



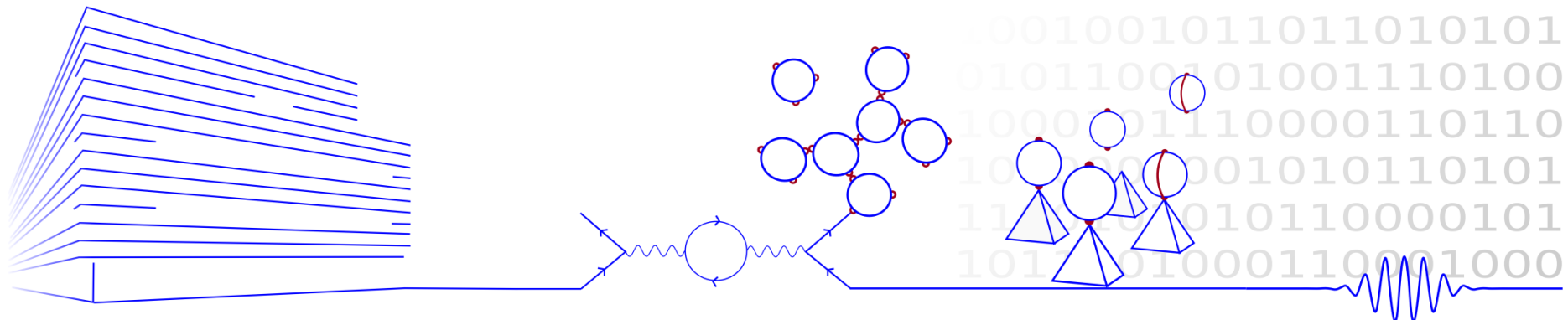
Ciências  
ULisboa

# Mecânica dos Meios Contínuos

Margarida Telo da Gama

Rodrigo Coelho

2020/21



# Apresentação do curso

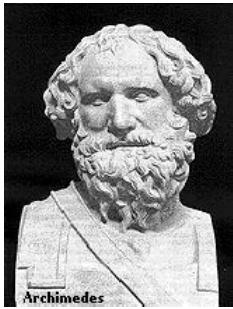
- Aulas teóricas por Zoom
  - Link permanente: <https://videoconf-colibri.zoom.us/j/95583477141>
  - Tirar dúvidas: durante as aulas e TPs, por email ([rccoelho@fc.ul.pt](mailto:rccoelho@fc.ul.pt)) ou por video no link acima;
  - Microfone desligado e câmera ligada;
  - Pode-se ligar o microfone e fazer perguntas a qualquer momento;
  - Não gravar a aula (pode-se usar captura de tela);
  - Notas de aula no Fenix;
- TPs presenciais/alternadas a partir do dia 22;
- Modelo de aula pode mudar de acordo com as recomendações.

# Apresentação do curso

- Avaliação contínua
  - (50%) Uma questão de cada uma das 5 séries entregue pelo Moodle + avaliação oral;
  - (50%) Uma apresentação final de 10 min com relatório de 10 páginas sobre tema relacionado com o curso (ex.: coração artificial, carros de fórmula 1, etc.);
- Avaliação não contínua: exame final (100%).
- Bibliografia
  - Fluid Dynamics for Physicists, by T E **Faber**, Cambridge University Press;
  - Elementary Fluid Dynamics (Oxford Applied Mathematics and Computing Science Series) by D J **Acheson**, Oxford University Press;
  - Fluid Mechanics: Fundamentals and Applications, by **Çengel & Cimbala**, McGraw-Hill series in mechanical engineering.

# History

## Faces of Fluid Mechanics



Archimedes  
(C. 287-212 BC)



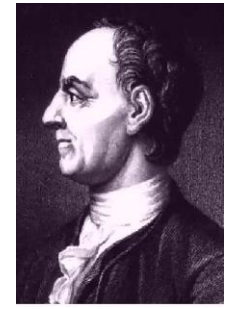
Newton  
(1642-1727)



Leibniz  
(1646-1716)



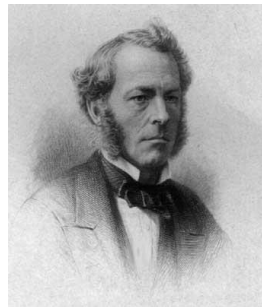
Bernoulli  
(1667-1748)



Euler  
(1707-1783)



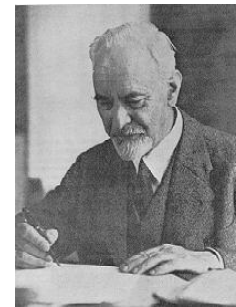
Navier  
(1785-1836)



Stokes  
(1819-1903)



Reynolds  
(1842-1912)



Prandtl  
(1875-1953)



Taylor  
(1886-1975)

# Significance

- Fluids everywhere
  - Weather & climate
  - Vehicles: automobiles, trains, ships, and planes, etc.
  - Environment
  - Physiology and medicine
  - Sports & recreation
  - Many other examples!

# Weather & Climate

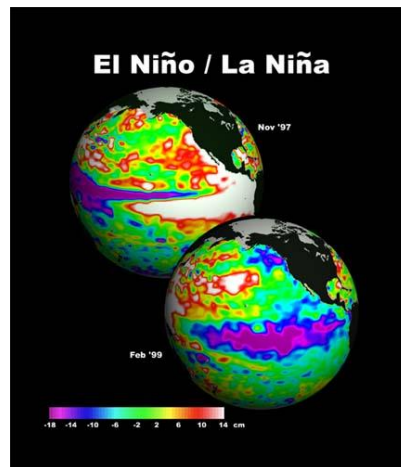
## Tornadoes



## Thunderstorm



## Global Climate



## Hurricanes



# Vehicles

Aircraft



Surface ships



High-speed rail



Submarines





# Environment

## Air pollution

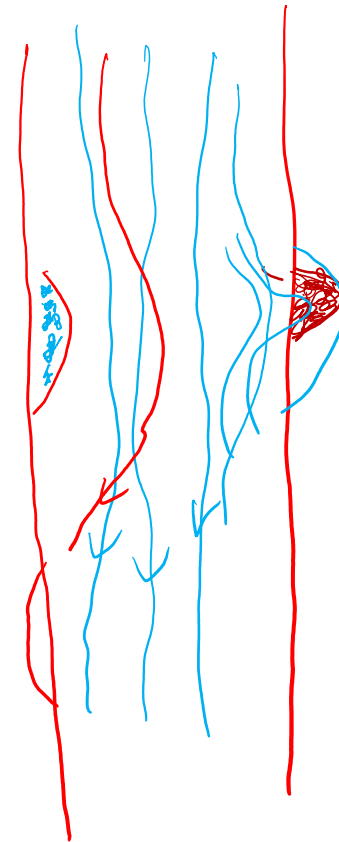


## River hydraulics





## Why do rivers curve?



[https://www.youtube.com/watch?v=8a3r-cG8Wic&feature=emb\\_title](https://www.youtube.com/watch?v=8a3r-cG8Wic&feature=emb_title)  
<https://physicstoday.scitation.org/doi/10.1063/PT.3.4523>

