

A technique for conducting point pattern analysis of cluster plot stem-maps

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Abstract

Point pattern analysis of forest inventory stem-maps may aid interpretation and inventory estimation of forest attributes. To evaluate the techniques and benefits of conducting point pattern analysis of forest inventory stem-maps, Ripley's $K(t)$ was calculated for simulated tree spatial distributions and for over 600 USDA Forest Service Forest Inventory and Analysis (FIA) plots in Minnesota and Wisconsin. A new technique for calculation of Ripley's $K(t)$ for cluster plot stem-maps was proposed that involves the truncation and combination of clustered, circular sub-plots (0.01 ha) into one square (0.04 ha) for each inventory plot. For Poisson and uniform simulated tree spatial distributions, combined sub-plots may possess nearly the same spatial properties as the entire plot area from which they were sampled. Although sub-plots may be too small for meaningful spatial analysis, combined sub-plots may permit spatial analysis regardless of how sub-plots are combined. The step-size (t) at which stem-map point patterns were most discernible as either clustered or uniform varied by forest type. Additionally, stand disturbances may increase $K(t)$. Although limitations exist, point pattern analysis of forest inventory stem-maps may permit refined ecological analysis of forest inventories.

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1. Introduction

The sampling designs commonly employed in forest inventories often reflect the user-group objectives of estimating volume, species composition, and growth/mortality rates. Many of the analyses resulting from conventional forest inventories such as these often lack quantification of stand-level spatial

attributes. Although numerous inventories map the locations of all sample trees, this information is primarily used to facilitate tree identification during remeasurements and has rarely been used for individual tree spatial pattern analysis. These stem-maps may provide data for a point pattern analysis of forest attributes across forest ecosystems.

Point pattern analysis, a branch of spatial statistics, can be used to quantify the spatial pattern of plant communities (Cressie, 1993). Information on the spatial pattern of individual plants within forests may refine our understanding of ecological processes such as forest establishment, growth, competition, reproduction, and mortality (Legendre and Fortin, 1989;

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Hasse, 1995; Dale, 1999). Spatial point pattern analysis can be used to investigate stand disturbance histories (Moeur, 1997; Mateu et al., 1998; Harrod et al., 1999; Mast and Veblen, 1999) and to describe the competitive interactions between trees in mixed-species forests (Duncan, 1991; Szwagrzyk, 1992; Ward et al., 1996). It may also offer an alternative to the traditional broad population density investigations (Weiner, 1982). In the temperate conifer-hardwood forests of Minnesota and Wisconsin, spatial analyses of regional forests have advanced successional theories and refined stand disturbance dynamics theories (Frelich and Reich, 1995; Frelich, 2002). However, to date most point pattern analysis of Minnesota and Wisconsin's forests have occurred outside of traditional forest inventories. Therefore, the potential exists to augment regional forest analyses in the upper Midwest by developing methodologies to conduct point pattern analysis of existing forest inventories.

The greatest obstacles to conducting point pattern analysis of inventory stem-maps are usually the sampling designs. Often, the relatively small size of most forest inventory plots and sometimes spatially disparate sub-plot sampling designs severely limit point pattern analysis. With this in mind, the goal of this study was to evaluate a new technique for conducting spatial point pattern analysis using the Ripley's $K(t)$ statistic on inventory data for the states of Minnesota and Wisconsin from the Forest Inventory and Analysis (FIA) program of the USDA Forest Service. FIA sample plots consist of four 7.31 m radius circular sub-plots configured as one central sub-plot and three peripheral sub-plots with centers located at 36.58 m and azimuths of 0, 120, and 240° from the center of the central sub-plot. The FIA program currently maps the location of every tree (diameter at breast height ≥ 12.7 cm) by bearing and distance from each

sub-plot center. However, since FIA inventory plots are a configuration of four of these spatially disparate sub-plots, a new methodology consisting of sub-plot truncation and combination was employed and evaluated using both simulated and actual inventory data. The Ripley's $K(t)$ statistic was then assessed in terms of reflecting forest type and stand disturbance attributes of FIA plots.

