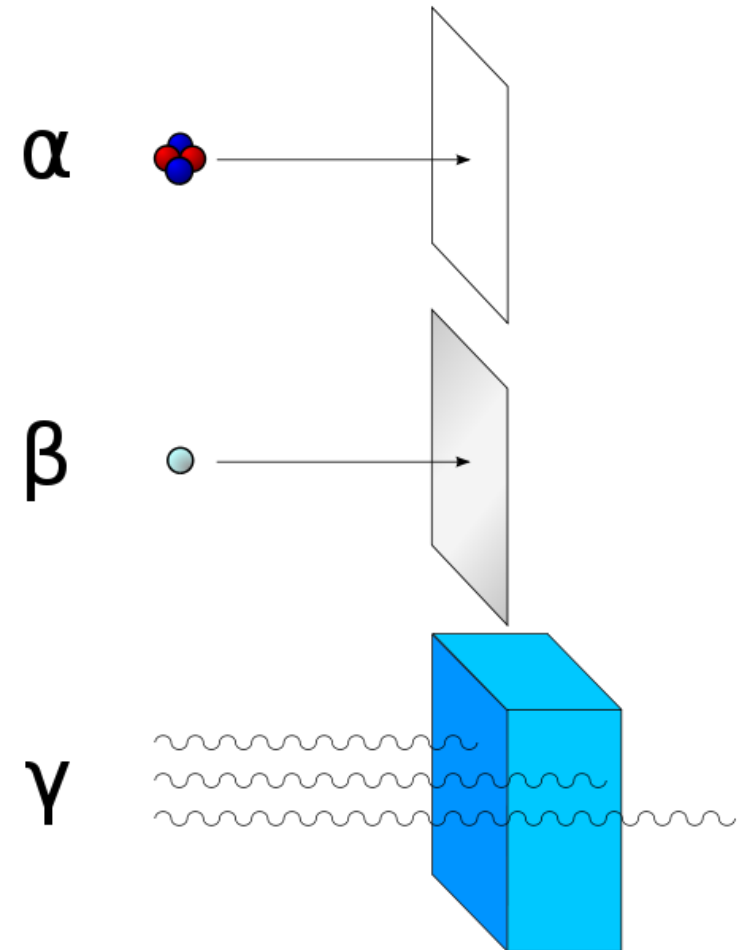


TYPES OF RADIOACTIVITY

Three types of radiation:

- α (Helium nuclei)
- β (electrons or positrons)
- γ (electromagnetic waves)

They have ranges, origins, **applications** and **effects** that are very different. The danger that a given radiation poses to human beings depends on many factors; it is difficult to make generalisations about the greater risk of one type of radiation compared to another.



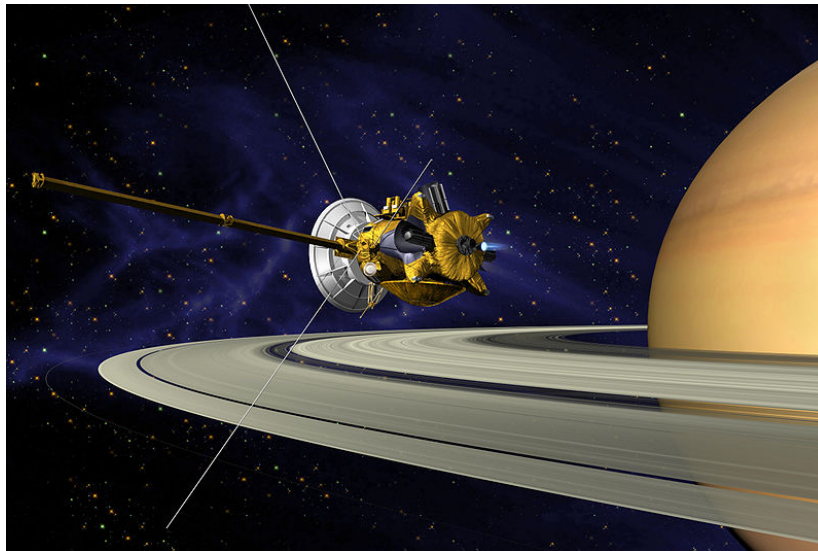
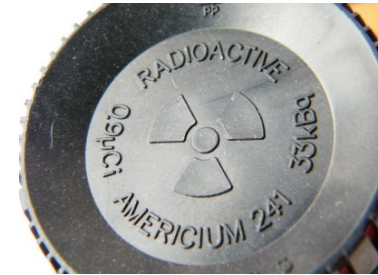
RANGES OF RADIATION

We will treat the calculation of the ranges of the different types of radiation, and their interaction with matter, in a rigorous and quantitative way.

- The **alpha** radiation is the one with the **SMALLEST** range, and the **LOWEST penetrating power**. Alpha particles can be blocked even by a sheet of paper, and even in air alpha particles travel very short distances (of the order of millimetres) before stopping. As a consequence, they deposit all their energy in a very small volume (potentially causing greater damage to living tissue).
 - **Beta** radiation has a greater range than alpha, and greater penetrating power. Even so, it is generally possible to block beta particles with thin sheets of metal. They can travel considerable distances in air (tens of centimetres).
 - **Gamma** radiation has a very large range and great penetrating power. Its intensity and energy are only slightly attenuated in air or in low-density materials. To completely block gamma radiation, a considerable thickness of dense metal (lead, for example) may be needed.
- A **more penetrating** radiation is not necessarily more dangerous – it may deposit **less energy** in an organism than another, **less penetrating** radiation, which deposits all its energy in a **small volume**.

Applications of alpha radiation

Smoke detectors

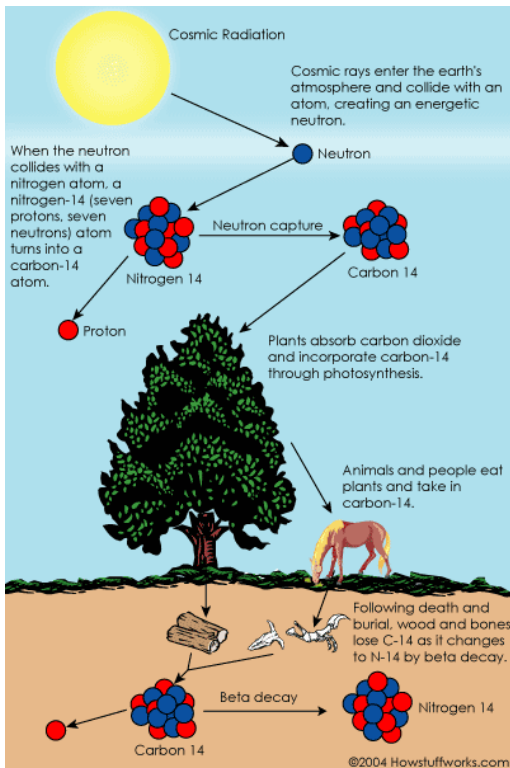


Thermoelectric generators for space probes

(for example, the *Galileo* and *Cassini* probes)

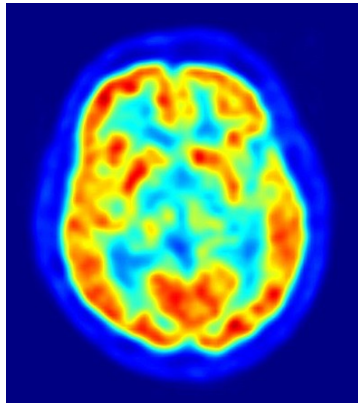
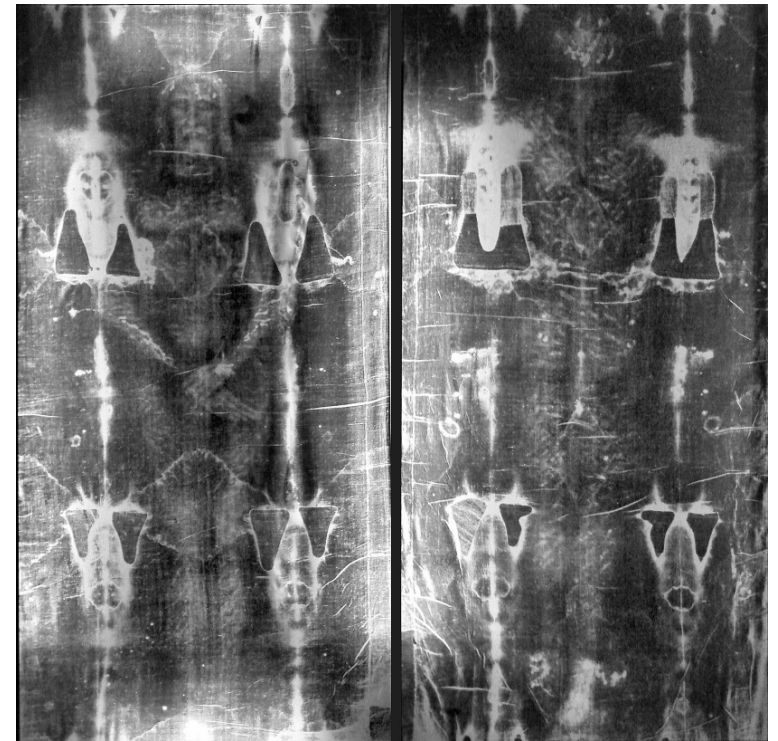
Applications of beta radiation