# Physiology and Medicine

Blood pump



A BVS blood pump

Ventricular assist device

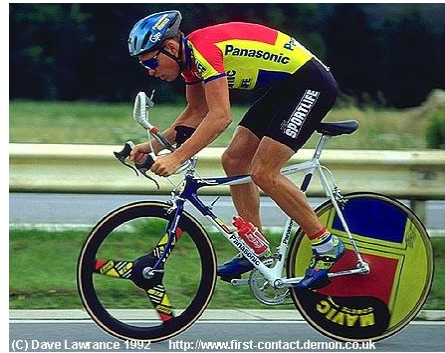


# Sports & Recreation

## Water sports



## Cycling



## Offshore racing



## Auto racing



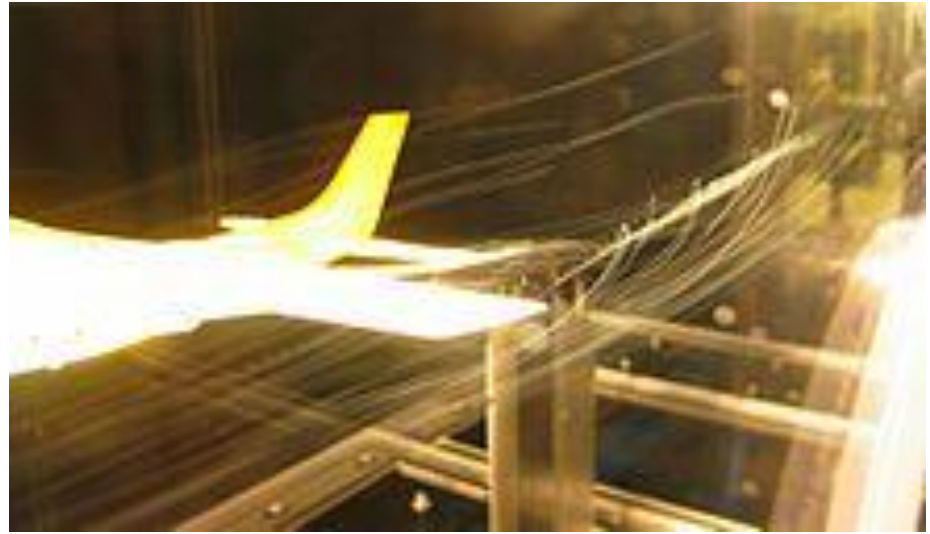
## Surfing



# Analytical Fluid Dynamics

- The theory of mathematical physics problem formulation
- Control volume & differential analysis (RTT)
- Exact solutions only exist for simple geometry and conditions
- Approximate solutions for practical applications
  - Linear
  - Empirical relations using EFD data

# Full and model scales: wind tunnel



- Scales: full-scale and model
- Selection of the model scale: governed by dimensional analysis and similarity

$$Re = \frac{\overbrace{L \cdot V}^{\text{tamanho velocidade}}}{\underbrace{\nu}_{\text{viscosidade cinemática}}} = C_{re}$$



# Computational Fluid Dynamics

- CFD is use of computational methods for solving fluid engineering systems, including modeling (mathematical & Physics) and numerical methods (solvers, finite differences, and grid generations, etc.).
- Rapid growth in CFD technology since advent of computer



ENIAC 1, 1946



IBM WorkStation

# Purpose

- The objective of CFD is to model the continuous fluids with Partial Differential Equations (PDEs) and discretize PDEs into an algebra problem, solve it, validate it and achieve **simulation based design** instead of “build & test”
- Simulation of physical fluid phenomena that are difficult to be measured by experiments: **scale simulations** (full-scale ships, airplanes), **hazards** (explosions, radiations, pollution), **physics** (weather prediction, planetary boundary layer, stellar evolution).



# Modeling

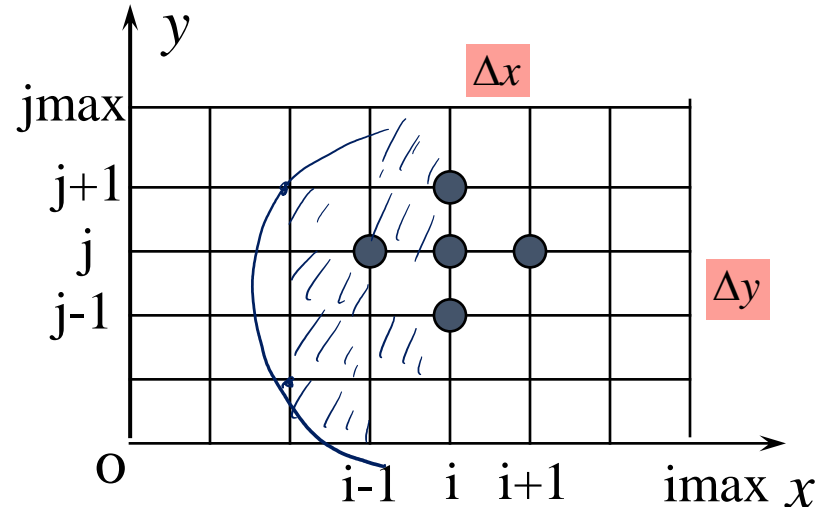
- Mathematical physics problem formulation of fluid engineering system
- **Governing equations**: Navier-Stokes equations (momentum), continuity equation, pressure Poisson equation, energy equation, ideal gas law, combustions (chemical reaction equation), multi-phase flows (e.g. Rayleigh equation), and turbulent models (RANS, LES, DES).
- **Coordinates**: Cartesian, cylindrical and spherical coordinates result in different form of governing equations
- **Initial conditions** (initial guess of the solution) and **Boundary Conditions** (no-slip wall, free-surface, zero-gradient, symmetry, velocity/pressure inlet/outlet)
- **Flow conditions**: Geometry approximation, domain, Reynolds Number, and Mach Number, etc.

