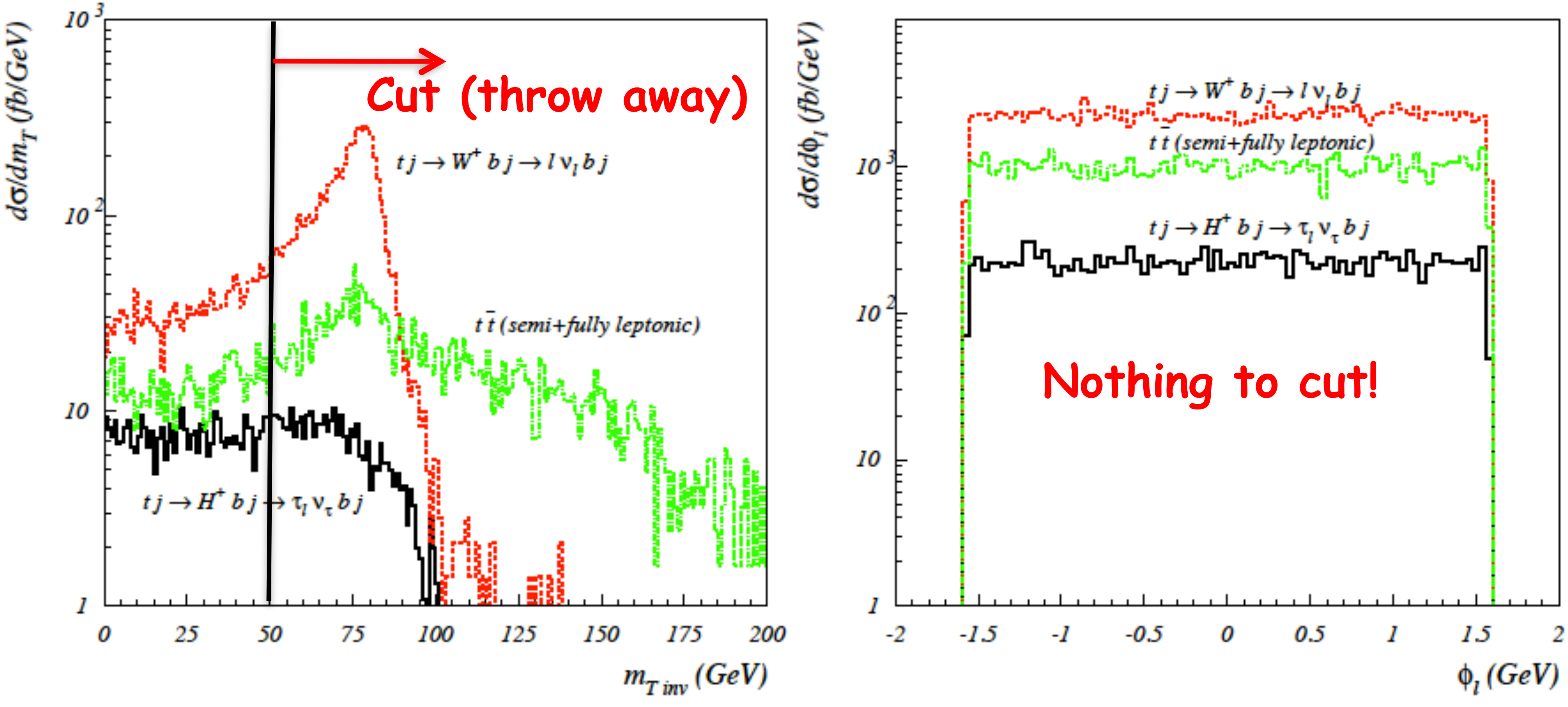
b) Pretend you understand what is happening (by mimicking the experimental analysis)

Trigger – how efficiently are “our” events recorded? One lepton!

Electron – how efficient is electron recognition?

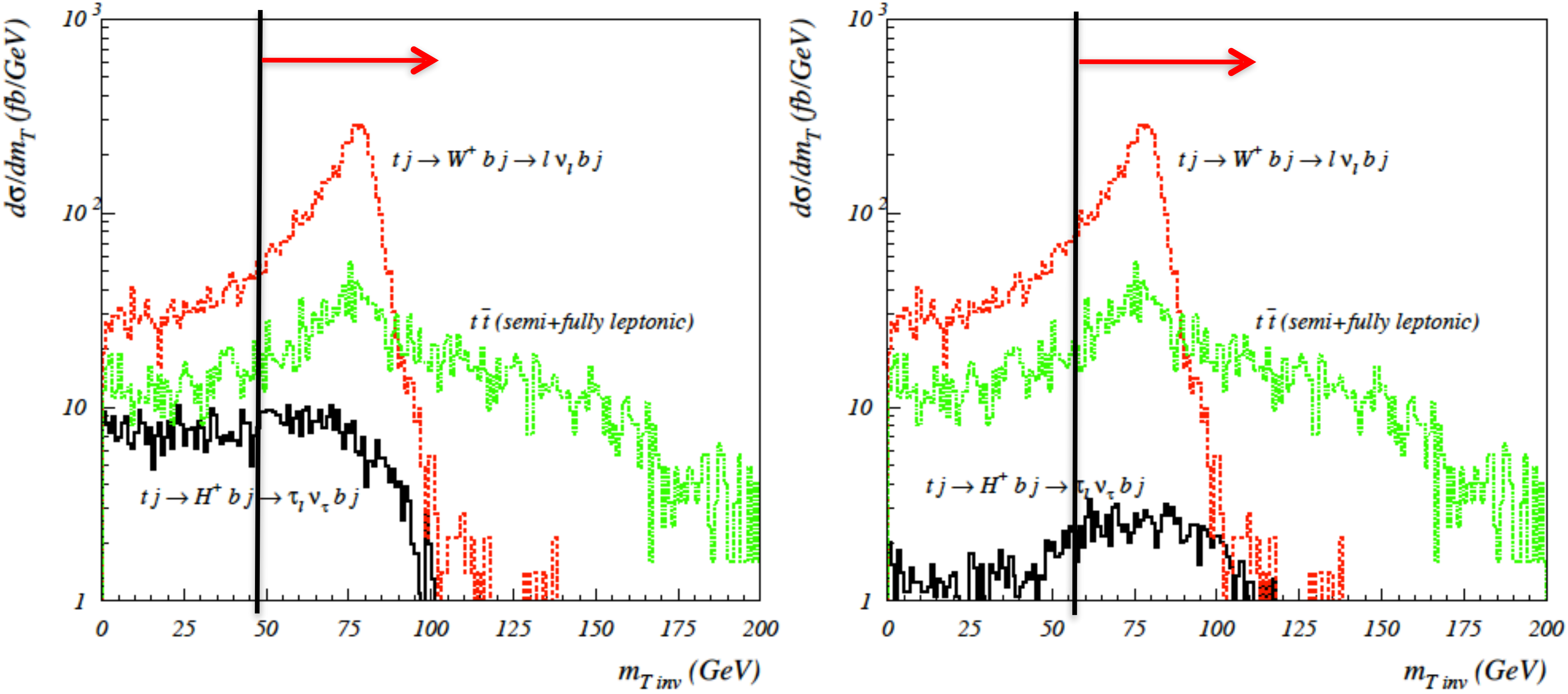
Levels of the search - folklore in traditional approach

c) Plot distributions and spot the differences (loose signal but loose even more background)



Two types of variables: the transverse mass and the lepton azimuthal angle (cut-based).