2. Methods

2.1. Data

To investigate the effects of the sub-plot truncation and combination methodology proposed for Minnesota and Wisconsin forest inventories, tree locations were simulated (649 tph) for an entire square plot (87.8 m \times 87.8 m, 0.77 ha) containing the FIA field plot sample design for three spatial distributions (250 plots each): uniform, Poisson, and clustered (Fig. 1). The three simulated spatial distributions were chosen to reflect the range of spatial patterns common to inventoried forests: uniform (even-aged plantations), Poisson (random tree locations), and clustered (old-growth/uneven-aged). Uniform spatial distributions had tree locations spaced approximately 4 m from each other with a random perturbation of up to 0.3 m. Poisson spatial distributions had tree locations with randomly assigned coordinates. Clustered spatial distributions had cluster centers (10 per plot) randomly located within the square plot (0.77 ha). These randomly located tree clusters contained 50 trees randomly located within 4.9 m of the cluster center. The random components of all spatial distributions were derived from a uniform distribution. The clustered and uniform spatial distributions were labeled accordingly to reflect the dispersive (uniform) and

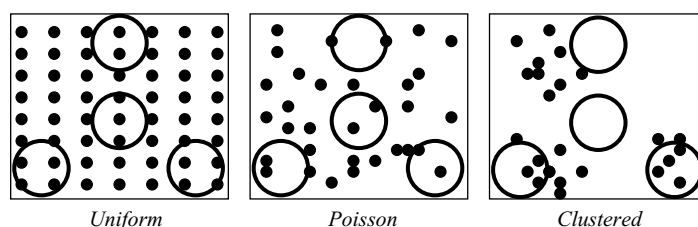


Fig. 1. Simulated uniform, Poisson, and clustered spatial patterns of individual trees (FIA plot design superimposed).

attractive (clustered) attributes of the spatial patterns, although random components were inherently required in their creation.

Data from FIA plots from Minnesota (337 plots) and Wisconsin (333 plots) measured in 1999 and 2000 were also used in this study. Only plots with all four sub-plots entirely in forested conditions were included in the study data set. Additionally, only plots in the following common forest types were included in the study data set: red pine (*Pinus resinosa*), black spruce (*Picea mariana*), oak/hickory (*Quercus* spp., *Carya* spp.), maple/beech/birch (*Acer* spp., *Fagus grandifolia*, *Betula* spp.), and aspen (*Populus tremuloides*).

Two attributes of the current FIA sampling design complicate point pattern analysis using the Ripley's $K(t)$ statistic. First, the sampling design of four, spatially disparate circular sub-plots does not allow for toroidal edge correction. Second, the FIA sub-plots may be individually too small to allow for robust spatial analysis. Therefore, for this study, the sub-plots for each FIA plot were truncated to a square by excluding all tree locations outside a superimposed square (Fig. 2A and B) and all four truncated sub-plots were combined into one square for each FIA plot. This truncation and combination method generates a larger overall area providing more spatial information. First, the locations (azimuth and distance from sub-plot center) of trees within a superimposed square are extracted from every sub-plot (Fig. 2A). Next, the distance and azimuth from plot center to each tree 12.7 cm or greater in DBH are converted to x , y coordinates (using the lower left corner as the origin). Finally, the tree coordinates from each truncated sub-plot are combined to form a larger square with the

lower left corner as the origin and 20.85 m sides (Fig. 2B). Additionally, to investigate the effect of the sub-plot configurations on Ripley $K(t)$ values for the entire combined plot, all 24 possible configurations of sub-plots were created.

2.2. Ripley's $K(t)$

Ripley's $K(t)$ is a widely used second order descriptive statistic in two-dimensional point pattern analysis (Hasse, 1995; Dale, 1999). The term “second order” refers to the analysis of all point-to-point distances, as opposed to first-order analyses such as “nearest neighbor” methods that use only mean inter-point distances (Hasse, 1995; Dale, 1999). Mathematically, the edge-corrected estimated Ripley's K -function for detecting departures from complete spatial randomness is defined as

$$\hat{K}(t) = \hat{\lambda}^{-1} \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n \frac{w(S_i, S_j)^{-1} I(\|S_i - S_j\| \leq t)}{N, t > 0} \quad (1)$$

where t is the step-size, N the number of trees in the study area, $|A|$ the size of the study area, $\hat{\lambda} = N/|A|$ the estimated tree density, $w(S_i, S_j)$ the proportion of the circumference of a circle centered at S_i , passing through S_j , and that is inside the study region A (Cressie, 1993).