# Numerical methods

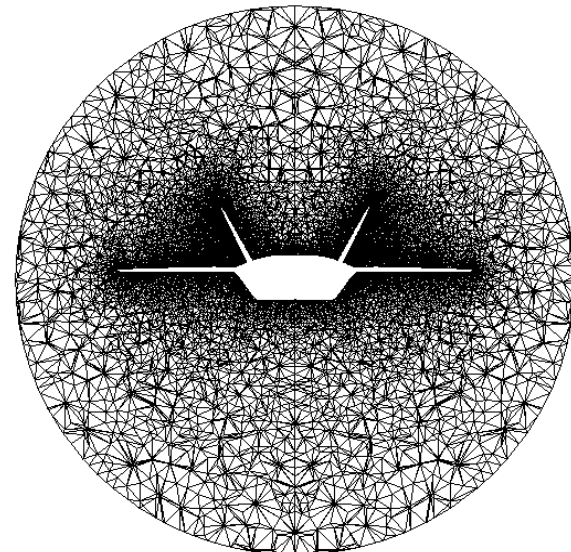
- **Finite difference methods:** using numerical scheme to approximate the exact derivatives in the PDEs

$$\frac{\partial^2 P}{\partial x^2} = \frac{P_{i+1} - 2P_i + P_{i-1}}{\Delta x^2}$$

$$\frac{\partial^2 P}{\partial y^2} = \frac{P_{j+1} - 2P_j + P_{j-1}}{\Delta y^2}$$



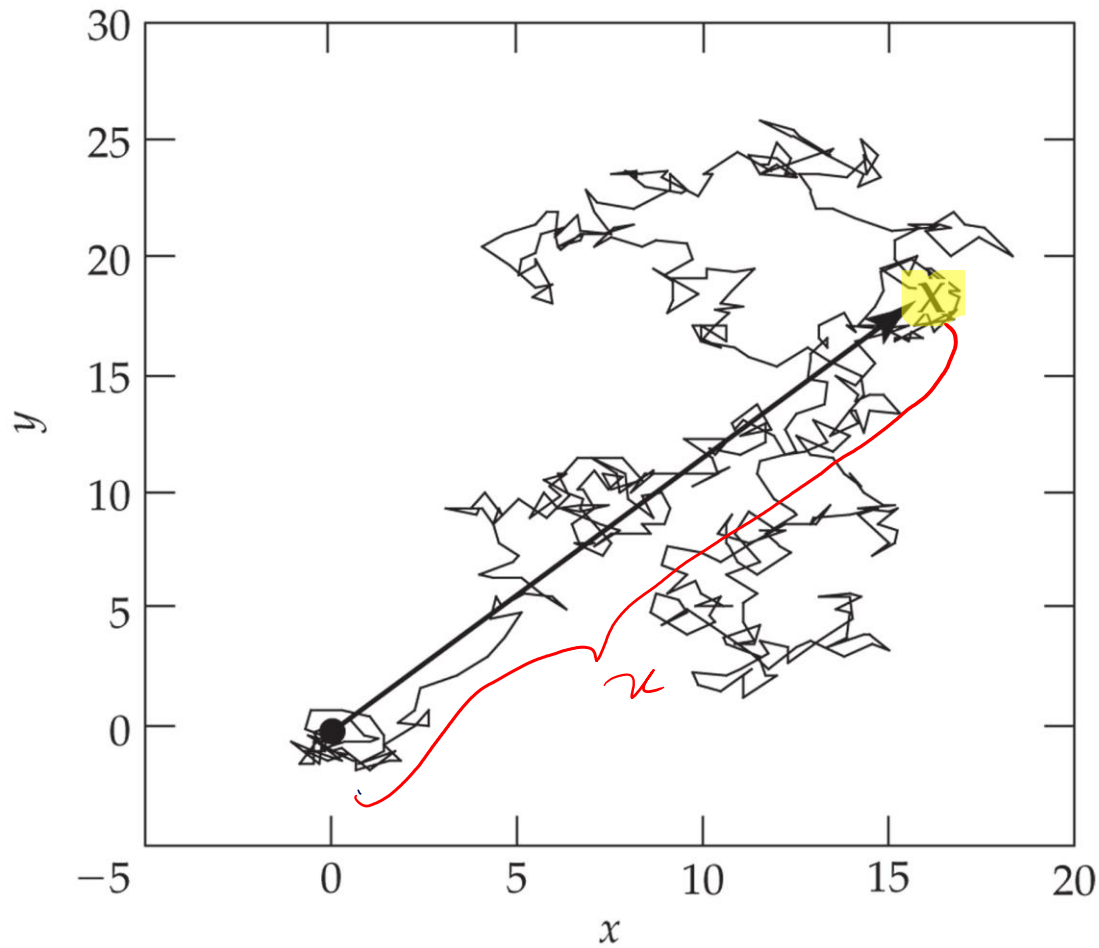
- **Finite volume methods**
- **Grid generation:** conformal mapping, algebraic methods and differential equation methods
- **Grid types:** structured, unstructured
- **Solvers:** **direct methods** (Cramer's rule, Gauss elimination, LU decomposition) and **iterative methods** (Jacobi, Gauss-Seidel, SOR)



Slice of 3D mesh of a fighter aircraft

# Diffusion & Convection

# Diffusion: Random walk



$$t \approx \frac{x^2}{2D}$$

*Coef. de difusão*

Table 1. Diffusion coefficient values for selected ions and small and large molecules.

Ion/Molecule	Atomic/Molecular Weight (g/mol)	Diffusion Coefficient (cm <sup>2</sup> /s)
H <sup>+</sup>	1.008	9.31 × 10 <sup>-5</sup>
Na <sup>+</sup>	22.990	1.33 × 10 <sup>-5</sup>
K <sup>+</sup>	39.098	1.96 × 10 <sup>-5</sup>
Ca <sup>2+</sup>	40.078	0.79 × 10 <sup>-5</sup>
Cl <sup>-</sup>	35.453	2.03 × 10 <sup>-5</sup>
Ammonia (NH <sub>3</sub> )	17.031	1.51 × 10 <sup>-5</sup>
Oxygen (O <sub>2</sub> )	31.999	2.10 × 10 <sup>-5</sup>
Carbon dioxide (CO <sub>2</sub> )	44.01	1.97 × 10 <sup>-5</sup>
Urea	60.055	1.38 × 10 <sup>-5</sup>
Glucose	180.156	5 × 10 <sup>-6</sup>
Sucrose	342.296	5.23 × 10 <sup>-6</sup>
Hemoglobin	68,000	6.9 × 10 <sup>-7</sup>
DNA	≈ 6,000,000	1.3 × 10 <sup>-8</sup>