Radioactive dating using Carbon-14



The Turin Shroud

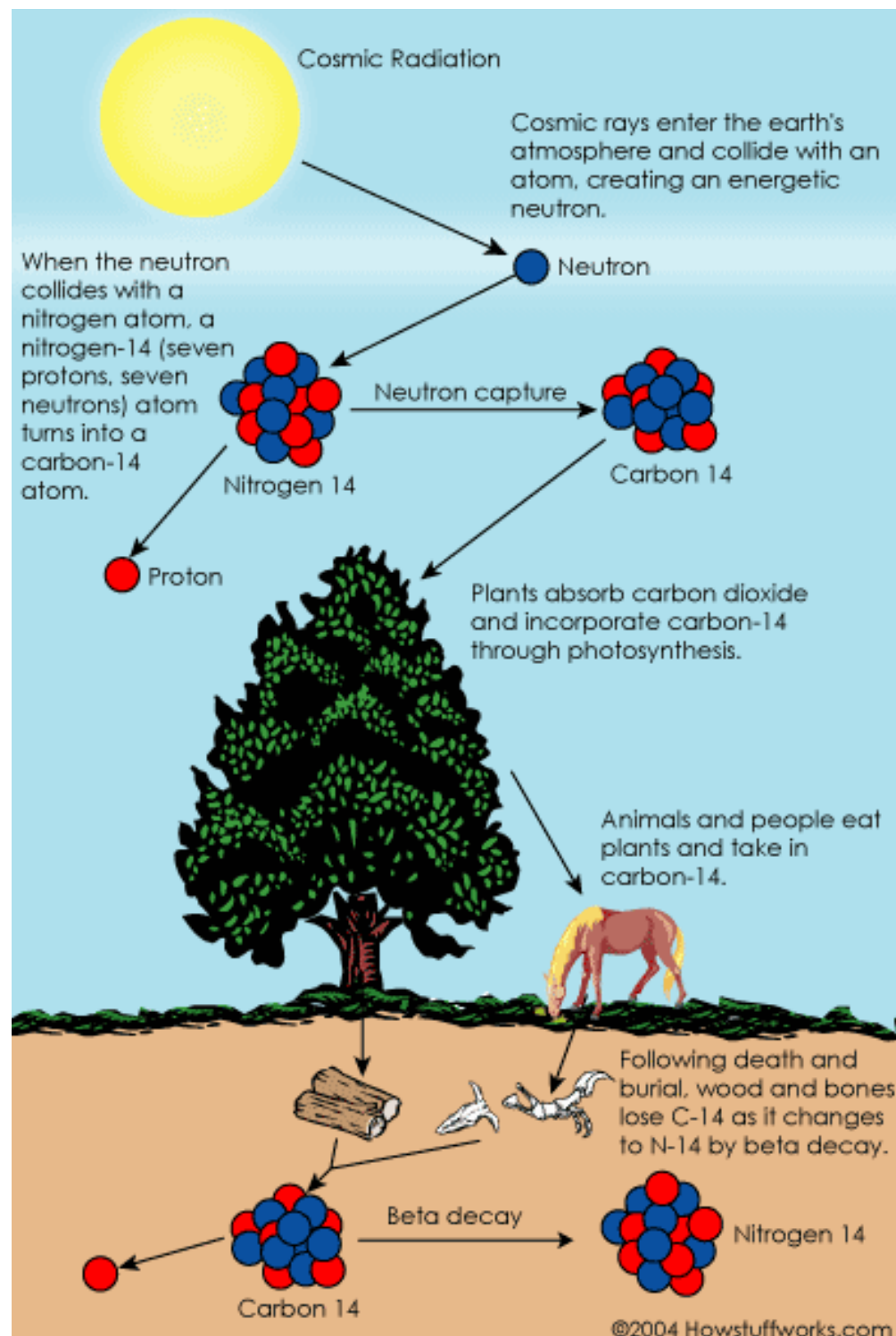
(supposedly the burial shroud of Christ, actually made with fabric from the Medieval era...)



Positron Emission Tomography (PET)

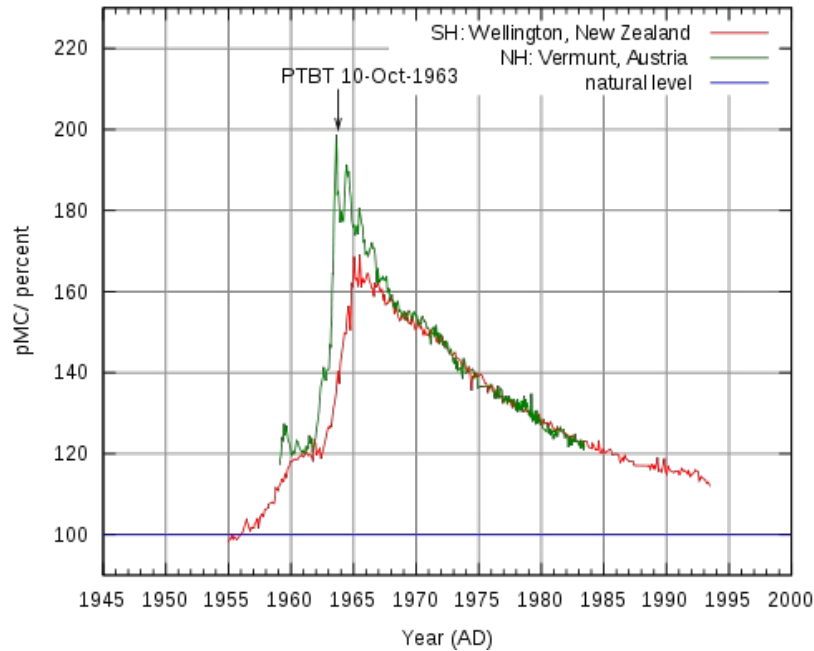
Uses antimatter to save lives...





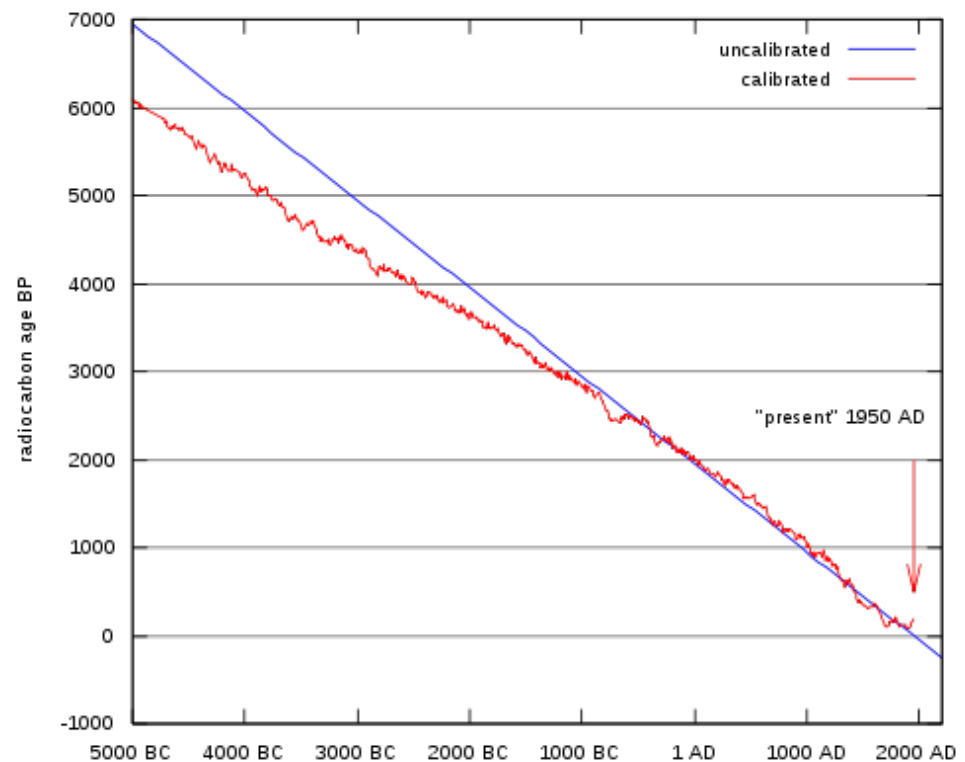
The Carbon-14 Cycle





The concentration of ^{14}C shows historical variations, associated with different periods of solar activity and climate changes on Earth – calibrations are needed, obtained through different methods (for example, tree rings).