Levels of the search - folklore in traditional approach



Distributions depend on the model parameters. The higher the charged Higgs mass the lower the cross section. There is also a "peak" shift.

d) e) f) Radiation, Detector and finally Data.

Levels of the search - folklore in traditional approach

	$\sigma_{m_{H^\pm}=100\,\mathrm{GeV}}$	$\sigma_{m_{H^\pm}=110\,\mathrm{GeV}}$	$\sigma_{m_{H^\pm}=120\,\mathrm{GeV}}$	$\sigma_{m_{H^\pm}=130\,\mathrm{GeV}}$	$\sigma_{m_{H^\pm}=140\,\mathrm{GeV}}$
Process					
Signal	379.4 fb	274.4 fb	202.7 fb	118.9 fb	65.5 fb
Bg (single-top)	1705.4 fb				
Bg ($t\bar{t}$ semi-leptonic)	683.1 fb				
Bg ($t\bar{t}$ leptonic)	393.6 fb				
σ_S/σ_B	0.14	0.098	0.073	0.042	0.023
$\sigma_S/\sqrt{\sigma_B}$ ($\mathrm{fb}^{1/2}$)	7.19	5.20	3.84	2.25	1.24

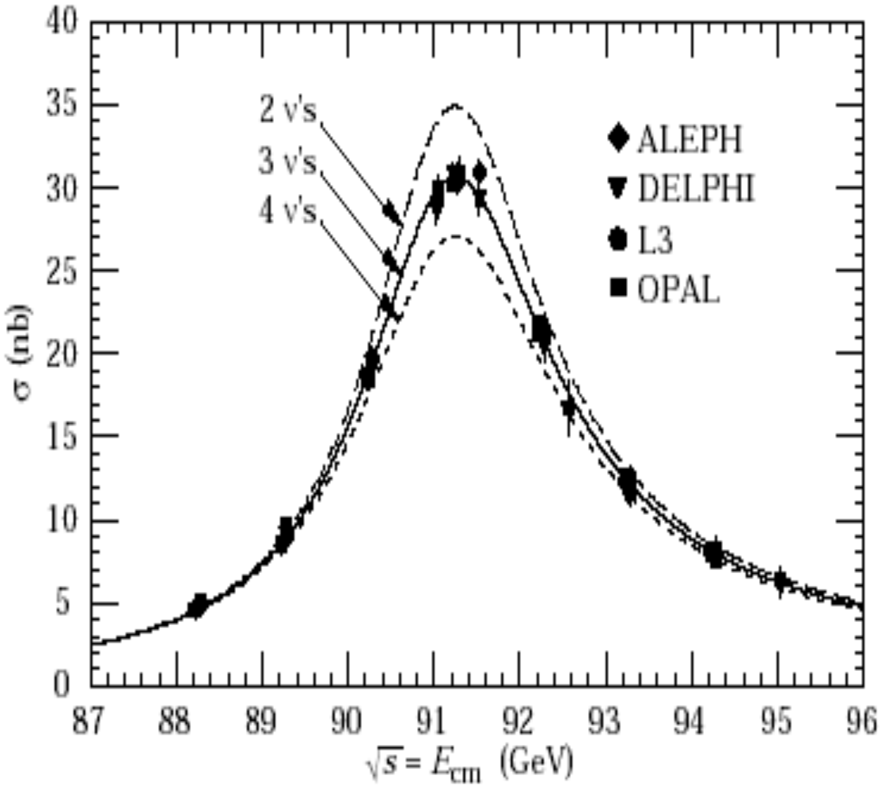
The analysis is done. We now have, for a given luminosity, S signal events and B background events.

Discovery - $S/B^{1/2} > 5$

An exclusion (absence of signal) is usually shown for 95 % C.L.

We see mass!

