

Project Topics

Complementary Quantum Mechanics

The projects are intended as assessed group coursework. The scope and possible directions below define a scientific area rather than a prescribed checklist. Each group should formulate its own central question and task list within the chosen project, in discussion with the lecturer. The suggested directions are examples only; students may combine them, replace them, or propose a closely related direction.

In addition to the written paper/report, each group must also prepare and deliver a 30-minute presentation on the project.

1. Quantum Algorithms: Interference as Computation

Scope: Explore how superposition, interference, entanglement and measurement can be organised into a computational procedure. The project should go beyond a catalogue of quantum gates and develop a clear physical or computational question about why a chosen quantum algorithm behaves differently from its classical counterpart.

Possible directions:

- A detailed analysis of one algorithm, such as Deutsch–Jozsa, Grover search or quantum phase estimation, following the quantum state through the main stages of the computation.
- The role of constructive and destructive interference in producing the final result, rather than treating the algorithm as a black-box circuit.
- A small numerical circuit simulation, for example to explore success probability, scaling with problem size, or sensitivity to imperfect operations.
- A comparison between what is genuinely quantum in the chosen algorithm and what could be reproduced by a classical probabilistic procedure.

Deliverables: A coherent written paper and oral presentation centred on a question chosen by the group. The project should contain enough explicit quantum-state or circuit analysis to demonstrate the mechanism of the selected algorithm; analytical, numerical or comparative work may be used as appropriate.

2. Quantum Cryptography: Detecting an Eavesdropper

Scope: Study how quantum state preparation and measurement can be used to establish a secret key and to place quantitative bounds on information that may have leaked to an eavesdropper. The project may use BB84 Protocol as the central example, but the group should formulate its own question within quantum key distribution.

Possible directions:

- How non-orthogonal states and incompatible measurement bases enter the security logic of BB84.
- An intercept–resend or another simple eavesdropping strategy and the quantum bit error rate that it produces.
- The distinction between observed errors, which may have several physical causes, and the conservative security assumption that part of the error may be attributed to an adversary.
- A comparison with an entanglement-based protocol, practical imperfections, or a small simulation of key generation and error testing.

Deliverables: A written analysis and presentation built around a clearly stated security question. The work should include explicit quantum-state reasoning or probability calculations, while leaving the group free to choose whether the emphasis is theoretical, numerical or protocol-oriented.

3. Bell Inequalities: Quantum Correlations Beyond Local Hidden-Variable Models

Scope: Investigate Bell's theorem and the distinction between correlations allowed by local hidden-variable models and correlations predicted by quantum mechanics. The central aim is to understand what a violation of a Bell inequality establishes, and what it does not establish.

Possible directions:

- A derivation of the Clauser–Horne–Shimony–Holt (CHSH) inequality from the assumptions of a local hidden-variable description.
- Quantum correlations for an entangled spin or photon-polarisation state and choices of measurement settings that violate the local hidden-variable bound.
- Finite statistics, experimental loopholes, or the analysis of simulated coincidence-count data.
- The relation between Bell nonlocality, entanglement and the no-signalling principle.

Deliverables: A coherent paper and presentation centred on a question chosen by the group. At least one explicit comparison between a local hidden-variable bound and a quantum prediction should be included, but the detailed theoretical or experimental emphasis is left open.

4. Symmetries and Conservation Laws: Hidden Symmetry in the Hydrogen Atom

Scope: Explore how continuous symmetries are represented in quantum mechanics and how their generators are related to conserved observables. The hydrogen atom offers an opportunity to go beyond ordinary rotational symmetry and examine the additional dynamical symmetry of the non-relativistic Coulomb problem.

Possible directions:

- The relation between unitary transformations, infinitesimal generators and commutators with the Hamiltonian.
- Translations and rotations as examples leading to momentum and angular momentum conservation.
- The Laplace–Runge–Lenz vector and the enlarged symmetry of the Coulomb Hamiltonian.
- How the additional symmetry helps explain the degeneracy of hydrogenic levels with different orbital angular momenta at fixed principal quantum number.

Deliverables: A written paper and presentation that develops a symmetry question selected by the group. The work should include explicit operator or commutator analysis and may focus on hydrogen or compare several symmetry examples.