Concentration of ^{14}C different in both hemispheres, and highly variable in recent times

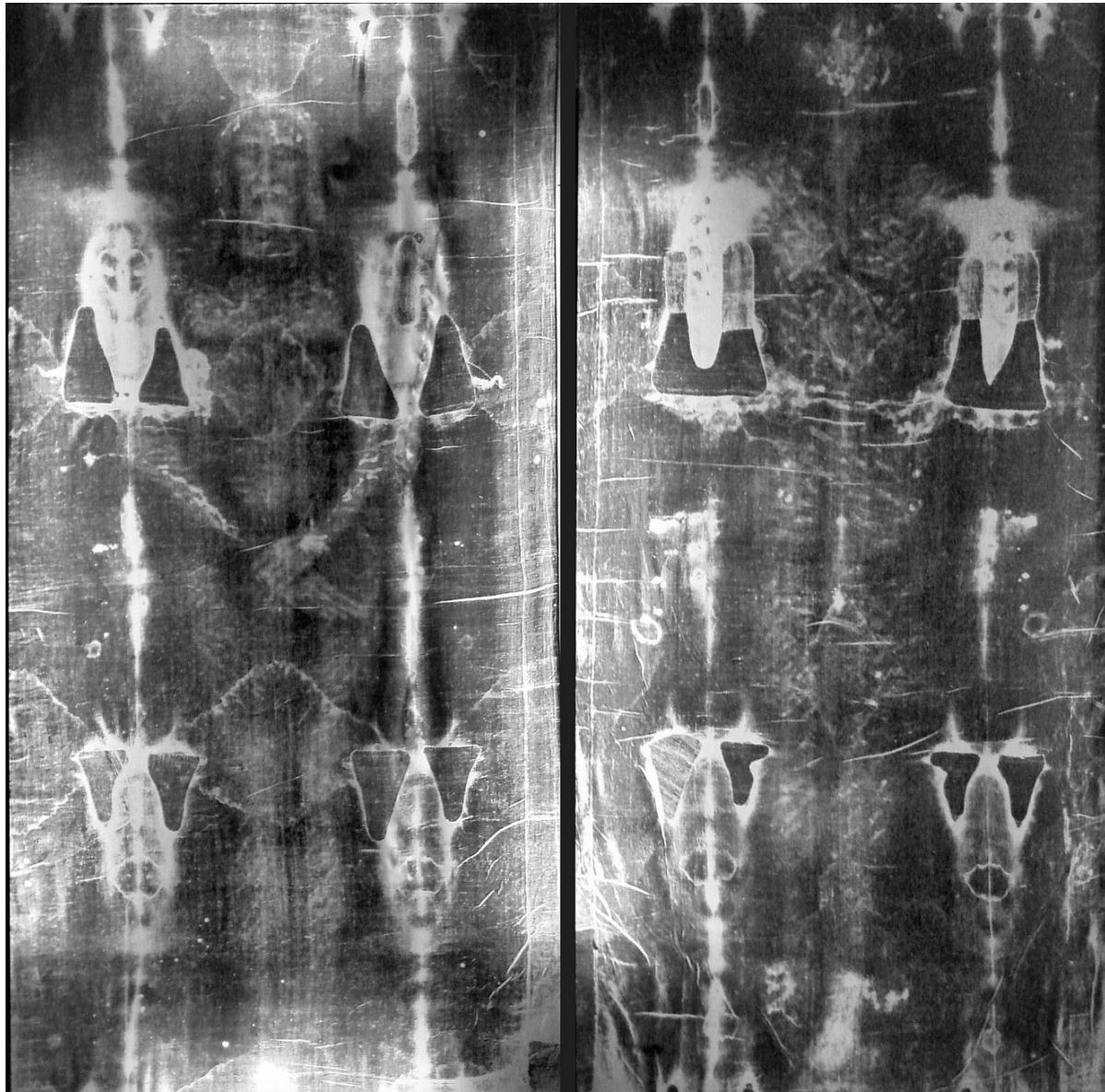


Examples



The Vinland Map

(supposedly from the Middle Ages and already showing Greenland, but actually written with 20th-century pigments)



The Turin Shroud

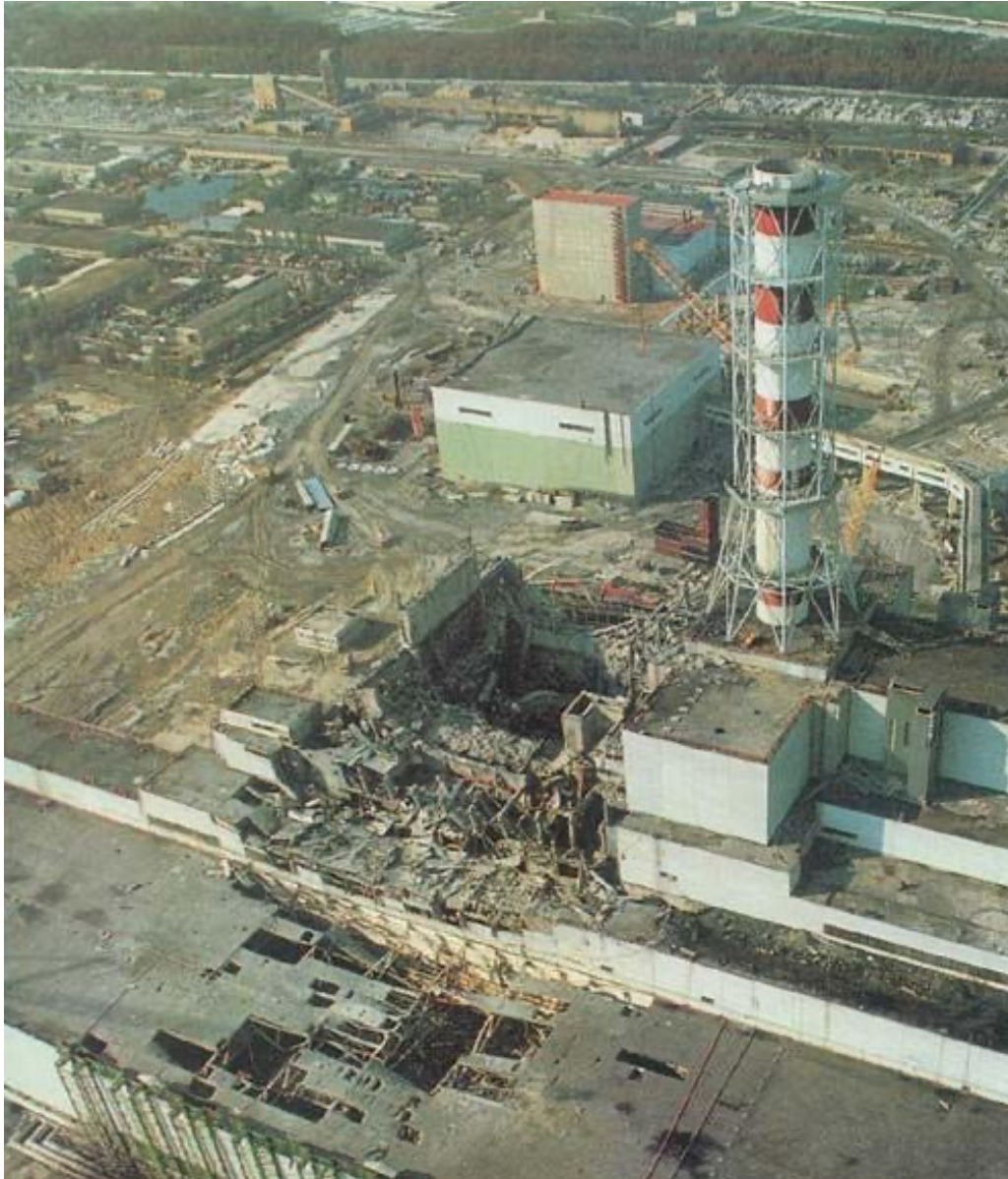
**(supposedly the burial
shroud of Christ, but
actually made with
fabric from the
Medieval era...)**