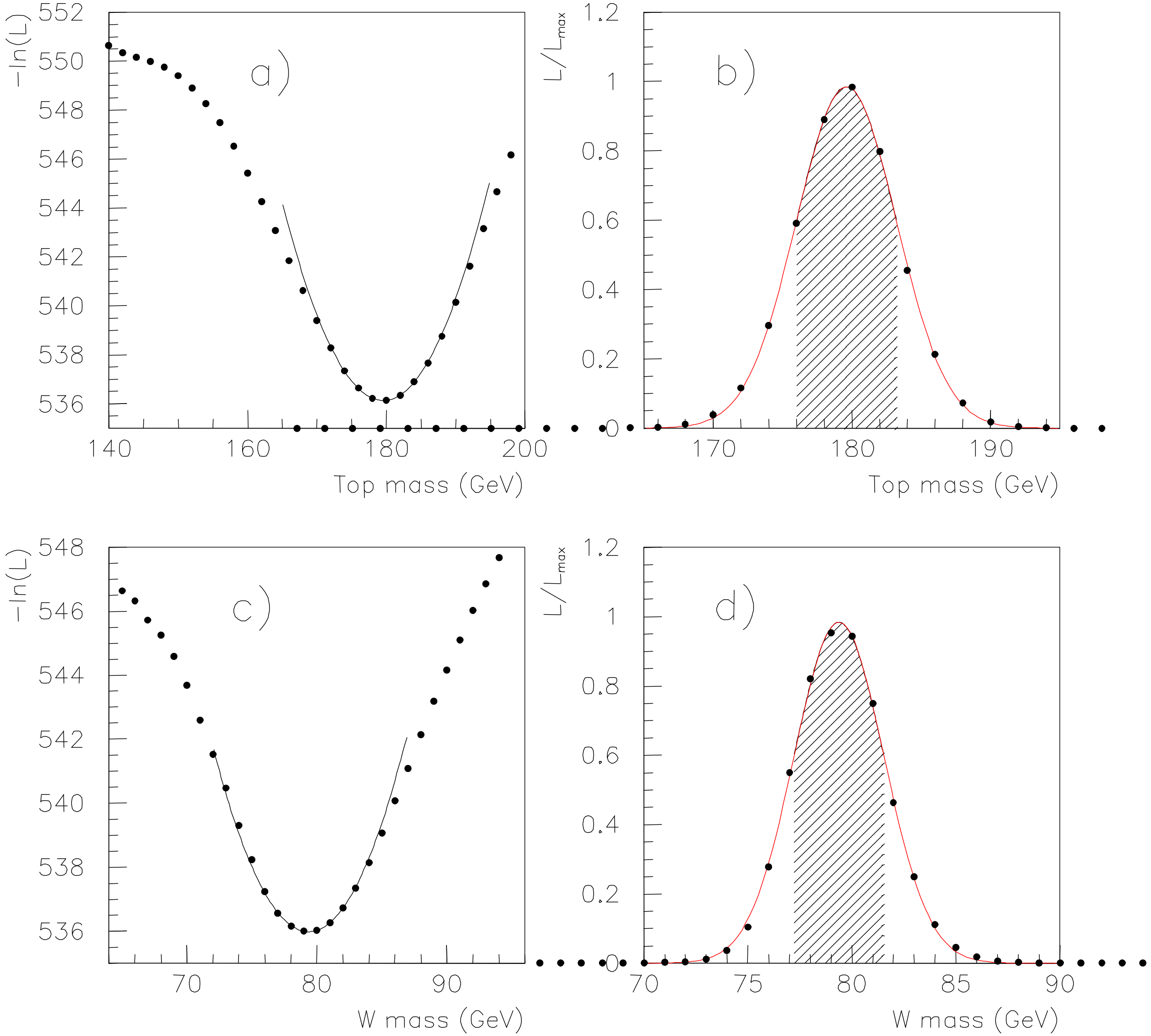
Weak gauge bosons have mass (LEP). The number of neutrinos can be counted in a given model



$$e^+e^- \to Z^0$$

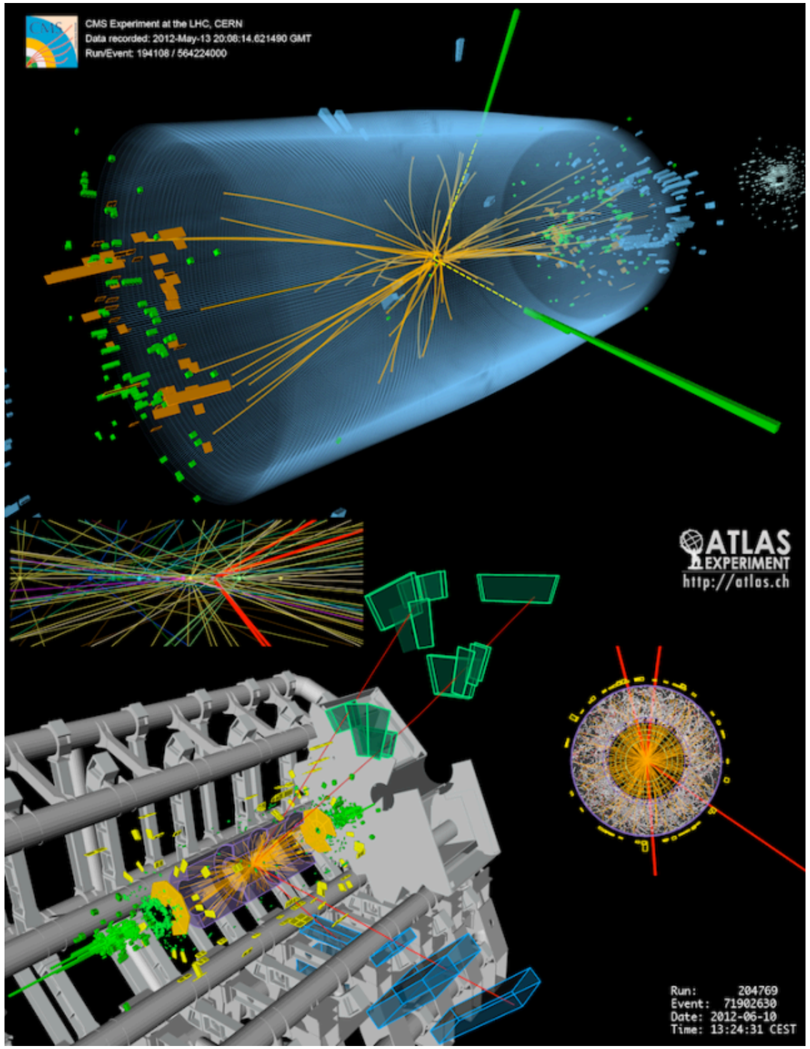
We see mass!

- Fermions have mass



top and W mass measurements by D0 (Tevatron Run I)

Some history - The Higgs discovery



There is a model, the Standard Model, that is based on symmetries.

With the symmetries, all particles emerge without a mass. But most particles have mass. Brout, Englert and Higgs proposed a mechanism that gives mass to the particles via the interaction with a field we now call the “Higgs” field.

Just after the Big Bang the Higgs field was zero but as the temperature fell below a critical value, it spontaneously grew and particles interacting with it got a mass. The larger the interaction the heavier the particle. No coupling to the photon.