Note: The diffusion coefficient varies with temperature and is also a function of the medium in which diffusion occurs. The values shown are for diffusion in water (H<sub>2</sub>O) at 25 °C.

$$D_{O_2} = 2.1 \cdot 10^{-5} \text{ cm}^2/\text{s}$$

$$\Delta x = 1 \text{ cm}$$

$$t = \frac{(1 \text{ cm})^2}{2.1 \cdot 10^{-5}} = 6.6 \text{ h}$$

**Table 2. Time required for diffusion of O<sub>2</sub> over a range of distances.**

Distance of Diffusion	Approximate Time Required
10 nm	23.8 ns
50 nm	595 ns
100 nm	2.38 μs
1 μm	238 μs
10 μm	23.8 ms
100 μm	2.38 s
1 mm	3.97 min
1 cm	6.61 hours
10 cm	27.56 days

In mammals, the circulatory system is such that no cell is more than approximately 10 μm from a capillary. This ensures proper nourishment and waste removal for all cells of the body.

$$Re = \frac{L \cdot V}{\nu}$$

### Range of Values for Viscosity, Density, and Kinematic Viscosity at Room Temperature

	Viscosity, $\mu$ (g cm <sup>-1</sup> s <sup>-1</sup> )	Density, $\rho$ (g cm <sup>-3</sup> )	Kinematic viscosity, $\nu = \mu/\rho$ (cm <sup>2</sup> s <sup>-1</sup> )
Gases	10 <sup>-4</sup>	0.001	0.1
Liquids			
Water	0.01	1.0	0.01
Glycerol	10	1	10
Blood	0.03	1.2	0.025

$\mu$  is dynamic viscosity



D - coeficiente de difusão

Peclet number

Relative Importance of Diffusion and Convection				
Molecule	MW (g mol <sup>-1</sup> )	$D_{ij}$ (cm <sup>2</sup> s <sup>-1</sup> )	Diffusion time, $L^2/D_{ij}$ (s)	$Pe = Lv/D_{ij}$
Oxygen	32	$2 \times 10^{-5}$	5	0.05
Glucose	180	$2 \times 10^{-6}$	50	0.50
Insulin	6,000	$1 \times 10^{-6}$	100	1.0
Antibody	150,000	$6 \times 10^{-7}$	167	1.67

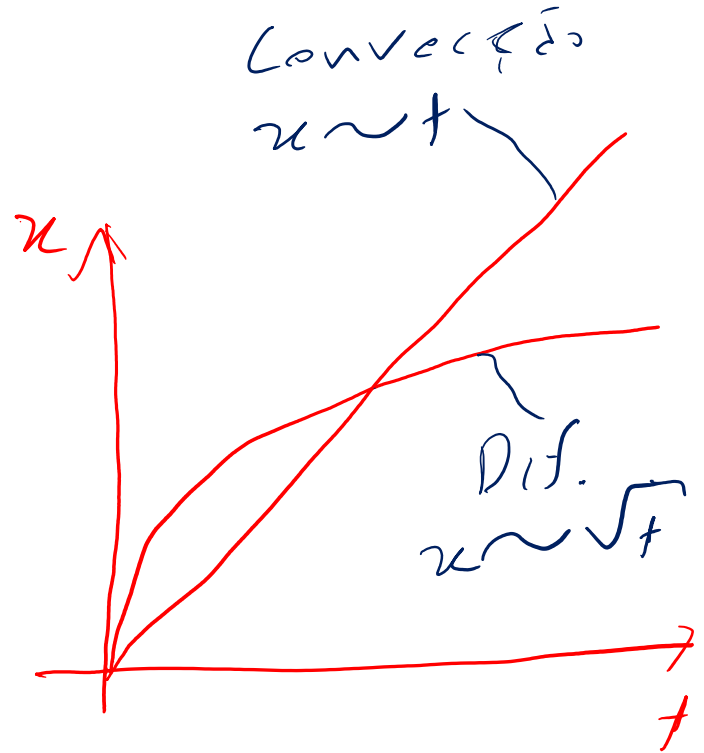
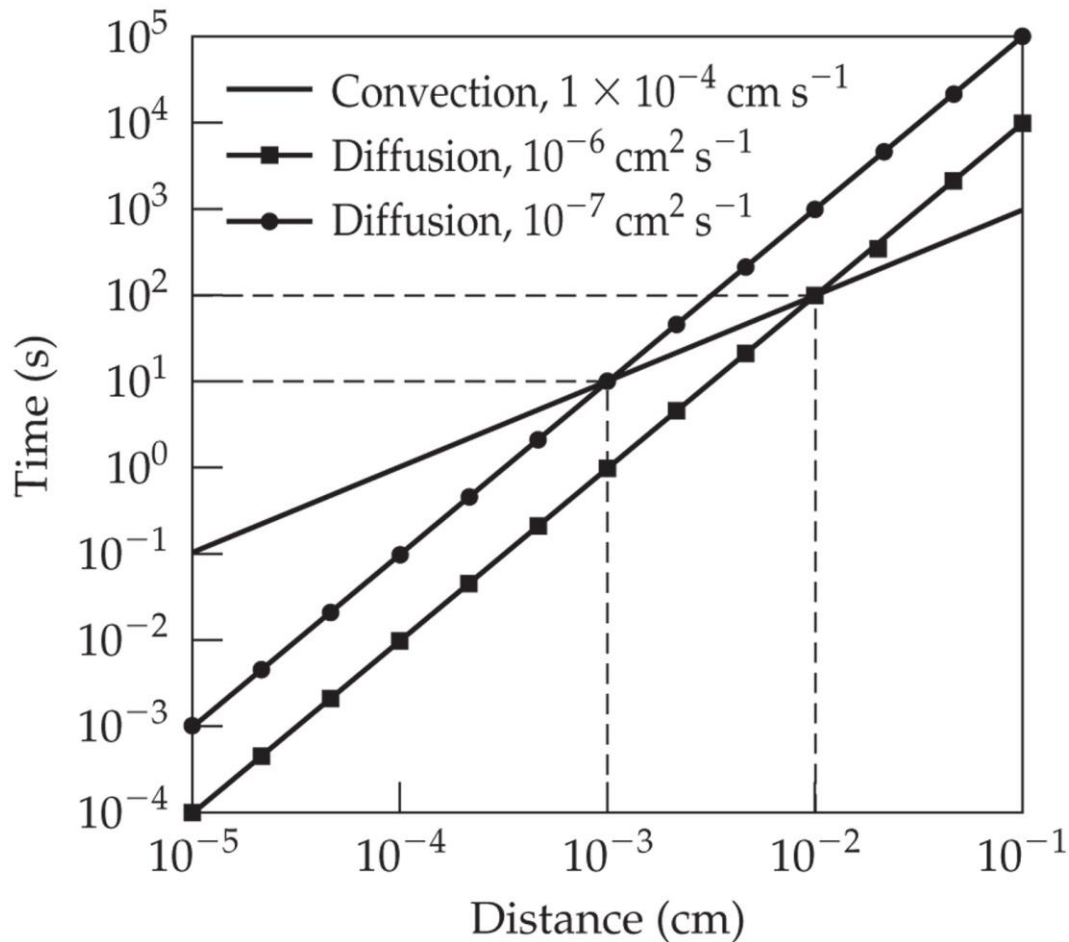
Particle	Diameter	$D_{ij}$ (cm <sup>2</sup> s <sup>-1</sup> )	Diffusion time (s)	Pe
Virus	0.1 $\mu$ m	$5 \times 10^{-8}$	2,000	20
Bacterium	1 $\mu$ m	$5 \times 10^{-9}$	20,000	200
Cell	10 $\mu$ m	$5 \times 10^{-10}$	200,000	2,000