Ripley's $K(t)$ estimates are evaluated with respect to step-size distance values (t). Larger Ripley's $K(t)$ values at a certain (t) may indicate attractive properties (clustering) among individual trees, while smaller Ripley's $K(t)$ values at the same (t) may indicate

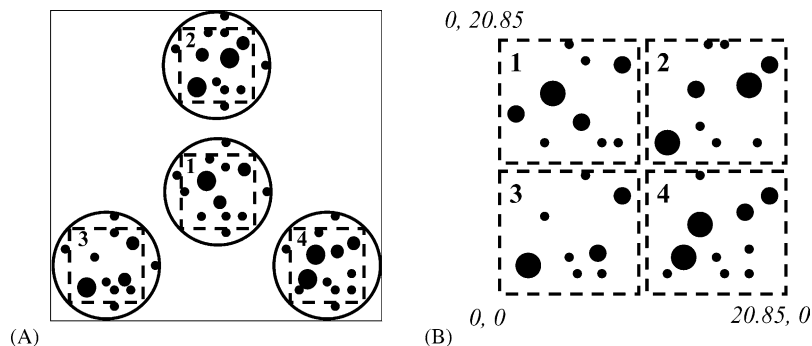


Fig. 2. To allow buffer creation, all circular sub-plots are truncated to a square (A), then all square sub-plots combined to one square plot (B) and all bearings and distances transformed to a coordinate system (x , y in meters).

dispersive spatial properties (regularity) among individual trees. Ripley's $K(t)$ is often determined using toroidal edge corrections (Ripley, 1977; Boots and Getis, 1988; Duncan, 1990; Mast and Veblen, 1999), which involves wrapping the plot around a torus such that opposite sides of the plot connect, creating a data set with no boundary (Ripley, 1977; Hasse, 1995). The common use of a torus assumption may validate the truncation and combination idea explored in this study. Moeur (1993), Hasse (1995), and Mast and Veblen (1999) provide more detailed descriptions of Ripley's $K(t)$, including its formulation and application to forest stem-maps.

2.3. Analysis

For this study, Ripley's $K(t)$ was calculated for all stem-maps, whether simulated or actual, using spatial analysis programs written by Duncan (1990). Step-sizes (t) used in these analyses were approximately half the length of the smallest plot size used in each analysis. Since some analyses required comparison of Ripley's $K(t)$ values across various scales (t), a derived Ripley's (t) statistic was used where appropriate (2) (Hasse, 1995):

$$\text{Derived } K(t) = \sqrt{\left[\frac{k(t)}{\pi}\right]} - t \quad (2)$$

Ripley's $K(t)$ values for simulated tree point patterns were used to test the hypothesis that the combined sub-plot spatial patterns may possess the same spatial pattern as the forest it samples. Mean standardized differences between Ripley's $K(t)$ values for the entire simulated stem-map areas (0.77 ha), the combined sub-plots (0.04 ha), and individual sub-plots (0.01 ha) were examined using

$$K_{\text{diff}} = \frac{|K_i - K_j|}{K_i} \quad (3)$$

where K_{diff} is the standardized difference, K_i the Ripley's $K(t)$ value for the total plot area, and K_j the Ripley's $K(t)$ value for the associated combined or individual truncated sub-plots.

Ripley's $K(t)$ results for actual inventory stem-maps were examined in terms of variation among individual inventory sub-plots and all possible configurations of truncated sub-plots into the combined plots. First,

coefficients of variation were determined for Ripley's $K(t)$ values between the four sub-plots of each inventory plot. Second, coefficients of variation were determined for each of the 24 possible configurations of sub-plots for each inventory plot. Lastly, mean Ripley's $K(t)$ values and associated standard errors were determined for each set of plots stratified according to forest type (red pine, black spruce, oak/hickory, maple/beech/birch, and aspen) and stand disturbance history (disturbed or undisturbed).

3. Results and discussion

The study hypothesis was that the dispersive and attractive properties of point patterns would be maintained during sub-plot truncation and combination. For simulated uniform and Poisson tree spatial distributions, combined sub-plots may possess nearly the same spatial character as the entire forest area from which they were sampled (S.E.'s <0.015) (Fig. 3). For clustered tree spatial distributions, there were larger differences between Ripley's $K(t)$ values for the entire forest area and the combined sub-plots indicating possible bias (S.E.'s <0.018) (Fig. 3). Individual sub-plots consistently had Ripley's $K(t)$ values far different from the 0.77 ha plot values (Fig. 3). Based on Ripley's $K(t)$ analyses using simulated spatial distributions, the proposed methodology for conducting point pattern analysis of relatively small, spatially disparate inventory sub-plots appears valid for uniform and Poisson tree distributions. Although this