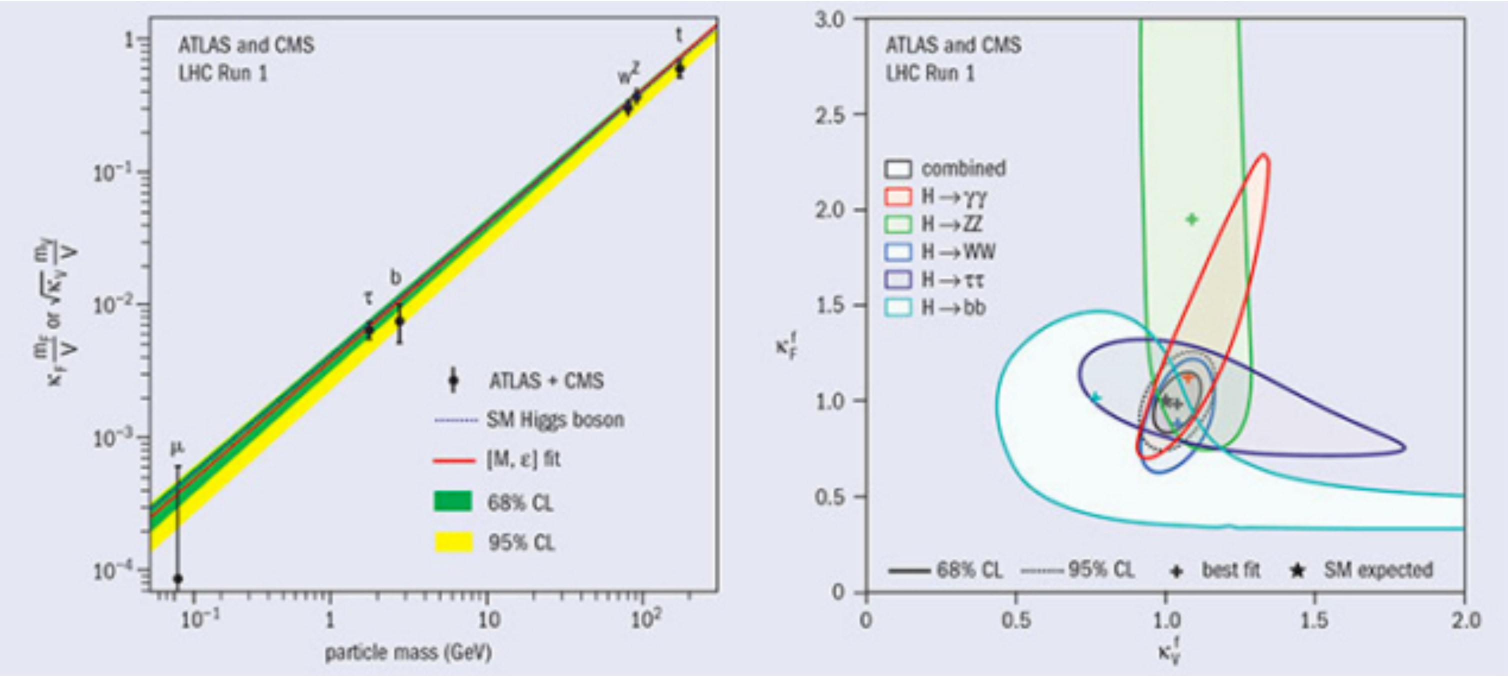
On July 4 2012, the ATLAS and CMS experiments at CERN’s Large Hadron Collider observed a new particle in the mass region around 125 GeV, consistent with the Standard Model Higgs boson. Is it the Higgs boson predicted by the Standard Model?

Some history - The Higgs discovery

There is a potential - the Higgs potential

$$V_{SM} = m_{11}^2(\Phi_1^\dagger\Phi_1) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2$$

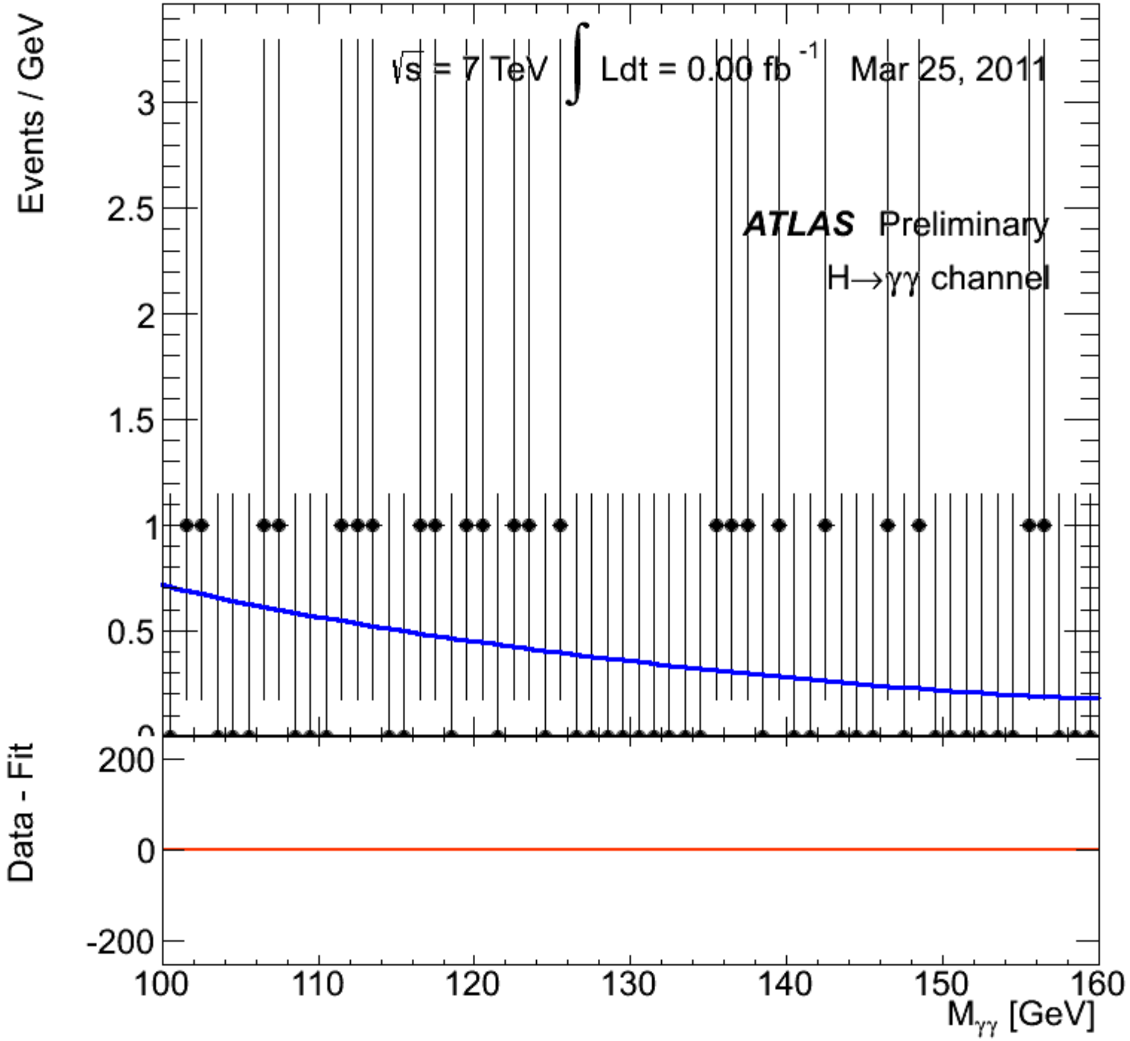
- The Higgs field
- Couples to boson fields - mass of the bosons
- Couples to fermion fields - mass of the fermions



$$g_{NP}^{hVV} = \kappa_V g_{SM}^{hVV}$$

So, 12 years after the discovery, the 125 GeV scalar looks very much like the SM Higgs