5. Angular Momentum, $SO(3)$ and $SU(2)$: Why Does a 2π Rotation Change the Sign of a Spinor?

Scope: Study the quantum theory of rotations and the relation between ordinary three-dimensional rotations and spinor states. The project should connect angular-momentum algebra with the group-theoretical distinction between $SO(3)$ and its double cover $SU(2)$.

Possible directions:

- Angular-momentum commutation relations, ladder operators and the spectrum of J^2 and J_z .
- Spin-1/2 rotation operators and the sign change of a spinor under a 2π rotation, together with the recovery of the original spinor after 4π .
- How a relative sign can become observable in an interference experiment even though an isolated state vector and its negative represent the same ray.
- Addition of angular momenta, Clebsch–Gordan coefficients, or the singlet–triplet decomposition of two spin-1/2 particles.

Deliverables: A coherent paper and presentation centred on a selected aspect of quantum rotations. The project should contain explicit angular-momentum or rotation-operator calculations, with the balance between algebra, group theory and physical interpretation chosen by the group.

6. The Helium Atom: Exchange Symmetry and the Two-Electron Spectrum

Scope: Use helium as a laboratory for approximation methods, electron–electron interaction and the consequences of indistinguishability. The project should clarify how antisymmetrisation changes the allowed two-electron states without introducing an additional classical “exchange force”.

Possible directions:

- A variational estimate of the helium ground-state energy, for example using hydrogen-like orbitals with an effective nuclear charge.
- Construction of symmetric and antisymmetric spatial states and their combination with singlet and triplet spin states.
- The origin of the energetic difference between orthohelium and parahelium states.
- Metastable helium states, spectroscopy, or comparison between simple independent-particle and correlated descriptions.

Deliverables: A written paper and presentation organised around a two-electron question chosen by the group. The work should include an explicit quantum-mechanical calculation or state construction and a careful distinction between Coulomb interaction and exchange symmetry.

7. Van der Waals Forces: Why Do Neutral Atoms Attract?

Scope: Investigate how an attractive interaction emerges between neutral atoms that have no permanent electric dipole moment. The project should connect stationary perturbation theory with a physically important long-range interaction in the non-retarded regime.

Possible directions:

- The multipole expansion and the leading dipole–dipole interaction between well-separated neutral atoms.
- Why the first-order energy correction vanishes for non-degenerate atomic ground states with zero permanent dipole moment.
- How second-order perturbation theory produces an attractive interaction proportional to $-1/d^6$ (d is the center of mass separation) .
- Connections to molecular binding, dispersion forces, or the crossover to retarded Casimir–Polder behaviour at sufficiently large separations.

Deliverables: A coherent paper and presentation on an aspect of dispersion forces selected by the group. The project should include enough perturbation-theory detail to establish the origin, sign and distance dependence of the interaction considered.

8. Molecules and Chemical Bonds: Why Is H_2 Strongly Bound while He_2 Is Only Weakly Bound?

Scope: Explore how quantum superposition, orbital symmetry and electron filling determine molecular bonding. The contrast between H_2 and He_2 provides a useful way to distinguish strong covalent bonding from the much weaker residual binding produced by dispersion forces.

Possible directions:

- The Born–Oppenheimer approximation and the construction of bonding and antibonding molecular orbitals.
- H_2^+ or He_2 . as simple examples of how orbital overlap and electron occupation lower the molecular energy.
- Why the elementary molecular-orbital picture gives zero covalent bond order for He_2 .
- How the observed weak He_2 . dimer can nevertheless arise through van der Waals attraction, or how more accurate electronic-structure descriptions modify the simple picture.

Deliverables: A written paper and presentation centred on a molecular bonding question chosen by the group. The work should combine a clear quantum-mechanical construction with an interpretation of what the chosen approximation captures and what requires additional physics.

9. Spin and Statistics: From the Pauli Principle to Degenerate Matter

Scope: Study how permutation symmetry constrains identical-particle states and how Fermi statistics can have macroscopic consequences. The spin–statistics connection may be taken as a fundamental relativistic result, while the project focuses on its consequences within many-particle quantum mechanics.

Possible directions:

- Symmetric and antisymmetric states, permutation operators and the Pauli exclusion principle.
- The filling of one-particle states in an ideal Fermi gas and the emergence of the Fermi momentum and Fermi energy.
- Degeneracy pressure as a consequence of quantum statistics rather than an additional microscopic force.
- Applications to electrons in solids, white-dwarf matter, neutron-star matter, or another system chosen by the group.