Note: For  $L = 100 \mu\text{m}$ , and if  $v = 1 \mu\text{m s}^{-1}$ , the time for convection is always equal to  $L/v = 100$  s for all molecules and particles.

$Pe > 1 \Rightarrow$  convecção + importante

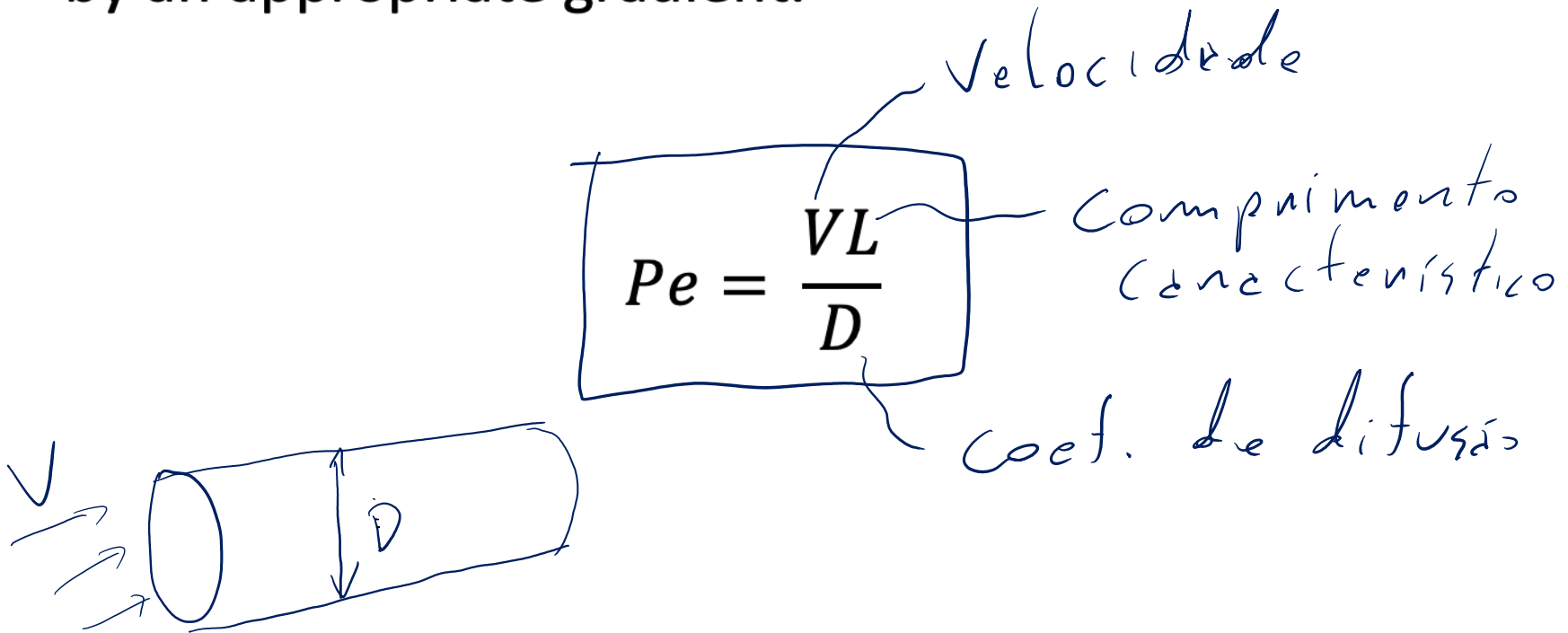
$Pe < 1 \Rightarrow$  difusão " "

# Diffusion and convection times



# Peclet number

The Peclet number is the ratio of the rate of advection of a physical quantity by the flow to the rate of diffusion of the same quantity driven by an appropriate gradient.