Some history - The Higgs discovery

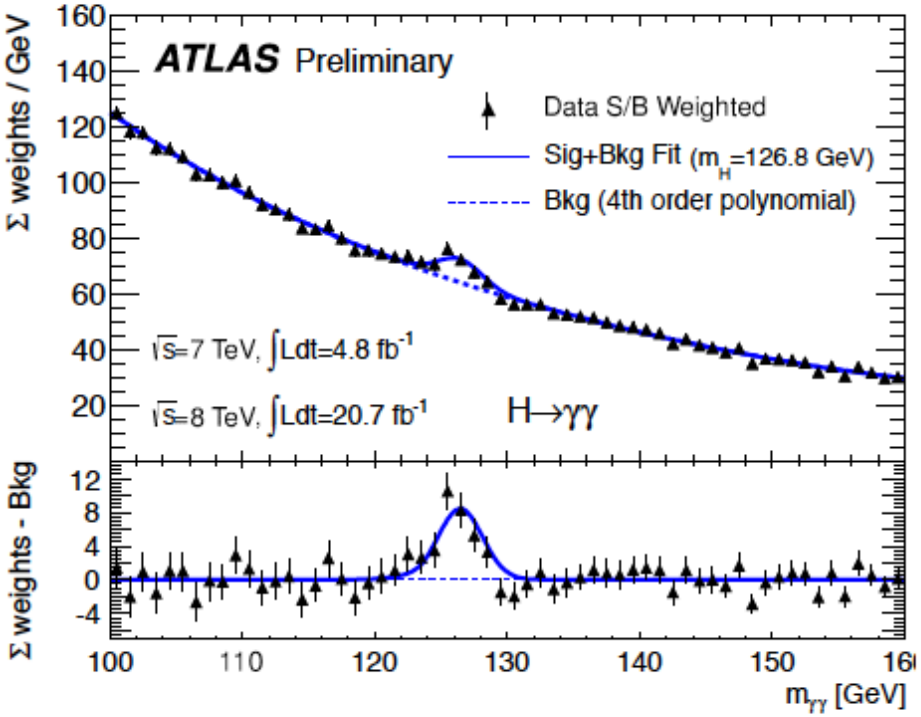


5 sigma and the Higgs discovery!

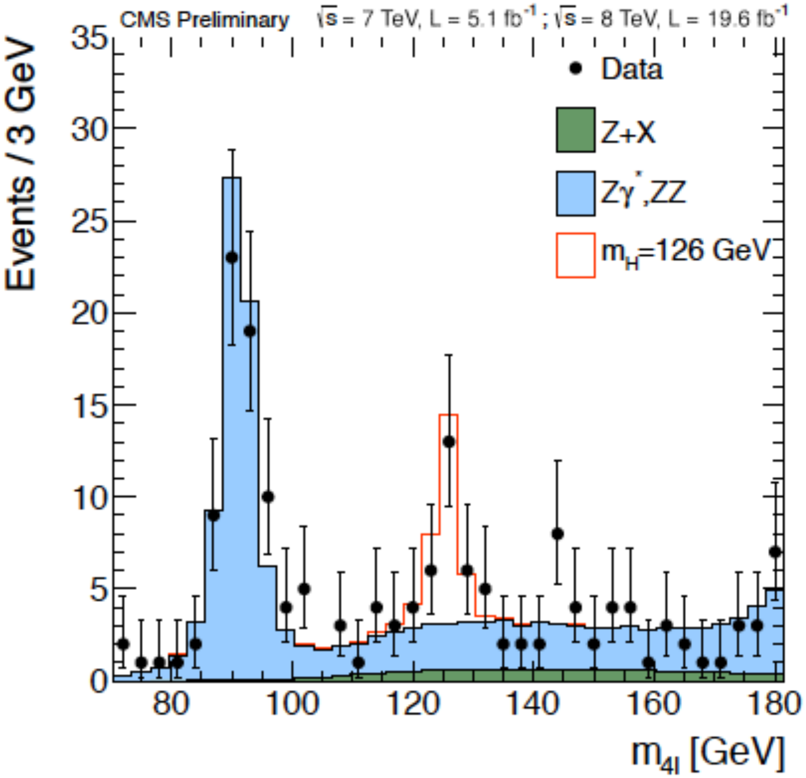
LHC collects a huge number of collisions data and counts how many times two given photons have the Higgs mass. When 5 sigma was reached the Higgs was considered to have been discovered.