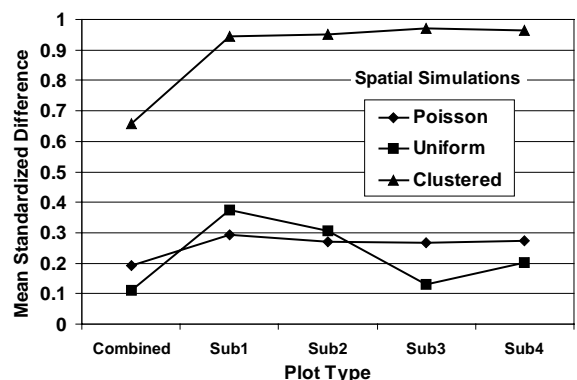


Fig. 3. Mean standardized differences between $K(t)$ for each plot and the associated, combined sub-plots and individual sub-plots for simulated distributions ($t = 4.88$ m).

proposed methodology may produce biased results for some clustered tree distributions, these methods may warrant future consideration in the analysis of forest inventories.

Results using actual inventory data from Minnesota and Wisconsin may further elucidate the behavior of Ripley's $K(t)$ values among: individual sub-plots, the 24 possible configurations of combined sub-plots, different forest types, and disturbed/undisturbed stands. Among individual sub-plots, for each plot, Ripley's $K(t)$ values varied by as much as 200% (Fig. 4). Most plots had coefficients of variation (CV) greater than 0.25 for their respective Ripley's $K(t)$ values between sub-plots. The level of variation among the sub-plots of any particular plot depended on the number of trees within the plot (Fig. 4). All plots with over 35 trees had CV's less than 0.5 (Fig. 4). It appears that as the number of trees increases per unit area, the variation decreases in the possible spatial patterns that trees can assume. There may be less available room for spatial dissimilarity to be expressed. Spatial investigations of forest inventory plots of relatively small sizes may yield no meaningful results other than displaying the wide variation in tree spatial patterns at small scales.

There are 24 possible sub-plot configurations for each combined plot stem-map (Fig. 2B). CV's for Ripley's $K(t)$ values, between all 24 sub-plot configurations for each plot, were all under 0.6 with the CV decreasing exponentially as the number of trees

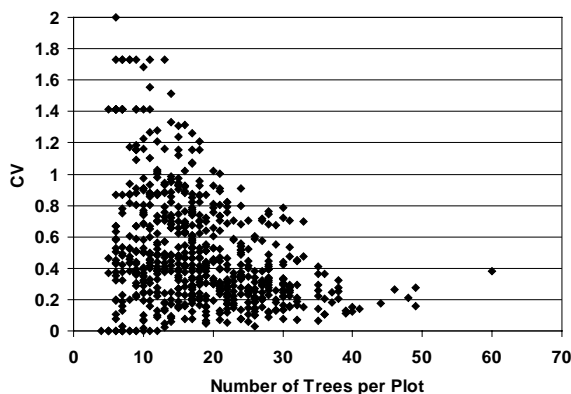


Fig. 4. Coefficients of variation for Ripley's $K(t)$ between individual sub-plots for each sample plot ($t = 4.88$ m).

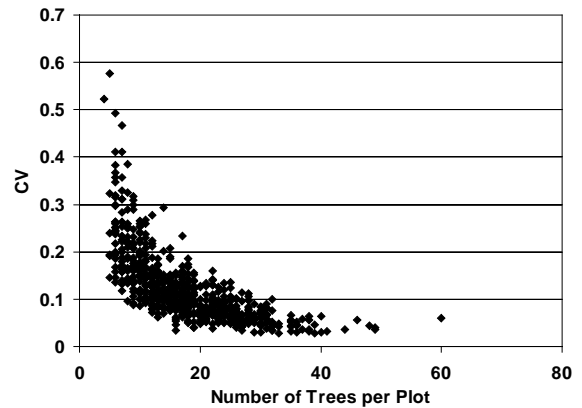


Fig. 5. Coefficients of variation for Ripley's $K(t)$ between all 24 different configurations of sub-plots by number of trees per plot ($t = 7.32$ m).

increased (Fig. 5). For each individual plot, there is less variation in Ripley's $K(t)$ values between the 24 combined plot configurations than between individual sub-plots. As long as plots had a minimum number of trees, the effect of configuration of sub-plots was deemed minimal. If computational resources allow, a mean or median of all the 24 possible sub-plot configurations for each plot may be preferable to the selection of just one of the sub-plot configurations.