The eruption of Thera/Santorini (*circa* 1600 BC)



**(a possible cause of the decline of
the Mycenaean Empire, and a
possible inspiration for
Atlantis...)**

Effects of beta radiation

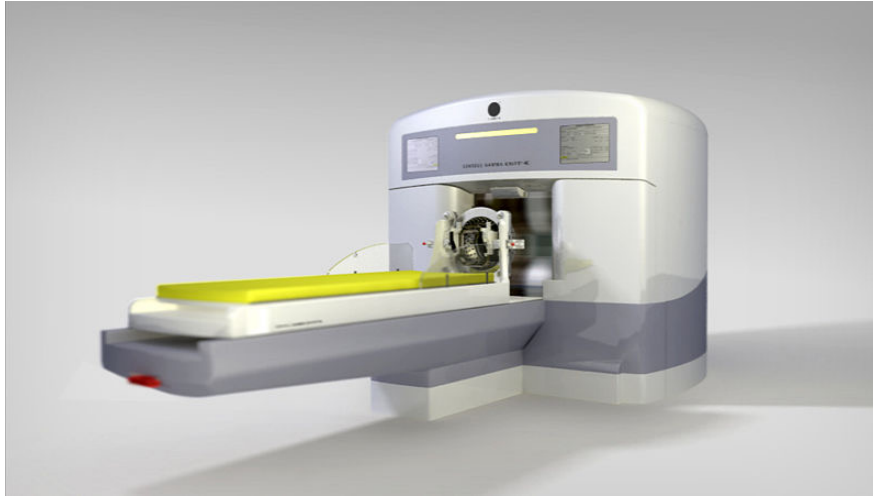


Chernobyl, 1986

One of the isotopes with the longest mean lifetime (**~30 years**) in nuclear accidents is the beta emitter ^{137}Cs .

Another common beta emitter in nuclear reactors, ^{131}I , has a much shorter mean lifetime (**about 8 days**) but is very dangerous to humans because it is so easily absorbed by the thyroid.

Applications of gamma radiation



**“Gamma Knife” used to
kill cancerous cells in
malignant tumours
through ionisation**



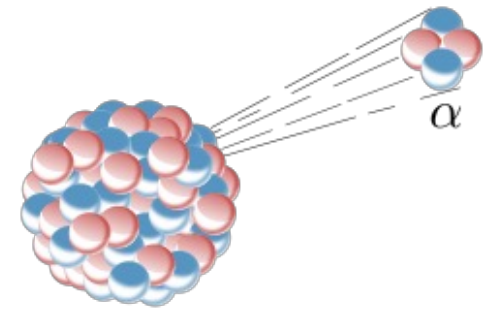
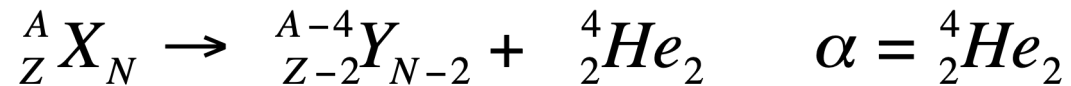
**Conversion of white topaz
into blue topaz**



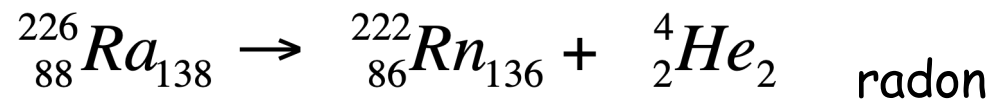
Radioactivity

The three most common types of radioactivity are

- α decay – characterised by the emission of an α particle, that is,

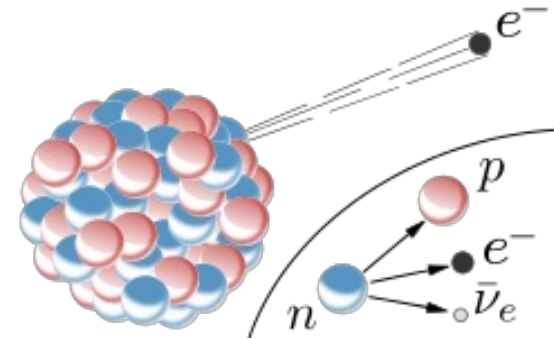


- The mass number A decreases by 4 units
- The atomic number Z decreases by 2 units (the particle carries charge $q = +2$)
- The α particle comes from inside nucleus X (it is not created)
- The forces acting here are the strong force and the electromagnetic force

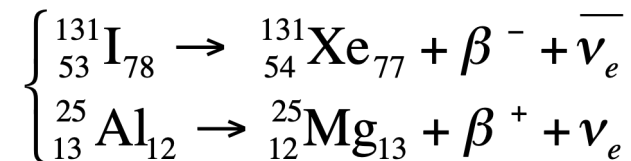


- **β decay** – occurs due to the conversion of a proton into a neutron or vice versa, with the emission of a β particle (positive or negative, i.e. a positron or an electron) and a neutrino (which is generally not detected).

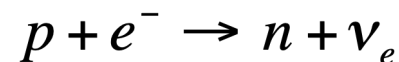
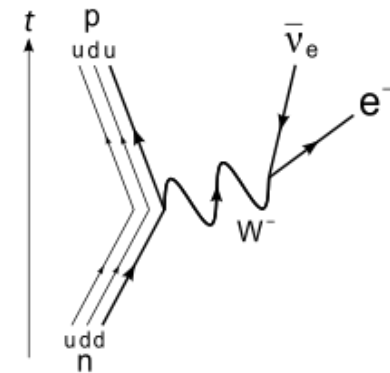
$$\begin{cases} n \rightarrow p + e^- + \bar{\nu}_e & \text{declínio } \beta^- \\ p \rightarrow n + e^+ + \nu_e & \text{declínio } \beta^+ \end{cases}$$



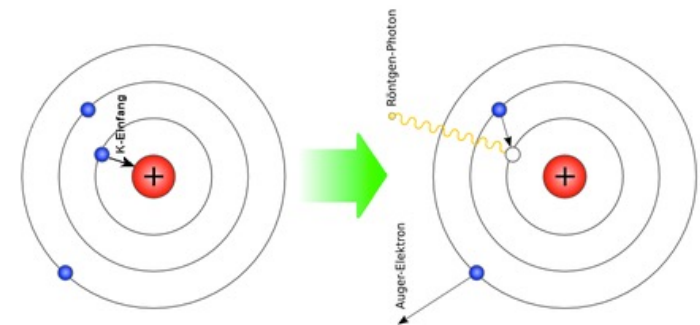
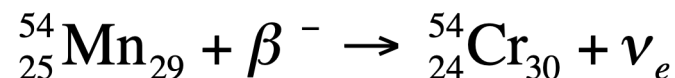
Examples



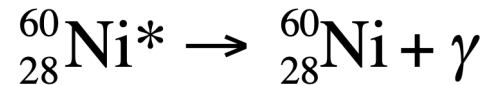
The inverse process, **electron capture**, can also occur. In this case the proton captures an electron from the electron shell.