Deliverables: A coherent paper and presentation based on a question chosen within identical-particle physics. The project should include explicit many-particle or Fermi-gas reasoning and may develop a condensed-matter, atomic or astrophysical application.

10. Bose–Einstein Condensation in Trapped Gases: Macroscopic Matter Waves

Scope: Investigate how Bose statistics allows macroscopic occupation of a quantum state and how condensation appears in experimentally realised trapped gases. The group may choose to focus on thermodynamics, coherence, finite-size effects or weak interactions.

Possible directions:

- The condensation condition and critical temperature in a uniform gas or a harmonic trap.
- Finite-size effects or the dependence of the condensate fraction on trap geometry and particle number.
- Matter-wave interference after release from a trap as a probe of coherence.
- The role of weak interactions, the Gross–Pitaevskii description, or comparison with landmark ultracold-atom experiments.

Deliverables: A written paper and presentation organised around a question selected by the group. The work should contain a quantitative quantum or statistical-mechanical component, which may be analytical, numerical or data-oriented.

11. Second Quantisation: From Fock States to Two-Photon Interference

Scope: Develop the occupation-number formulation of quantum mechanics and use it to analyse a phenomenon in which particle indistinguishability is directly observable. Hong–Ou–Mandel interference is one possible focal example, but the project may develop another application of second quantisation.

Possible directions:

- Bosonic and fermionic Fock states and the action of creation, annihilation and number operators.
- One-body and two-body Hamiltonians in second-quantised form.
- Two indistinguishable photons incident on an ideal 50:50 beam splitter and the suppression of coincidence events in the perfectly indistinguishable limit.
- Partial distinguishability, many-particle interference, field quantisation, or another application chosen by the group.

Deliverables: A coherent paper and presentation demonstrating effective use of second-quantised notation. The group should include at least one explicit operator calculation and is free to choose the physical application around which the project is built.

12. Atomic Transitions and Stellar Spectral Lines: Reading Quantum Mechanics in Starlight

Scope: Connect quantum transitions and angular-momentum selection rules with the absorption and emission lines observed in stellar spectra. The project may focus on the underlying transition theory, the structure of a particular atom or ion, or the interpretation of astronomical spectra.

Possible directions:

- Time-dependent perturbation theory for an atom interacting with an oscillating electromagnetic field.
- Electric-dipole matrix elements, parity and angular-momentum selection rules.
- A chosen atomic or ionic multiplet and the origin of its allowed and forbidden lines.
- Comparison with a stellar spectrum or line catalogue to show how quantum transitions are used to identify chemical species and physical conditions.

Deliverables: A written paper and presentation centred on a spectroscopic question chosen by the group. Explicit transition or selection-rule analysis should support the physical interpretation, but the precise atomic and astronomical example is left open.

13. Microscopic Origin of Stellar Opacity: From Atomic Cross Sections to Radiation Transport

Scope: Explore how microscopic quantum processes determine the interaction of radiation with stellar matter. Rather than attempting a complete theory of stellar atmospheres, the group should select a focused question connecting quantum cross sections or transition rates with macroscopic opacity.

Possible directions:

- The roles of bound–bound, bound–free, free–free and scattering processes in different temperature and density regimes.
- A detailed quantum treatment of one selected process, such as photoionisation, resonant absorption or electron scattering.
- Ionisation edges, strong spectral lines, frequency-dependent transparency or the relation between a cross section and a photon mean free path.
- How changes in composition, ionisation state, level populations, temperature or density alter the dominant opacity mechanism.

Deliverables: A coherent paper and presentation built around one well-defined opacity question. The group should make an explicit link between a microscopic quantum process and a macroscopic radiative effect, while choosing its own balance of analytical calculation, modelling and literature analysis.

14. Hyperfine Structure and the 21-cm Hydrogen Line: Mapping Invisible Hydrogen

Scope: Investigate how the coupling of electron and proton spins produces the hyperfine splitting of ground-state hydrogen and why an extremely weak magnetic-dipole transition has become one of the most powerful probes of neutral hydrogen in astronomy.