The variation in Ripley's $K(t)$ across various spatial scales (t) and forest types was examined with the idea that distinct point patterns may be most evident at characteristic scales by forest type. Ripley's $K(t)$

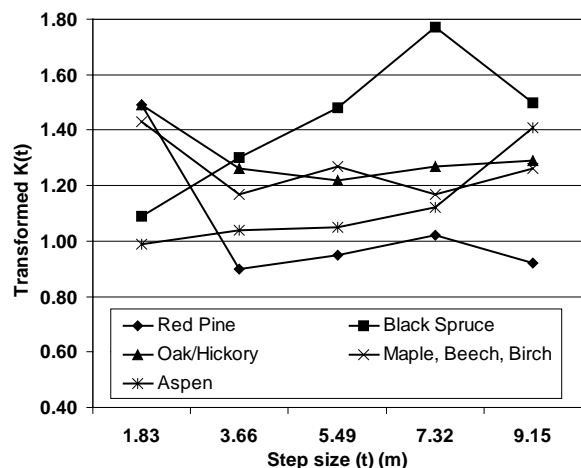


Fig. 6. Transformed $K(t)$ for various forest types using various step-sizes (t) in Minnesota (no. of trees ≥ 20).

statistics reached their largest values at differing spatial scales by forest type (Fig. 6). Aspen and black spruce forest types displayed their most attractive spatial patterns at larger spatial scales (t), while red pine, oak/hickory, and maple/beech/birch forest types displayed similar attractive patterns at the smallest spatial scales (t) (Fig. 6). When interpreting spatial patterns across the myriad of stand conditions present in the forest inventories of Minnesota and Wisconsin, one may need to be cognizant of the scale of observation and the forest type. Spatial patterns may be more discernible in a 0.1 ha plot in a red pine plantation than in a 0.5 ha plot in an uneven-aged mixed-species hardwood stand.

Using the following step-sizes for each forest type where attractive spatial patterns were most evident (aspen: 9.15 m, red pine: 7.32 m, black spruce: 1.83 m, oak/hickory: 1.83 m, and maple/beech/birch: 1.83 m), the difference in mean Ripley's $K(t)$ values between plots with and without past disturbances were examined. Plots with relatively recent stand disturbances (wind, fire, insect/disease, flood, snow/ice, and harvest) had a larger mean, derived Ripley's $K(t)$ value ($\bar{x} = 3.32$, S.E. = 0.51) than undisturbed plots ($\bar{x} = 2.53$, S.E. = 0.14). It appears that stand disturbances may increase the degree of clustering among individual trees within a plot. Small-scale disturbances, such as some wind and ice storms, may kill trees in random clusters, resulting in an overall clustered spatial tree distribution for entire stands.

Spatial analysis of forest inventory stem-maps, using methodology described by this study, may enhance inventory analyses. However, numerous limitations and caveats must be observed. First, the actual area of analysis is reduced, due to sub-plot truncation, resulting in a substantial loss of data. Second, Ripley's $K(t)$ may be calculated only on data from inventories using fixed-radius plots. Third, for sampling designs such as FIA's, Ripley's $K(t)$ may be calculated only for trees with a minimum DBH, prohibiting investigation of the spatial dynamics of regeneration. Fourth, because the study plots are a combination of spatially disparate sub-plots, plots located in ecotonal/multi-use areas may confound tree point pattern analysis; e.g., where one sub-plot is located on cropland while the other three are on forestland. Fifth, stand densities may affect the efficacy of conducting point pattern analysis of inventory plots with extremely low stand

densities. Six, the methodology of sub-plot truncation and combination may produce misleading Ripley's $K(t)$ output for clustered tree distributions. Finally, to ease the data management requirements for spatial analysis of large inventories, a single step-distance (t) may need to be selected that may be most appropriate for the inventoried forest ecosystem.

4. Conclusions

Stem-mapped inventory data, often collected strictly for tree remeasurement, may provide a wealth of stand-level spatial information for forests across large regions. Unfortunately, inventory sample designs, such as FIA's, do not lend themselves well to point pattern analysis. As shown in this study, relatively small sample plots may not be of sufficient size to accurately estimate the point patterns of forests in Minnesota and Wisconsin. However, this study's methodology of sample plot truncation and combination provided larger sample areas for more accurate pattern analysis. Although constrained by numerous limitations, point pattern analysis of forest inventory stem-maps is feasible, expands the ecological analysis capability of inventory data sets, and may warrant future application and evaluation.

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