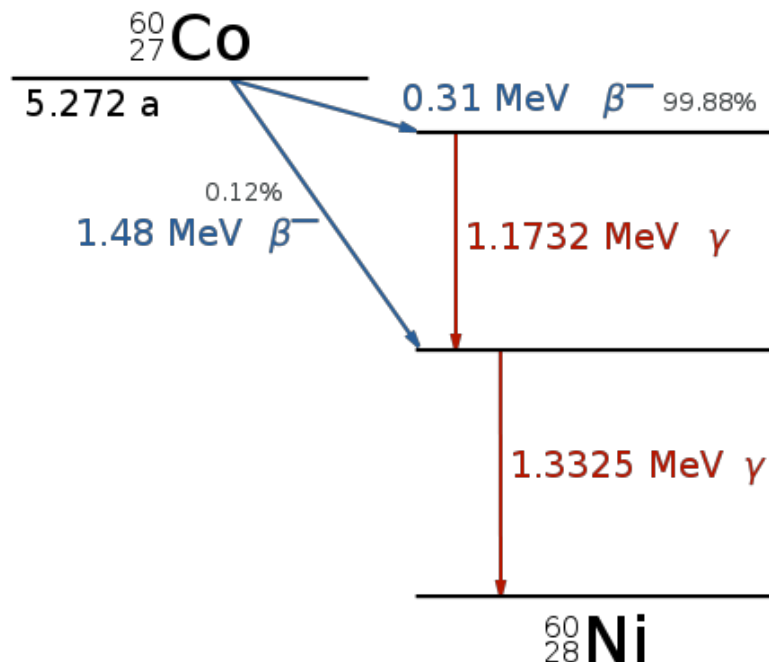
Example



- γ decay – occurs when an excited nucleus decays to a lower energy level with the emission of a highly energetic photon (from 0.1 MeV to 10 MeV). In this case A, Z and N do not change, because the nuclei are the same.



Decay scheme of ${}^{60}\text{Co}$

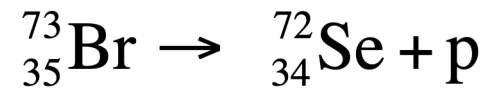
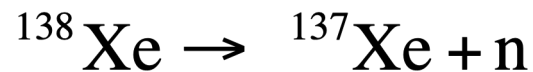


First, ${}^{60}\text{Co}$ decays to an excited nucleus of ${}^{60}\text{Ni}$ via beta decay, emitting an electron of 0.31 MeV. Next, the excited state of ${}^{60}\text{Ni}$ decays to its ground state, emitting gamma radiation in succession (1.17 MeV followed by 1.33 MeV). This path is followed 99.88% of the time.

Other less common types of radioactivity:

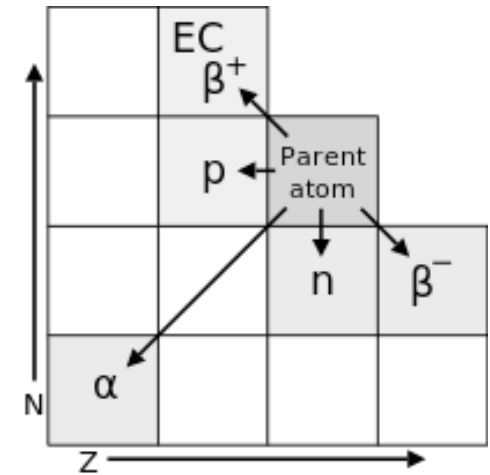
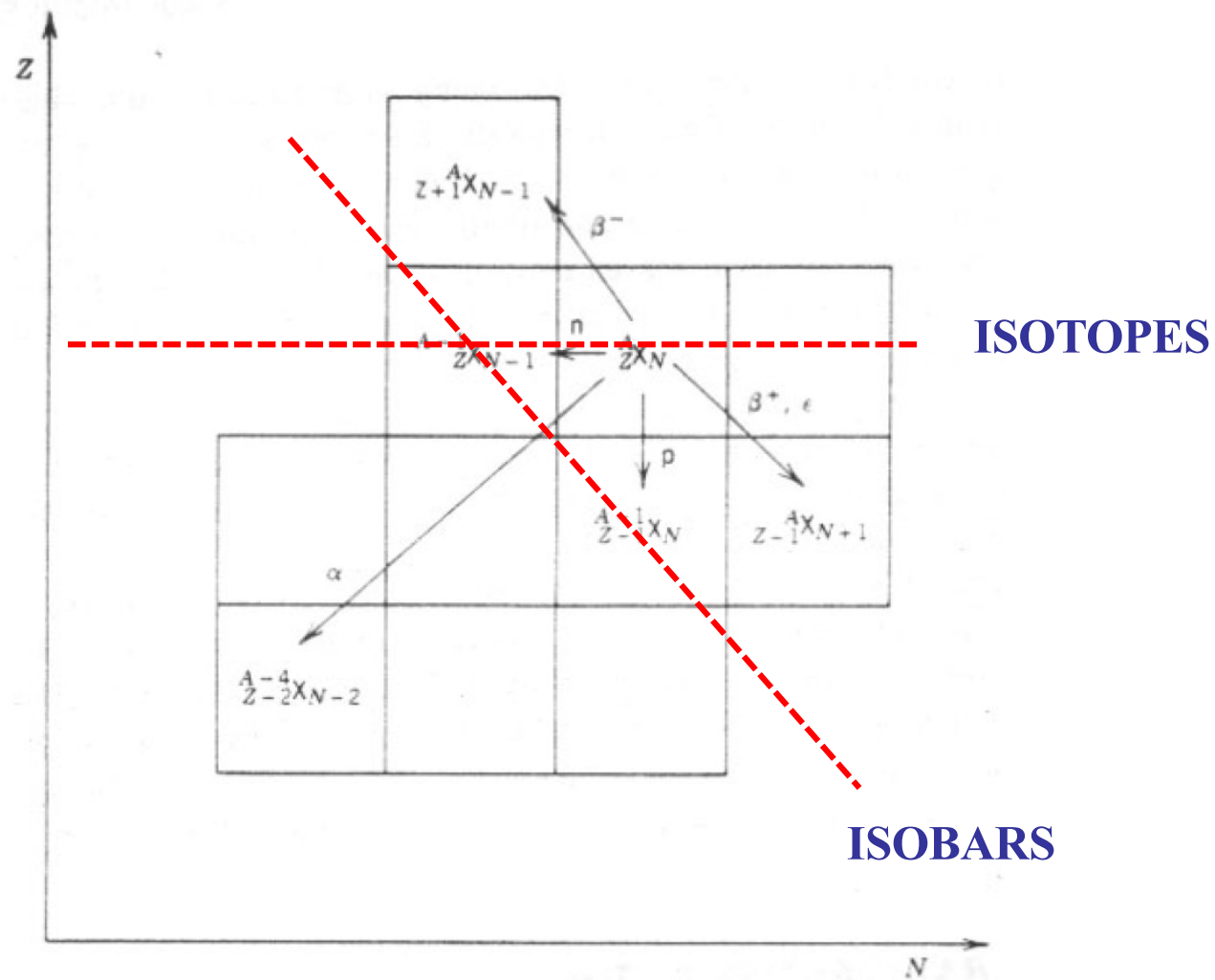
- Spontaneous fission giving rise to two different nuclei
- Nucleon emission – sometimes nuclei can simply eject a neutron or a proton to lose their excess energy (these are generally very unstable nuclei)