Possible directions:

- Addition of the electron and proton spin angular momenta and construction of the $F = 0$ and $F = 1$ hyperfine states.
- The scale of the hyperfine splitting and its relation to the 21-cm frequency and wavelength.
- Why the transition is magnetic-dipole rather than electric-dipole, and why its spontaneous transition rate is so small.

- How large columns of neutral hydrogen make the line observable and what information the line can provide about Galactic or cosmological hydrogen.

Deliverables: A written paper and presentation centred on a question chosen by the group. The work should connect the quantum angular-momentum structure of hydrogen with the observational significance of the 21-cm transition.

15. Zeeman Effect and Stellar Magnetic Fields: Measuring a Magnetic Field from a Spectral Line

Scope: Study how an external magnetic field modifies atomic energy levels and spectral transitions, and how these changes can be used to infer magnetic fields in stars or other astronomical objects.

Possible directions:

- The linear Zeeman effect, angular-momentum coupling and the Landé g -factor.
- Selection rules and the $\sigma+$, $\sigma-$ and π components of Zeeman-split spectral lines, including their polarisation properties.
- The relation between magnetic-field strength and the frequency or wavelength splitting of a selected atomic transition.
- How magnetic fields can be inferred when individual Zeeman components are unresolved, using line broadening or spectropolarimetric measurements.
- For a more advanced treatment, the transition from the Zeeman to the Paschen–Back regime at stronger fields.

Deliverables: A coherent paper and presentation centred on a question chosen by the group. The project should establish at least one explicit quantitative connection between the magnetic perturbation of an atomic level and an observable spectroscopic or polarimetric signature.

16. Neutrino Oscillations in Vacuum and Matter: The Solar Neutrino Problem

Scope: Use neutrino flavour conversion as a quantum-mechanical example of coherent evolution when the states in which neutrinos are produced and detected differ from the propagation eigenstates. The project may focus on vacuum oscillations, matter effects, the solar-neutrino problem, or the accuracy and limitations of the two-flavour approximation.

Possible directions:

- Two-flavour oscillations arising from the relation between flavour and mass eigenstates.
- Oscillation length and its dependence on neutrino energy, mixing angle and mass-squared difference.
- The effective Hamiltonian in matter, the MSW resonance condition, or adiabatic propagation through a varying density.
- Solar-neutrino survival probabilities and how observations established flavour conversion.
- The relation between the two-flavour description of solar neutrinos and the full three-flavour framework.

Deliverables: A written paper and presentation centred on a neutrino-oscillation question selected by the group. The work should include an explicit quantum-mechanical treatment of the evolution and may be complemented by numerical plots, comparison with solar-neutrino observations, or analysis of the limits of the chosen approximation.

17. Superfluidity and Quantised Vortices: How Does a Superfluid Respond to Rotation?

Scope: Explore the quantum mechanics of superfluid flow and the emergence of quantised circulation. A useful central question is how a superfluid, whose velocity field is locally irrotational away from vortex cores, responds to imposed rotation and develops states containing quantised vortices.

Possible directions:

- The phase of the macroscopic order parameter, superfluid velocity and quantisation of circulation.
- Structure and energetics of a vortex described by the Gross–Pitaevskii equation for a dilute weakly interacting Bose gas, including the role of the healing length.
- The conditions under which vortices become energetically favourable in a rotating trapped condensate, and the formation of vortex arrays or lattices.
- How a dense vortex lattice reproduces, on coarse spatial scales, aspects of classical rotational flow.
- Comparison with quantised vortices in superfluid helium and in dilute Bose–Einstein condensates, including similarities and differences in their description. The Gross–Pitaevskii equation may be used for dilute Bose gases, while superfluid helium requires a different microscopic description.

Deliverables: A coherent paper and presentation organised around a question selected by the group. The project should contain an explicit quantum-mechanical or quantum-fluid analysis and may include analytical estimates, numerical solutions, comparison with experiments, or investigation of vortex structure and rotation.

18. Quantum Phase Transitions in Optical Lattices: Superfluid or Mott Insulator?

Scope: Study a zero-temperature quantum phase transition controlled by a non-thermal parameter, using ultracold atoms in an optical lattice as a concrete realisation. The project should explore how the competition between particle tunnelling and on-site interactions reorganises the many-body ground state, producing a transition from a phase-coherent superfluid to a Mott-insulating state at commensurate filling.