# Reynolds number

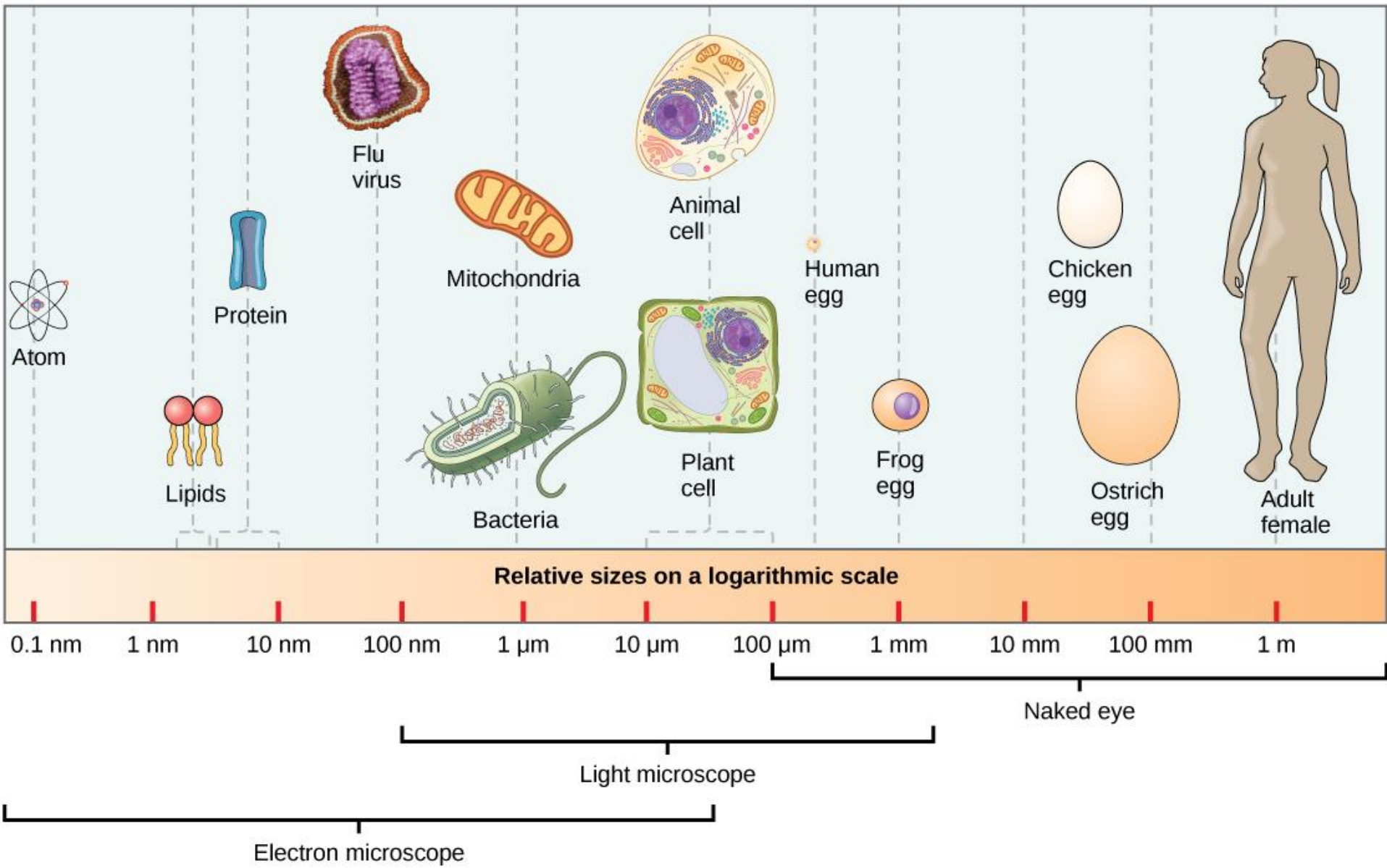
The Reynolds number is the ratio of inertial forces to viscous forces within a fluid which is subjected to relative internal movement due to different fluid velocities.