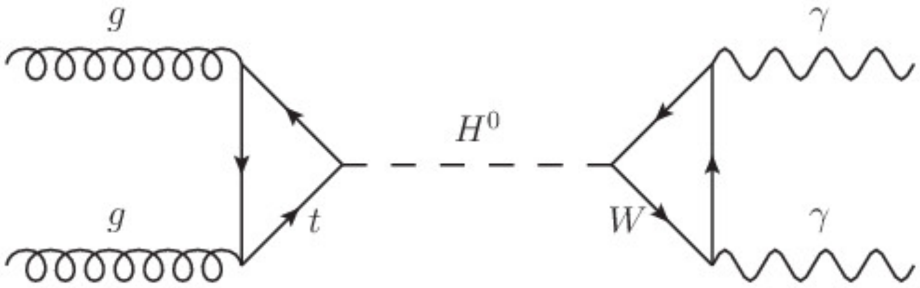
Some history - The Higgs discovery

ATLAS-CONF-2013-12

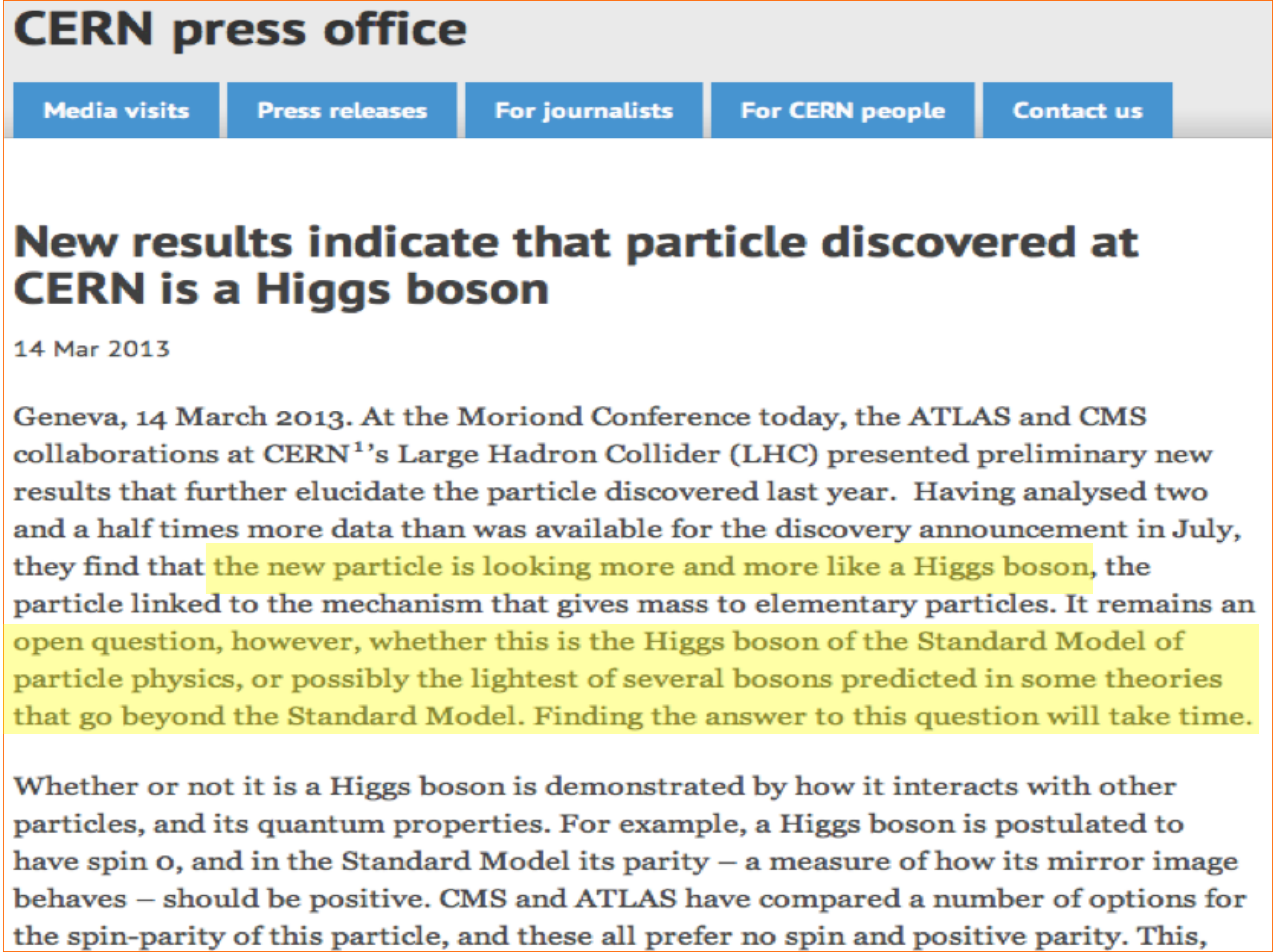


CMS-PAS-HIG-13-002





… a poster child for
quantum theory!



New results indicate that particle discovered at CERN is a Higgs boson

14 Mar 2013

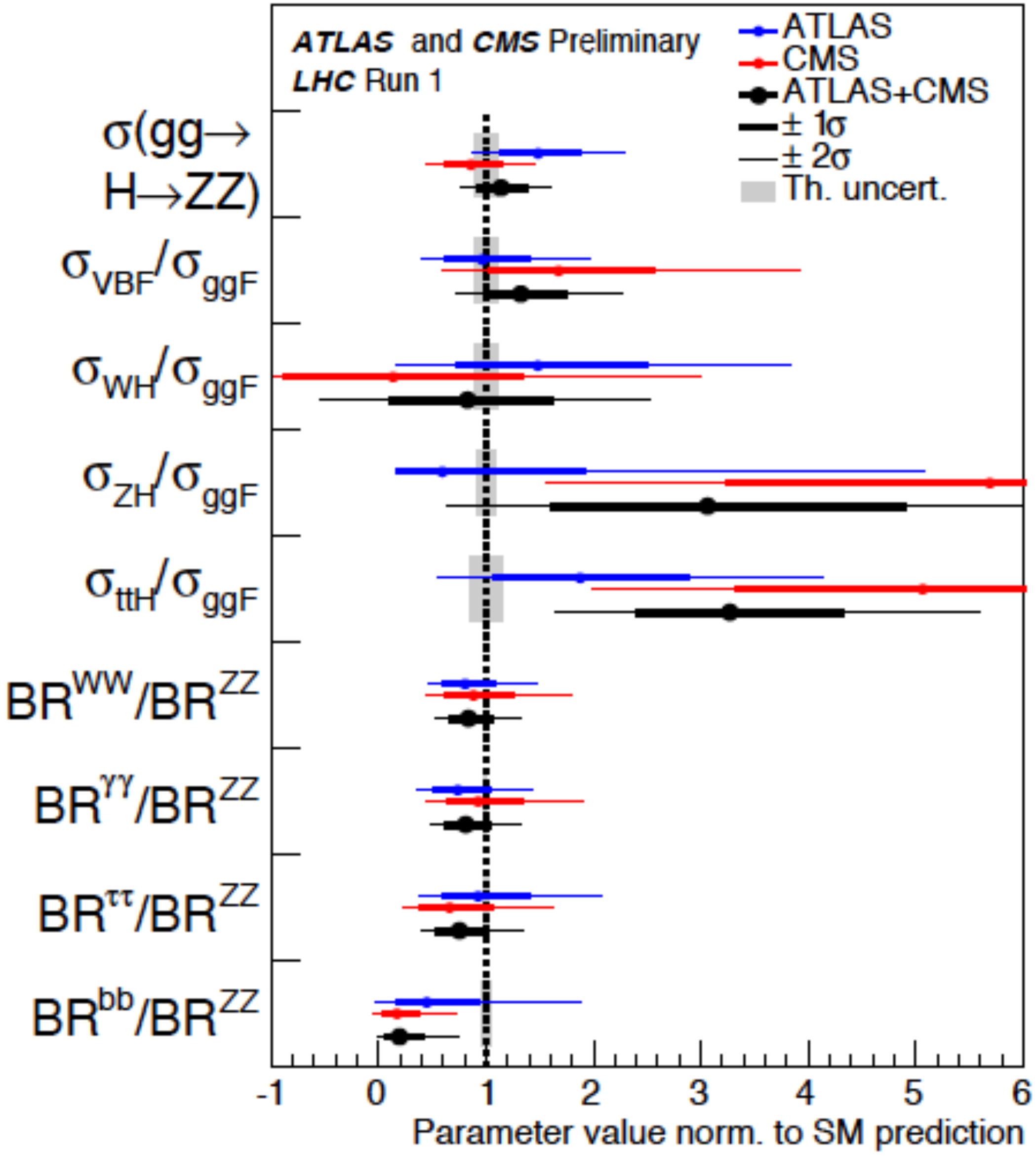
Geneva, 14 March 2013. At the Moriond Conference today, the ATLAS and CMS collaborations at CERN[1]'s Large Hadron Collider (LHC) presented preliminary new results that further elucidate the particle discovered last year. Having analysed two and a half times more data than was available for the discovery announcement in July, they find that the new particle is looking more and more like a Higgs boson, the particle linked to the mechanism that gives mass to elementary particles. It remains an open question, however, whether this is the Higgs boson of the Standard Model of particle physics, or possibly the lightest of several bosons predicted in some theories that go beyond the Standard Model. Finding the answer to this question will take time.