Examples



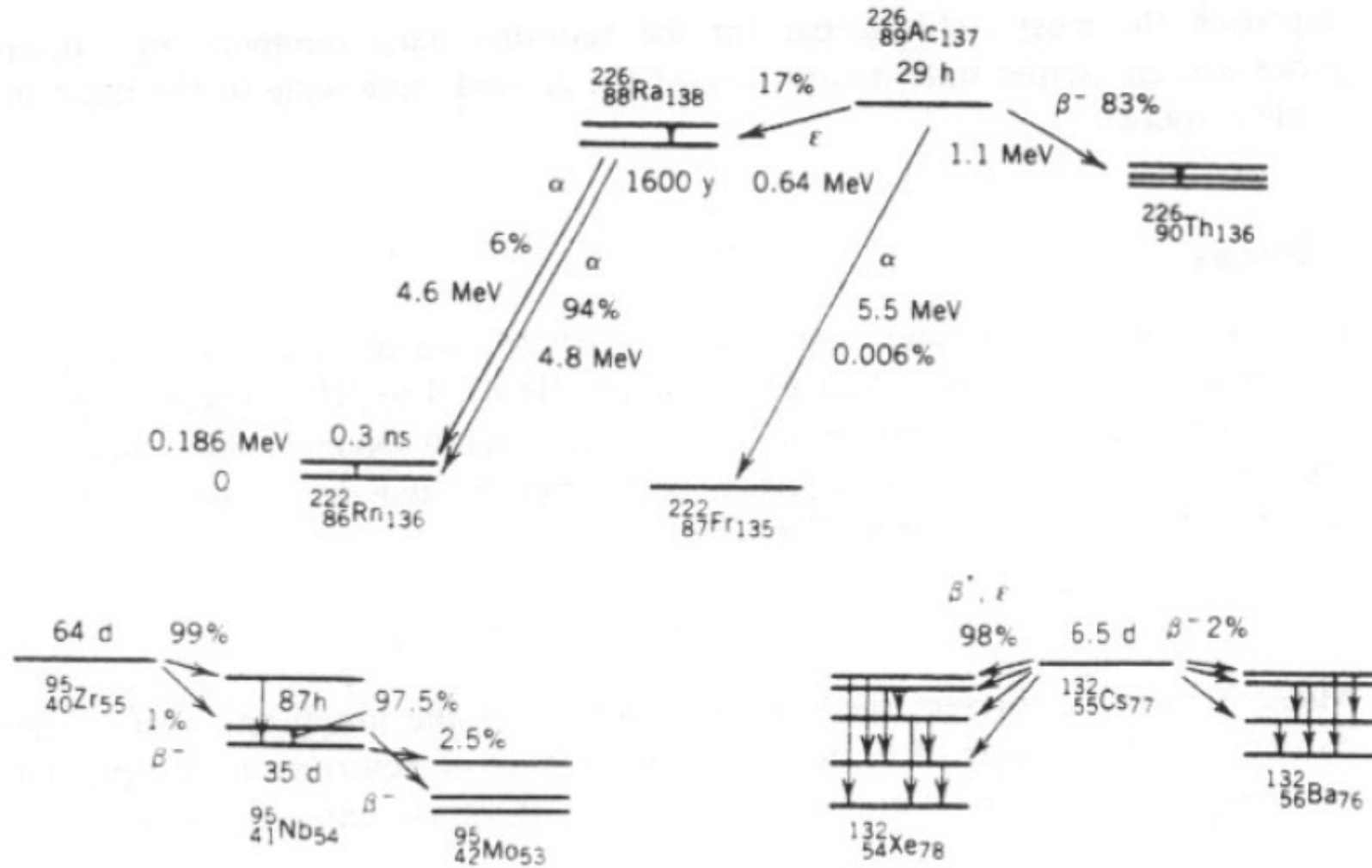
In the first case we have the emission of a neutron, so N and A decrease by one unit and Z remains constant – isotopes. In the second case a proton is emitted and Z and A decrease by one unit while N remains constant.

Decay tables



The figure shows the elements related by the various decays in the (Z,N) plane.

Some examples



The law of radioactive decay

- Radioactive decay is a **random** process – it is impossible to predict whether, at a given instant, an isolated radioactive nucleus will decay or not.
- But for large numbers of nuclei, it is possible to establish numerical laws governing how the number of nuclei evolves over time.
- The fundamental assumption is that the number of nuclei that decay in a given time interval is proportional to the number of nuclei still present:

The diagram shows the differential equation for radioactive decay: $\frac{dN(t)}{dt} = -\lambda N(t)$. Three arrows point from descriptive text to parts of the equation: one from 'Decay rate (decays/unit of time)' to the derivative term, one from 'The “-” sign is because the number of nuclei is decreasing' to the minus sign, and one from 'Decay constant (independent of time or the number of nuclei)' to the lambda symbol. A fourth arrow points from 'Number of nuclei present at instant t (have not yet decayed)' to the N(t) term.

$$\frac{dN(t)}{dt} = -\lambda N(t)$$

Decay rate
(decays/unit of time)

Number of nuclei present at instant t
(have not yet decayed)

Decay constant
(independent of time or the number of nuclei)

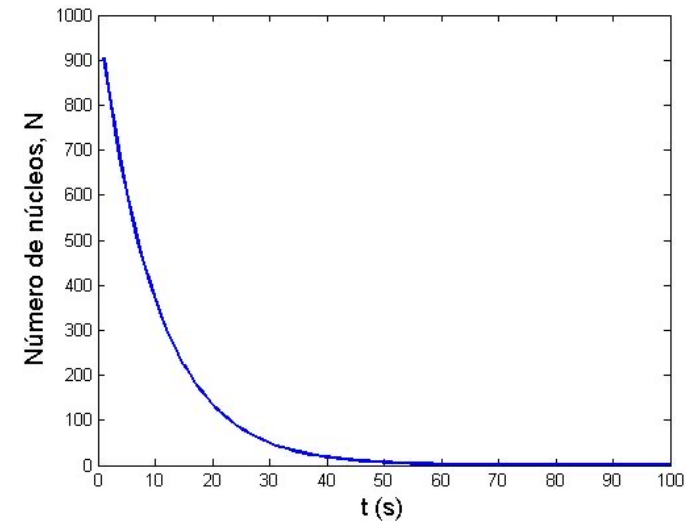
The “-” sign is because the number of nuclei is *decreasing*

- Integrating this equation, we obtain (N_0 is the number of nuclei present at the instant $t = 0$):

$$N(t) = N_0 e^{-\lambda t}$$

- The number of nuclei follows an **exponential** evolution, decreasing very rapidly over time (depending on the constant λ).

- The units of λ are those of inverse time: depending on the nucleus, they can appear as s^{-1} , m^{-1} , d^{-1} , y^{-1} , ...



- The radioactive decay constant λ can also be interpreted as the *probability of a radioactive nucleus decaying per unit of time*.

- The **half-life** $t_{1/2}$ (or **mean lifetime**, or **half-decay period**) is defined as the time interval needed for the number of radioactive nuclei to reduce by half.

$$N(t_{1/2}) = \frac{1}{2} N_0 \Leftrightarrow \frac{1}{2} N_0 = N_0 e^{-\lambda t_{1/2}} \Leftrightarrow e^{-\lambda t_{1/2}} = \frac{1}{2} \Leftrightarrow -\lambda t_{1/2} = \log\left(\frac{1}{2}\right)$$

- So, we have

$$t_{1/2} = \frac{\log 2}{\lambda} \simeq \frac{0.693}{\lambda}$$

- The units of $t_{1/2}$ are units of time, and vary depending on the nucleus: s, m, d, y, My, ...
- For each half-life that passes, the number of nuclei is reduced to half. After two half-lives, the number of nuclei has been reduced to a quarter. Three half-lives, 1/8. Four half-lives, 1/16, etc. – for n half-lives, the number of nuclei is reduced to $1/2^n$ of its initial value.
- Since it is very difficult to know, at every instant, the number of nuclei that still exist (it would require constantly performing chemical analyses, and the number of nuclei would keep changing anyway), it is more common to study the **ACTIVITY** ($\mathcal{A}$) of a radioactive decay: the *activity* is the **number of nuclei that decay at each instant**. Since λ is the probability of a nucleus decaying, $\lambda \times N(t)$ is exactly the number of nuclei that decay at instant t . So,

$$\mathcal{A}(t) = \lambda N(t) \quad \Leftrightarrow \quad \mathcal{A}(t) = \mathcal{A}_0 e^{-\lambda t}$$

where

$$\mathcal{A}_0 = \lambda N_0$$

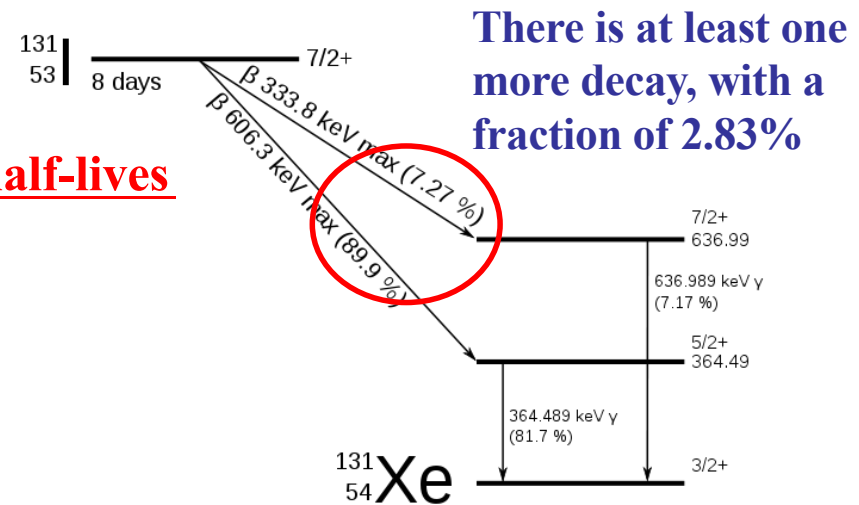
is the **initial activity** of the sample of nuclei. Although this change of variables may seem trivial, activity is what is actually measured in radioactivity experiments. Knowing the activity $\mathcal{A}$ at a given moment and the decay constant λ , it is easy to find the number of nuclei still present: **$N(t) = \mathcal{A}(t)/\lambda$** .