$$Re = \frac{\rho V L}{\eta}$$

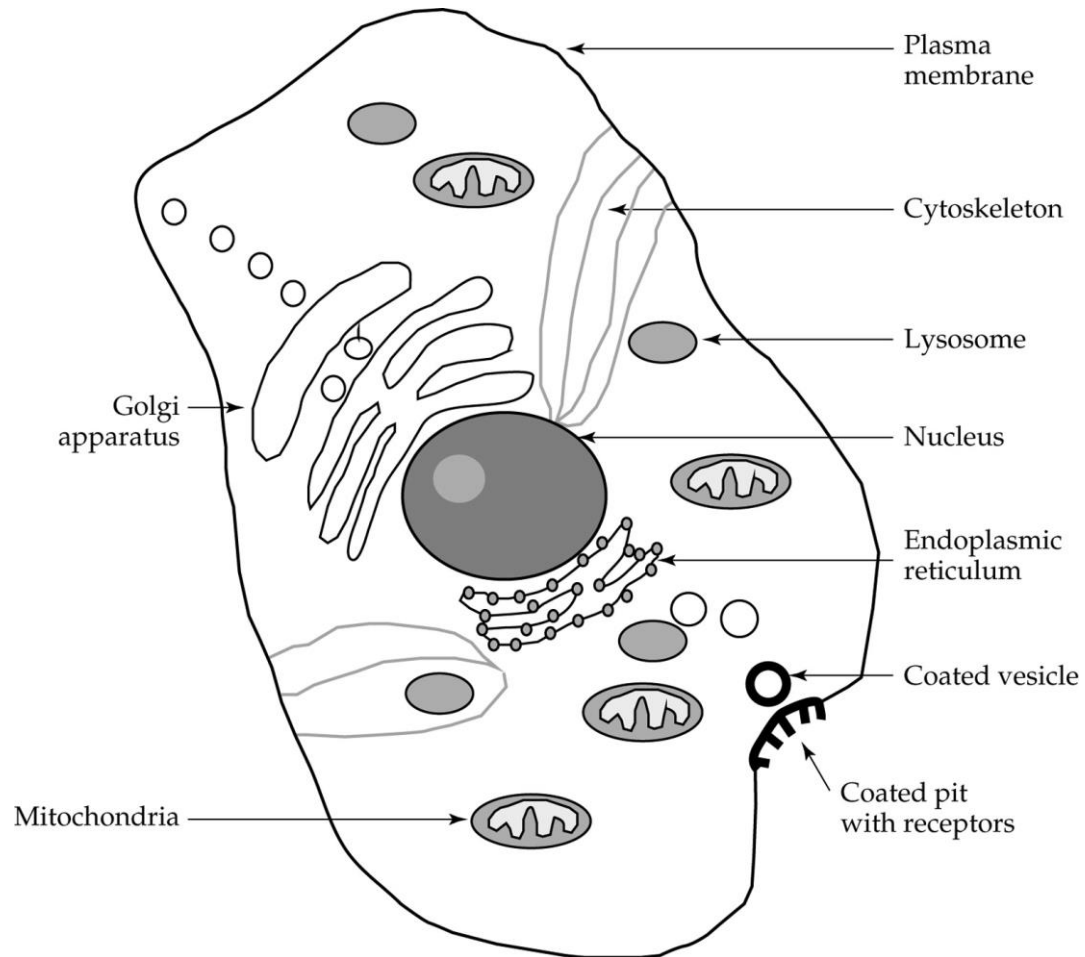
$$\nu = \frac{\mu}{\rho}$$

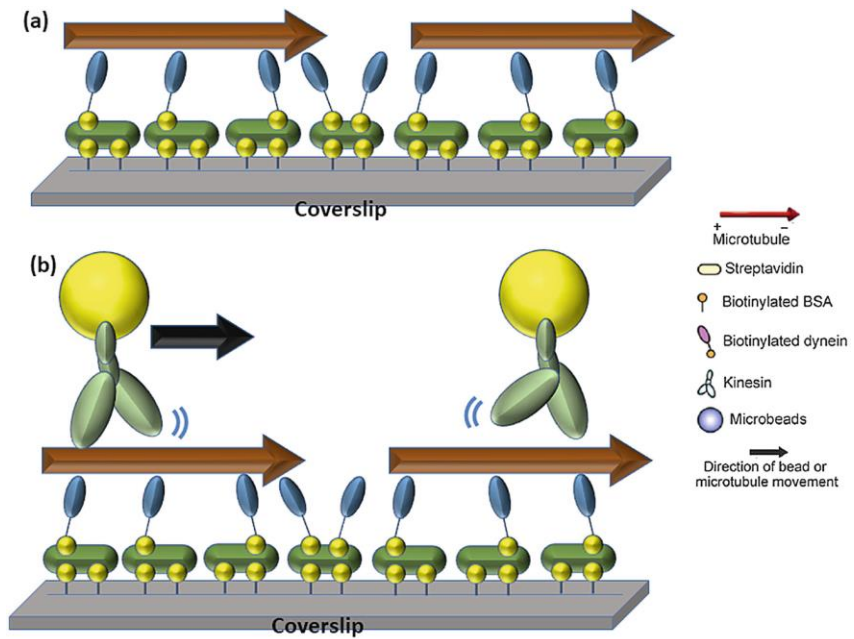


# Scales of living systems

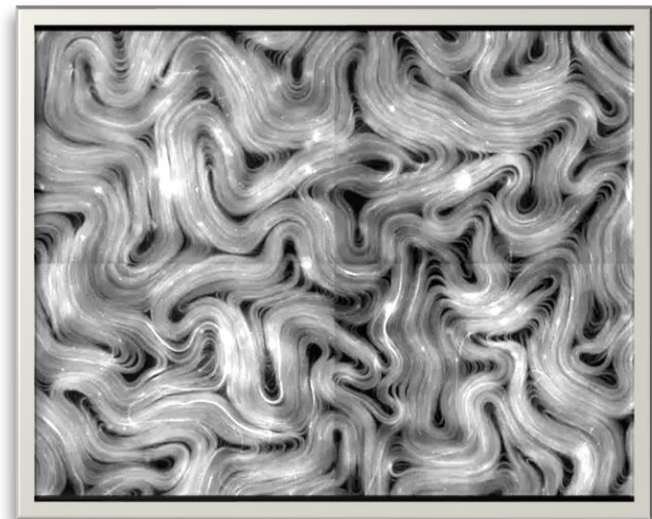


# Mammalian eukaryotic cell & organelles.





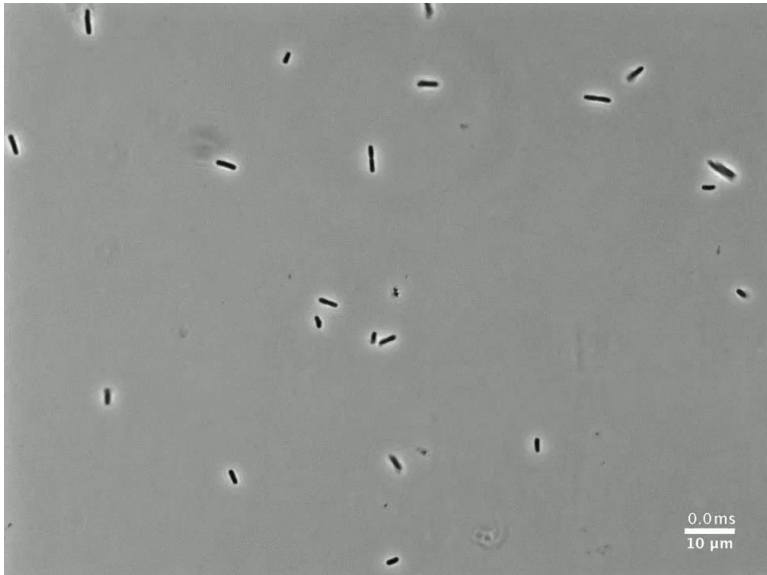
Schematic diagram of Molecular motors



Sanchez et. Al., Nature **491**, 431–434 (2012)