Whether or not it is a Higgs boson is demonstrated by how it interacts with other particles, and its quantum properties. For example, a Higgs boson is postulated to have spin 0, and in the Standard Model its parity – a measure of how its mirror image behaves – should be positive. CMS and ATLAS have compared a number of options for the spin-parity of this particle, and these all prefer no spin and positive parity. This,

1964
Brout-Englert-Higgs-
Mechanism

2013
Nobel Prize for Physics

The Standard Model
is complete.
Now what?



ATLAS/CMS combination with all run1 data.

ATLAS-CONF-2015-044
CMS-PAS-HIG-15-002

15th September 2015

The Higgs looks very SM-like because all couplings are well within the SM predictions. And there is no hint of new physics so far.

There are essentially two ways of showing we need new physics:

a) deviations from the SM predictions

b) finding something new (like with dark matter if it is indeed a particle)

Feynman Rules, Cross Sections and Branching ratios - calculations

10.1 Kinematics

Let the Higgs boson of mass M_H and momentum q decay into particles A and B of masses m_1 and m_2 and momenta p_1 and p_2 respectively: $H(q) \to A(p_1) + B(p_2)$. The decay rate summed over final polarisations and colours is:

$$d\,\Gamma = \frac{1}{2\,M}\,\frac{d^3p_1}{(2\,\pi)^3\,2\,E_1}\,\frac{d^3p_2}{(2\,\pi)^3\,2\,E_2}\,(2\,\pi)^4\,\delta^4\,(q - p_1 - p_2)\,\left|\bar{T}\right|^2 \;, \tag{10.1}$$

with $\left|\bar{T}\right|^2$ the invariant matrix element squared, summed over final colours and polarisations. Momentum conservation imposes $p_1.p_2 = (M_H^2 - m_1^2 - m_2^2)/2$ with $p_1^2 = m_1^2$ et $p_2^2 = m_2^2$. Thus $\left|\bar{T}\right|^2$ depends only on the external masses $\left|\bar{T}(m_1^2, m_2^2, M_H^2)\right|^2$ and the integral in eq. (10.1) can be done independently of the decay channel. Using $d^3p_2/2\,E_2 = d^4p_2\,\delta^+(p_2^2 - m_2^2)$ and carrying out the d^4p_2 integration it comes out

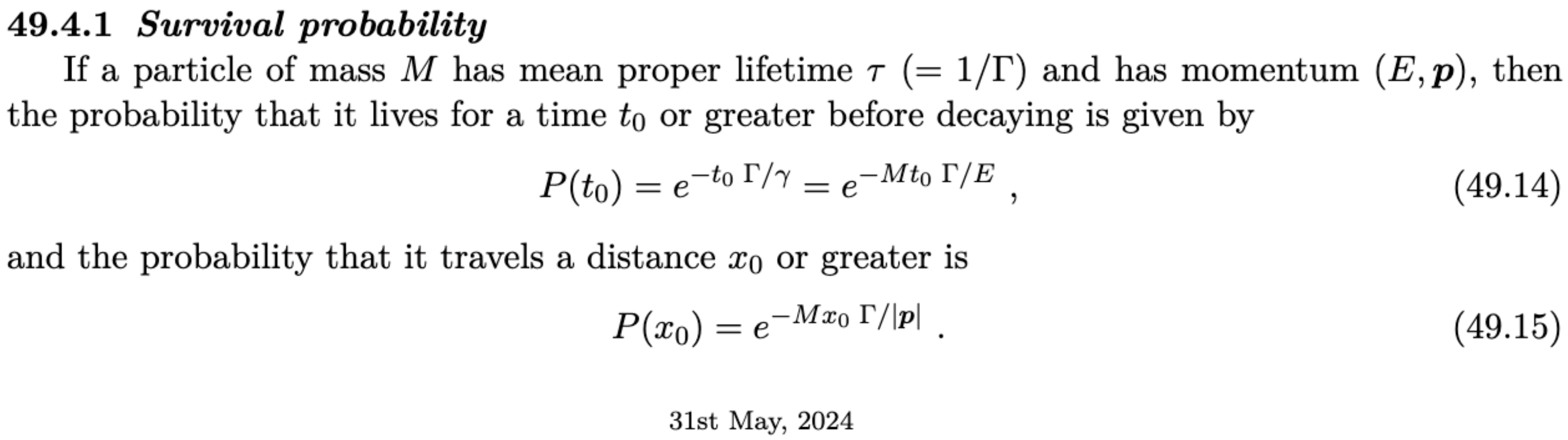
$$d\,\Gamma = \frac{1}{2\,M_H}\,\frac{\left|\bar{T}\right|^2}{(2\,\pi)^2}\int\frac{d^3p_1}{2\,E_1}\,\delta^+\left((q - p_1)^2 - m_2^2\right). \tag{10.2}$$

Going to the rest frame of the Higgs boson, $q = (M, 0, 0, 0)$, one finds that the argument of the δ^+ function reduces to $(M^2 - 2ME_1 + m_1^2 - m_2^2)$ independent of the angles. Since all cases we consider have $m_1 = m_2$ the expressions will simplify. Using $p_1\,dp_1 = E_1\,dE_1$ all integrations are easily done to get:

$$\boxed{\Gamma = \frac{1}{16\,\pi\,M_H}\,\left|\bar{T}\right|^2\,\sqrt{1 - \frac{4\,m^2}{M^2}}\;,} \tag{10.3}$$

with m the common mass of the decay products.