Units of activity:

- Decays per second (**dec/s** or simply **s⁻¹**)
- Becquerel (Bq): 1 Bq = 1 decay/second
- Curie (Ci): 1 Ci = 3.7×10^{10} dec/s

The most commonly used unit is the *Curie*, due to the large number of decays per second in a typical radioactive sample.

- Remember that nuclei can have several decays occurring in parallel, each one with different probabilities, and different half-lives.
- These probabilities are called *fractions* (or *probabilities*) of decay, whose sum always adds up to 100%.



The law of radioactive decay

Number of decays between t_1 and $t_1 + \Delta t$ is given by

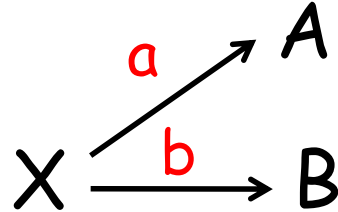
$$\Delta N = \int_{t_1}^{t_1 + \Delta t} \left| \frac{dN}{dt} \right| dt = \int_{t_1}^{t_1 + \Delta t} A(t) dt$$

If $\Delta t \ll t_{1/2}$ $A(t)$ can be considered approximately constant

$$\Delta N \cong A(t_1) \Delta t$$

- The Activity is therefore the number of decays of the sample per unit of time.
- The Activity tells us nothing about either the energy or the type of radiation. It only carries information about the intensity.

1. Parallel decays



The nucleus decays to A with constant λ_a and at the same time decays to B with a constant λ_b .

$$\begin{cases} \left(\frac{dN}{dt}\right)_A = -\lambda_a N & \text{decaimento para A} \\ \left(\frac{dN}{dt}\right)_B = -\lambda_b N & \text{decaimento para B} \end{cases} \quad \rightarrow \quad \frac{dN}{dt} = \left(\frac{dN}{dt}\right)_A + \left(\frac{dN}{dt}\right)_B = -(\lambda_a + \lambda_b)N$$

$$N(t) = N_0 e^{-\lambda_T t} \quad \text{com} \quad \lambda_T = \lambda_a + \lambda_b$$

This means that only the disintegration constant λ_T is observed, and not the individual constants λ_a and λ_b .

Let's see how many nuclei of A and B we have over time. The equation is

$$\frac{dN_A}{dt} = +\lambda_a N = +\lambda_a N_0 e^{-(\lambda_a + \lambda_b) t}$$

And the most general solution

is

$$N_A(t) = A + B e^{-(\lambda_a + \lambda_b) t} \quad \text{com} \quad N_A(0) = 0 \quad \Rightarrow \quad A = -B$$

and

d

$$\frac{dN_A}{dt} = -B(\lambda_a + \lambda_b) e^{-(\lambda_a + \lambda_b) t} = +\lambda_a N_0 e^{-(\lambda_a + \lambda_b) t} \quad \Rightarrow \quad B = -\frac{\lambda_a}{\lambda_a + \lambda_b} N_0$$

And so

$$N_A(t) = \frac{\lambda_a}{\lambda_a + \lambda_b} N_0 (1 - e^{-(\lambda_a + \lambda_b) t}) \quad N_B(t) = \frac{\lambda_b}{\lambda_a + \lambda_b} N_0 (1 - e^{-(\lambda_a + \lambda_b) t})$$

$$t \rightarrow +\infty \quad N_A \rightarrow \frac{\lambda_a}{\lambda_a + \lambda_b} N_0 \quad N_B \rightarrow \frac{\lambda_b}{\lambda_a + \lambda_b} N_0 \quad t \rightarrow +\infty \quad N \rightarrow 0$$

For any instant t we have

$$N_A + N_B + N = N_0$$

2. Series (chain) decays



Nucleus X_1 decays to X_2 with constant λ_1
which in turn decays to X_3 with constant λ_2 .

Let's assume that $N_1(t=0) = N_0$ e $N_2(t=0) = N_3(t=0) = 0$

and so
$$N_1(t) = N_0 e^{-\lambda_1 t}$$

For N_2 we have to consider the number of nuclei 2 being formed per unit of time (equal to the number of nuclei 1 disappearing per unit of time) and the number of nuclei 2 disappearing

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \text{with solution} \quad N_2(t) = A e^{-\lambda_1 t} + B e^{-\lambda_2 t}$$

$$\frac{dN_2}{dt} = -A\lambda_1 e^{-\lambda_1 t} - B\lambda_2 e^{-\lambda_2 t} = \lambda_1 N_0 e^{-\lambda_1 t} - A\lambda_2 e^{-\lambda_1 t} - B\lambda_2 e^{-\lambda_2 t}$$

$$N_2(0) = 0 = A + B$$

and so

$$A = -B = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0$$

and finally

$$N_1(t) = N_0 e^{-\lambda_1 t} \quad N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$$

Let us consider the special case where $\lambda_1 \ll \lambda_2$, that is, the parent nucleus has a mean lifetime much longer than the daughter nucleus. We further assume that $\lambda_1 \ll 1$.

$$N_1(t) \cong N_0 \text{ (constante)} \quad N_2(t) = \frac{\lambda_1}{\lambda_2} N_0 \left(1 - e^{-\lambda_2 t} \right)$$

Recalling that **Activity** is defined by $A_i = \lambda_i N_i$, as t tends to infinity we have

$$t \rightarrow +\infty \quad N_2(t) = \frac{\lambda_1}{\lambda_2} N_0 \quad \begin{cases} A_2(t) \rightarrow \lambda_1 N_0 \\ A_1(t) \rightarrow \lambda_1 N_0 \end{cases}$$

And this situation is called **secular equilibrium** – both nuclei progressively tend towards the same activity.

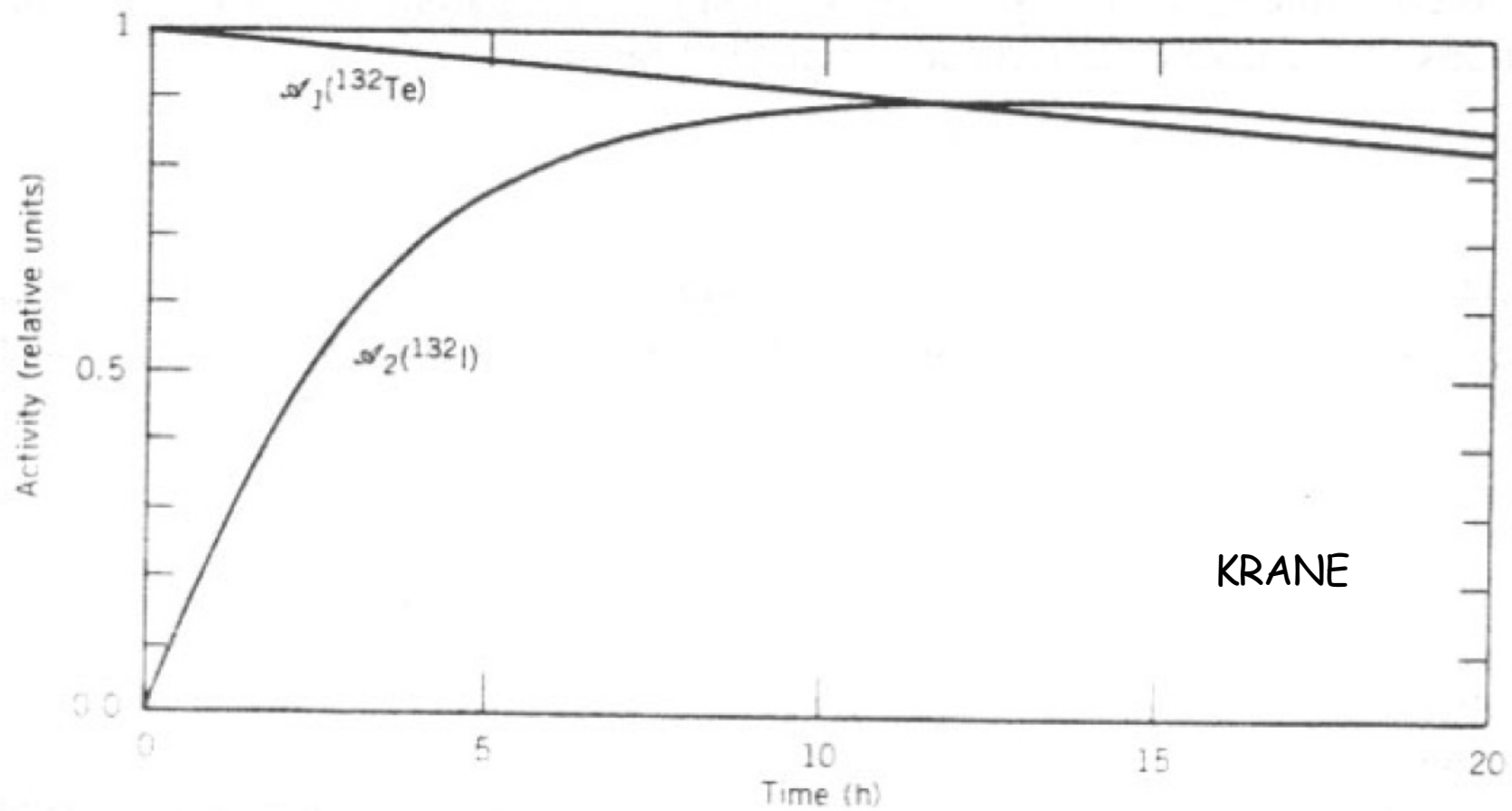
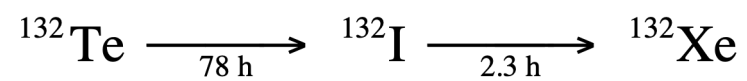