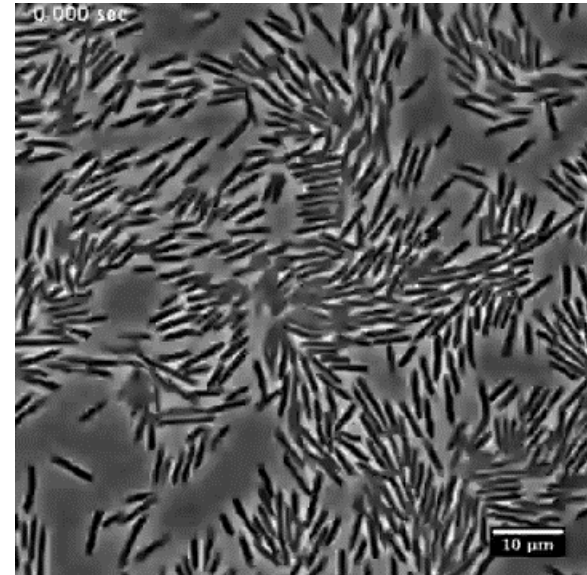


Swimming E. coli



Individual motion

Swarming E. coli

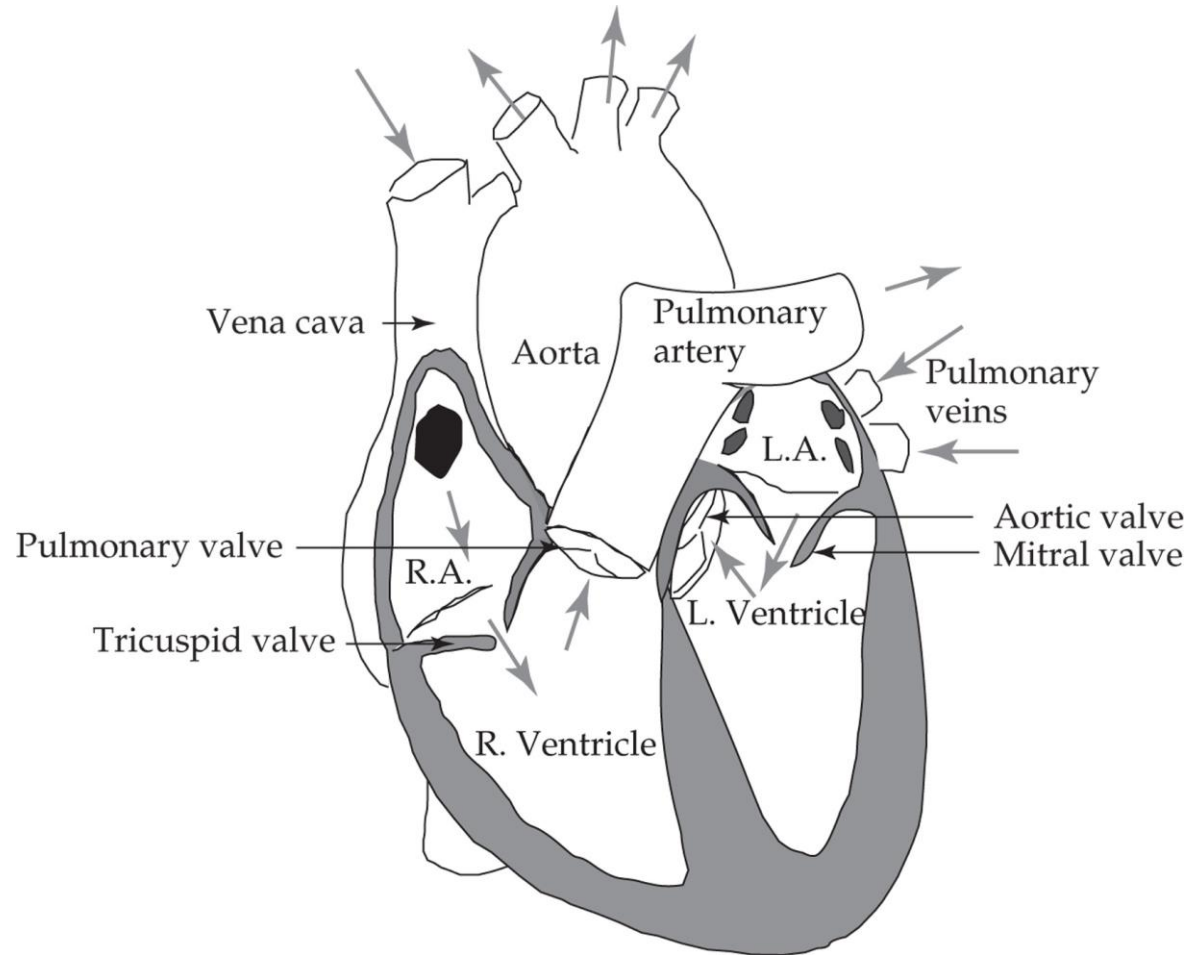


Collective motion

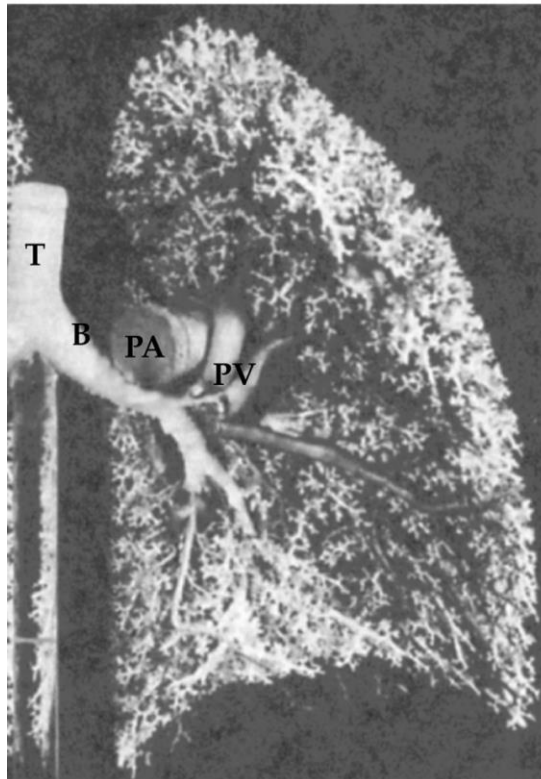
## Examples of Organs and Organ Systems with Transport Functions

Organ or organ system	Transport functions
Respiratory system	Delivery of oxygen from the lungs to the blood and transport of carbon dioxide in the opposite direction
Cardiovascular system	Transport of oxygen within red blood cells Removal of carbon dioxide Delivery of antibodies and cells of immune system to sites of infection Thrombosis and hemostasis
Gastrointestinal tract	Digestion and absorption of nutrients
Liver	Carbohydrate storage and release Cholesterol metabolism and lipoprotein synthesis and metabolism Synthesis of plasma and transport proteins (e.g., albumin, transferrin) Synthesis and export of molecules for tissue energy metabolism Urea synthesis
Kidneys	Metabolism of toxins Filtration of plasma Removal of urea and waste products Water reabsorption Maintenance of plasma volume and blood pH

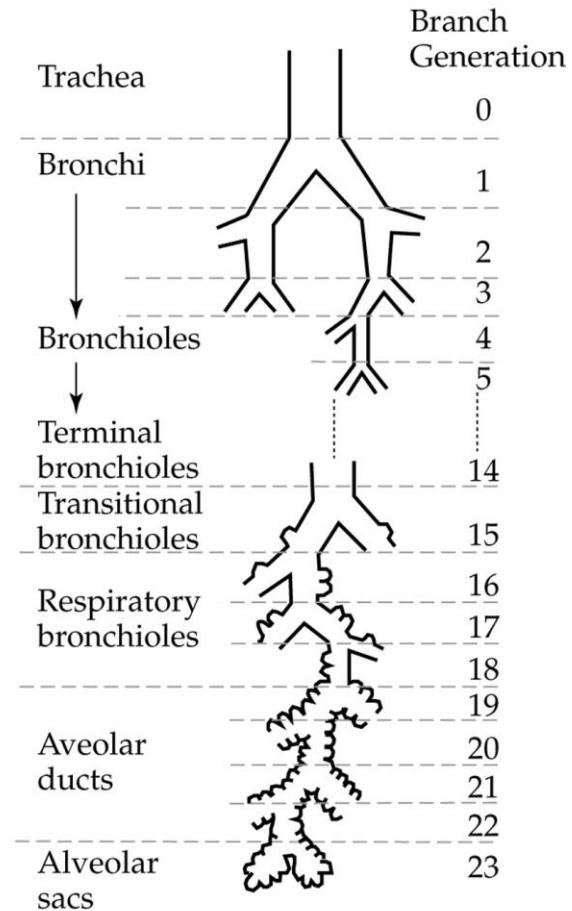
# Diagram of the heart showing valves, veins and arteries



# Schematic of the airways in the lung



(a)



(b)