Decays and lifetime



49.4.1 *Survival probability*

If a particle of mass M has mean proper lifetime τ $(= 1/\Gamma)$ and has momentum $(E, \boldsymbol{p})$, then the probability that it lives for a time t_0 or greater before decaying is given by

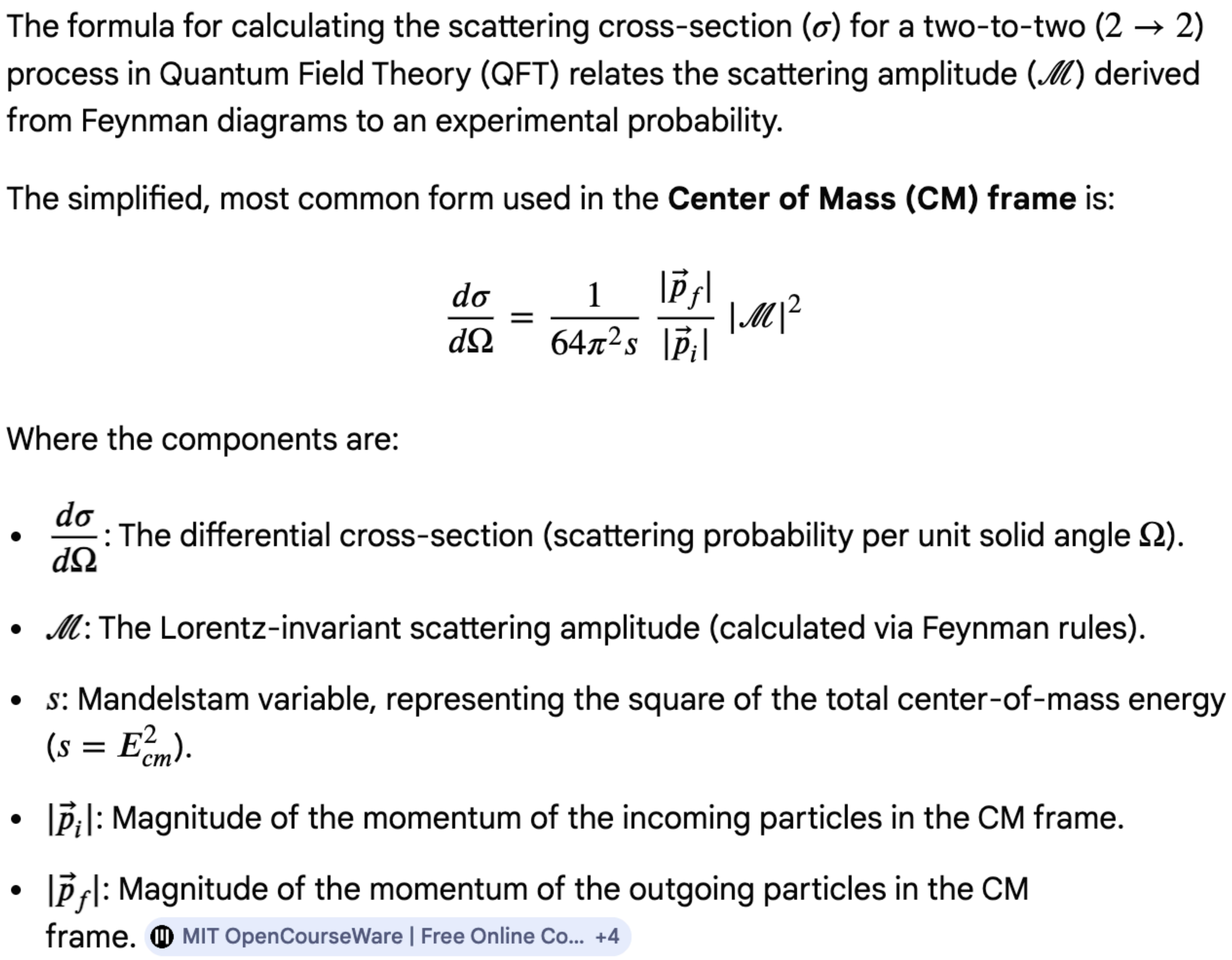
$$P(t_0) = e^{-t_0\,\Gamma/\gamma} = e^{-Mt_0\,\Gamma/E}\ , \tag{49.14}$$

and the probability that it travels a distance x_0 or greater is

$$P(x_0) = e^{-Mx_0\,\Gamma/|\boldsymbol{p}|}\ . \tag{49.15}$$

31st May, 2024

Cross Sections



The formula for calculating the scattering cross-section (σ) for a two-to-two ($2 \to 2$) process in Quantum Field Theory (QFT) relates the scattering amplitude ($\mathscr{M}$) derived from Feynman diagrams to an experimental probability.

The simplified, most common form used in the **Center of Mass (CM) frame** is:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_f|}{|\vec{p}_i|} |\mathscr{M}|^2$$

Where the components are:

- $\frac{d\sigma}{d\Omega}$: The differential cross-section (scattering probability per unit solid angle Ω).
- $\mathscr{M}$: The Lorentz-invariant scattering amplitude (calculated via Feynman rules).
- s: Mandelstam variable, representing the square of the total center-of-mass energy ($s = E_{cm}^2$).
- $|\vec{p}_i|$: Magnitude of the momentum of the incoming particles in the CM frame.
- $|\vec{p}_f|$: Magnitude of the momentum of the outgoing particles in the CM frame.

49.4.2 *Two-body decays*

In the rest frame of a particle of mass M, decaying into 2 particles labeled 1 and 2,

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M} \ , \tag{49.16}$$

$$|\boldsymbol{p}_1| = |\boldsymbol{p}_2| = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)} \ , \tag{49.17}$$

and

$$d\Gamma = \frac{1}{32\pi^2} \, |\mathscr{M}|^2 \, \frac{|\boldsymbol{p}_1|}{M^2} \, d\Omega \ , \tag{49.18}$$

where $\lambda(\alpha, \beta, \gamma) = \alpha^2 + \beta^2 + \gamma^2 - 2\alpha\beta - 2\alpha\gamma - 2\beta\gamma$ is the Källén function and $d\Omega = d\phi_1 d(\cos\theta_1)$ is the solid angle of particle 1. The invariant mass M can be determined from the energies and momenta using Eq. (49.2) with $M = E_{\rm cm}$.