Possible directions:

- The Bose–Hubbard Hamiltonian and the physical meaning of the hopping amplitude J , on-site interaction U , chemical potential and particle filling.
- The superfluid and Mott-insulating limits, and the qualitative or mean-field structure of the phase diagram.
- The role of integer filling, number fluctuations, phase coherence and the excitation gap in distinguishing the two phases.
- Small-system exact diagonalisation, finite-size effects, or experimentally measurable signatures such as momentum-distribution interference peaks or number fluctuations.
- The difference between the zero-temperature quantum phase transition and the effects of finite temperature in experiments.

Deliverables:

A written paper and presentation centred on a question chosen by the group about the Bose–Hubbard transition or a related lattice model. The work should contain explicit many-body quantum mechanics and may combine analytical arguments, numerical calculations or comparison with experiments.

19. Cooper Pairing and the Origin of Superconductivity: Why Does Weak Attraction Matter at the Fermi Surface?

Scope: Investigate the Cooper problem and the special role of a filled Fermi sea. In the idealised Cooper model, two electrons added above the Fermi surface can form a paired state for arbitrarily weak attraction, with an energy below that of two unpaired added electrons. The project may then explore how this two-particle result is related to the collective many-body pairing of Bardeen–Cooper–Schrieffer (BCS) superconductivity.

Possible directions:

- The Cooper-pair eigenvalue problem for two electrons above an inert filled Fermi sea.
- Why the presence of the Fermi surface, Pauli blocking and the density of available states near E_F qualitatively change the effect of a weak attractive interaction compared with the corresponding two-body problem in vacuum.
- Zero-total-momentum pairing, opposite momenta and spin-singlet structure in the simplest conventional s -wave model.

- The relation between the Cooper problem and the BCS ground state, in which many overlapping pairs participate collectively.
- The superconducting energy gap, isotope effect, coherence length or experimental signatures, provided the distinction between the elementary Cooper problem and full BCS theory is kept explicit.

Deliverables: A coherent paper and presentation organised around a pairing question chosen by the group. The project should contain an explicit calculation or model analysis and should distinguish clearly between the two-electron Cooper problem and the many-body BCS theory of superconductivity.

20. Casimir Effect: From Quantised Electromagnetic Modes to a Measurable Force

Scope: Study how material boundaries modify the allowed modes and fluctuations of the electromagnetic field, producing a measurable interaction between conducting bodies. The project may use the ideal parallel-plate calculation as a starting point and may explore how the result changes when more realistic material properties or geometries are considered.

Possible directions:

- Quantisation of the electromagnetic field between ideal parallel conducting plates.
- The regularised change in zero-point energy relative to infinitely separated plates and the resulting dependence of the Casimir force on plate separation.
- The role of finite conductivity, finite temperature or non-planar geometries in modifying the ideal result.
- Experimental measurements of the Casimir force and the comparison between theoretical idealisations and real materials.
- For a more advanced treatment, comparison between the vacuum-energy description and formulations based on electromagnetic fluctuations and material response.

Deliverables: A written paper and presentation centred on a Casimir-effect question selected by the group. The work should contain at least one quantitative derivation or estimate and should identify clearly the physical assumptions and idealisations used.

21. Quantum Scattering Resonances and Ramsauer–Townsend Transparency

Scope: Investigate how the wave nature of quantum particles can produce unexpectedly weak or strong scattering at particular energies. The project should explore how scattering phase shifts encode these effects, with Ramsauer–Townsend minima and resonant enhancement providing contrasting examples of energy-dependent quantum scattering.

Possible directions:

- Low-energy partial-wave scattering and the conditions under which a small number of partial waves dominate.
- Phase shifts and cross sections for a simple finite-range potential, such as a spherical square well.
- Ramsauer–Townsend minima and the relation between suppressed scattering and the phase of the scattered wave.
- Resonant enhancement of the cross section, including near-threshold states, shape resonances or the role of the scattering length.
- Comparison with a physical system such as low-energy electron–atom scattering or neutron scattering.
- Numerical exploration of how changing the incident energy or potential parameters moves scattering minima and resonances.

Deliverables: A coherent paper and presentation centred on a scattering question selected by the group. The work should include an explicit analysis of phase shifts, scattering amplitudes or cross sections, and may combine analytical calculations, numerical results and comparison with experimental data.