Figure 6.6 In the decay ^{132}Te (78 h) $\rightarrow$ ^{132}I (2.28 h) $\rightarrow$ ^{132}Xe , approximate secular equilibrium is reached at about 12 h.



We already know that

$$N_1(t=0) = N_0 \quad \text{e} \quad N_2(t=0) = N_3(t=0) = 0$$

$$N_1(t) = N_0 e^{-\lambda_1 t} \quad N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$$

Assuming that N_3 is stable, we have

$$\frac{dN_3}{dt} = \lambda_2 N_2 \quad \frac{dN_3}{dt} = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} N_0 \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$$

with solution

$$N_3(t) = \frac{N_0}{\lambda_2 - \lambda_1} \left(\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t} \right) + C$$

$$N_3(t) = \frac{N_0}{\lambda_2 - \lambda_1} \left(\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t} \right) + N_0$$

Artificial production of radioactive nuclei

Many radioactive isotopes of interest do not occur in nature, and are produced in *nuclear reactors*, or *cyclotrons*, in *nuclear reactions* involving beams of protons or neutrons (to be studied later). Since the isotopes produced are radioactive, they decay at the same time as they are produced, so the **reaction rate R** (number of nuclei produced per second) must be sufficiently high.

Imagine a nucleus X produced in a nuclear reactor at a rate R . As soon as it is produced, X immediately begins to decay, with a decay constant λ . The dynamical equation for this nucleus is

$$\frac{dN(t)}{dt} = R - \lambda N(t)$$

Rate of change
(nuclei/unit of time) →

↑
Production rate
(nuclei produced/unit of time)

← **Disintegration rate**
(nuclei X decaying)

Note that the **production rate** appears with a “+” sign (nuclei X being produced), while the contribution from **decays** appears with a “−” sign (nuclei X being destroyed).

The solution of this equation is

$$N(t) = \frac{R}{\lambda} (1 - e^{-\lambda t})$$

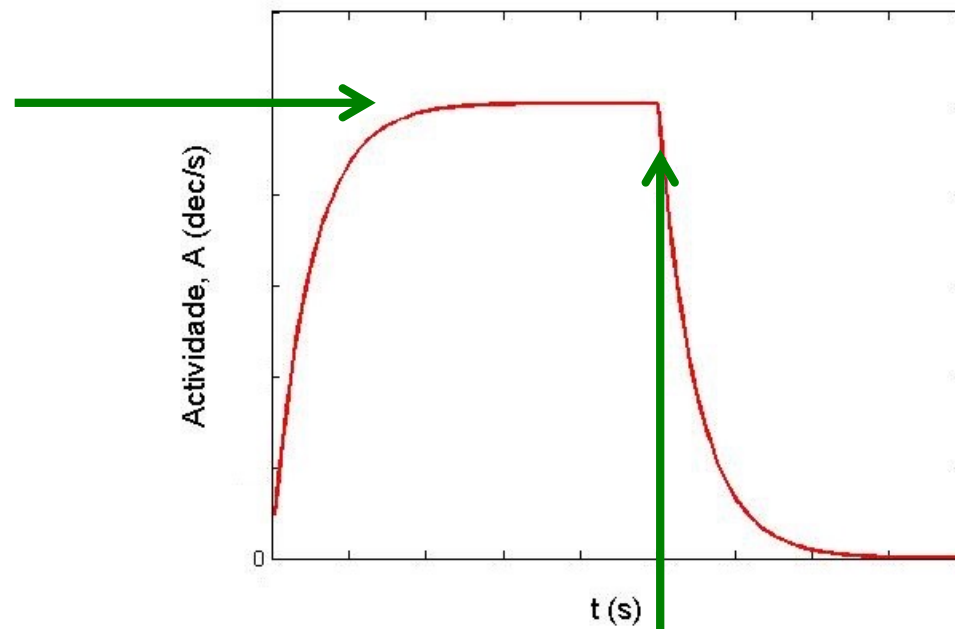
and so the activity of nucleus X is given by

$$\mathcal{A}(t) = R (1 - e^{-\lambda t})$$

Note that in these equations $N(0) = 0$ – that is, initially there are no nuclei X .

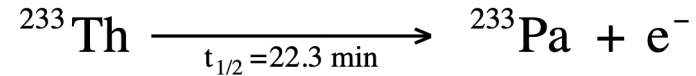
If, at a certain point, the reactor is switched off, the number of nuclei will begin to decay following its normal exponential evolution.

Maximum or
saturation activity



Instant at which the reactor is switched off – the number of nuclei decreases because those that disintegrate are not “replenished”

Example



Assuming the formation rate is $R = 2.0 \times 10^{11} \text{ s}^{-1}$. What is the activity of Thorium (half-life 22.3 min) after one hour? And that of Protactinium (half-life 27 days)?

$$t_{1/2} = \frac{\ln 2}{\lambda} \cong \frac{0.693}{\lambda} \Rightarrow \lambda = \frac{0.693}{22.3 \times 60} = 5.18 \times 10^{-4} \text{ s}^{-1}$$

$$A(t) = \lambda N(t) = 2.0 \times 10^{11} \left(1 - e^{-5.18 \times 10^{-4} \times 3600}\right) = 1.69 \times 10^{11} \text{ dec/s (4.57 Ci)}$$

For Protactinium we have

$$\lambda_2 = \frac{0.693}{27 \times 24 \times 3600} = 2.97 \times 10^{-7} \text{ s}^{-1}$$

We have already seen that

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \Rightarrow A_2(t) = \lambda_2 N_2(t) = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} N_0 \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

Example

This is a situation where $\lambda_2 \ll \lambda_1$

$$A_2(t) = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} N_0 (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \cong \frac{\lambda_1 \lambda_2}{-\lambda_1} N_0 (e^{-\lambda_1 t} - 1)$$

and since $R = \lambda_1 N_0$ we get

$$A_2(t) \cong \frac{\lambda_2}{\lambda_1} R (1 - e^{-\lambda_1 t}) = \frac{\lambda_2}{\lambda_1} A_1(t)$$

and so

$$A_2(t) = \frac{2.97 \times 10^{-7}}{5.18 \times 10^{-4}} A_1(t) = \frac{2.97 \times 10^{-7}}{5.18 \times 10^{-4}} 1.69 \times 10^{11} = 9.69 \times 10^7 \text{ dec/s (2.6 mCi)}$$