22. Isospin and $SU(2)$: Why Do Proton and Neutron Form an Isospin Doublet?

Scope: Use $SU(2)$ symmetry to understand the approximate proton–neutron symmetry of the strong interaction and the classification of nuclear states into isospin multiplets. The project should make clear that isospin is an internal symmetry, approximately respected by the strong interaction, rather than an exact symmetry of the full physical system.

Possible directions:

- The nucleon isospin doublet, with proton and neutron as the $I = \frac{1}{2}$ states, and the analogy between isospin algebra and ordinary angular momentum.
- Addition of isospins in two-nucleon or few-nucleon systems.
- Classification of nuclear states, selection rules or reaction channels using approximate isospin conservation.
- Sources of isospin-symmetry breaking, particularly electromagnetic interactions and the up–down quark mass difference.
- If desired, extension from nucleons to other hadronic isospin multiplets.

Deliverables: A written paper and presentation centred on an isospin question chosen by the group. The work should contain an explicit $SU(2)$ construction, isospin-addition calculation or state-classification example, and connect it to a physical application in nuclear or hadronic physics.

Summary Table

Project	Focus	Difficulty	Coding Effort	Balance	Best For
1. Quantum Algorithms	Interference and computation	★★☆	Medium	Theory + coding	Quantum information
2. Quantum Cryptography	Security from measurement	★★☆	Low–Medium	Theory + simulation	Quantum information
3. Bell Inequalities	Bell nonlocality	★★☆	Low–Medium	Theory + data	Foundations
4. Hidden Symmetry of Hydrogen	Symmetry and degeneracy	★★★	Low	Math-heavy	Group theory
5. Angular Momentum & SU(2)	Spinors and rotations	★★☆	Low	Analytical	Symmetry / spin
6. Helium Atom	Two-electron exchange symmetry	★★☆	Low	Analytical	Atomic physics
7. Van der Waals Forces	Dispersion attraction	★★☆	Low	Perturbation theory	Atomic / molecular
8. Molecules & Chemical Bonds	Bonding and residual binding	★★☆	Low–Medium	Analytical + diagrams	Molecular physics
9. Spin & Statistics	Degenerate fermionic matter	★★☆	Low	Analytical + applications	Many-particle QM
10. BEC in Trapped Gases	Macroscopic matter waves	★★☆	Medium	Theory + numerics	Ultracold atoms
11. Second Quantisation	Fock space and interference	★★☆	Medium	Operators + simulation	Many-body / quantum optics
12. Atomic Transitions & Stellar Lines	Quantum spectroscopy	★★☆	Low–Medium	Theory + data	Atomic / astrophysics
13. Stellar Opacity	Microscopic origin of opacity	★★★	Medium	Theory + modelling	Astrophysics
14. Hyperfine 21-cm Line	Spin coupling and astronomy	★★☆	Low	Analytical + application	Astrophysics
15. Zeeman Splitting	Atomic probe of magnetic fields	★★☆	Low–Medium	Theory + estimates	Spectroscopy
16. Neutrino Oscillations	Coherent flavour conversion	★★★	Medium	Theory + plotting	Particle / astrophysics
17. Superfluid Vortices	Quantised circulation	★★★	Medium–High	Theory + simulation	Quantum fluids
18. Quantum Phase Transitions	Mott–superfluid transition	★★★	Medium	Many-body + numerics	Condensed matter
19. Cooper Pairing	Pairing near the Fermi surface	★★★	Low–Medium	Analytical + numerics	Condensed matter
20. Casimir Effect	Quantum boundary effects	★★★	Low	Analytical	Quantum fields
21. Scattering Resonances	Transparency and resonances	★★☆	Medium	Theory + plotting	Scattering
22. Isospin & SU(2)	Internal symmetry	★★☆	Low	Group theory	Nuclear physics

The list spans:

- Quantum information and foundations (Projects 1–3, 11)
- Symmetry, groups and angular momentum (Projects 4–5, 14–15, 22)
- Atomic and molecular quantum mechanics (Projects 6–8, 12–15)
- Identical particles and quantum condensed matter (Projects 9–11, 17–19)
- Astrophysical and particle-physics applications (Projects 9, 12–16)
- Quantum fields and vacuum effects (Projects 11, 20)
- Scattering and wave phenomena (Project 21)