Ciências ULisboa

Fluid Kinematics

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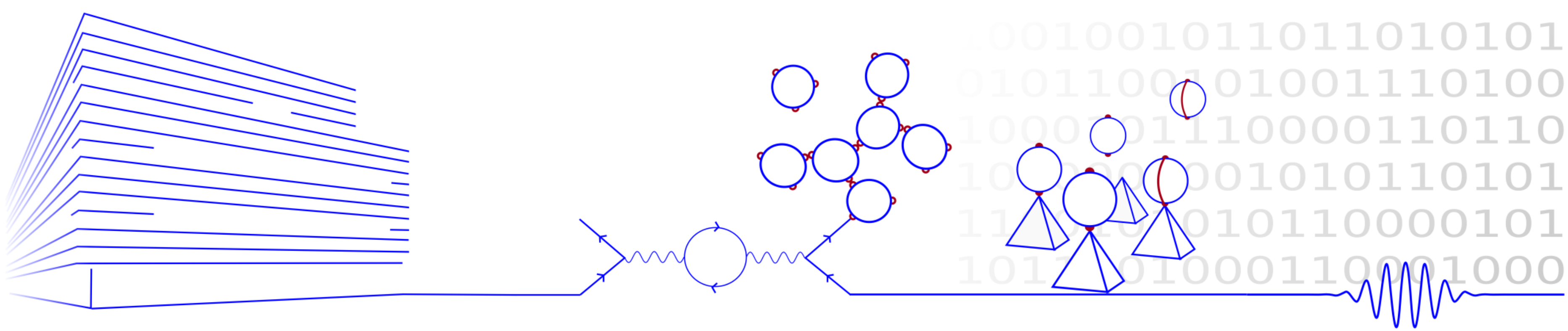


Overview

- Fluid Kinematics deals with the motion of fluids without considering the forces and moments which create the motion.

- Reference: (Chap. 4) Fluid Mechanics: Fundamentals and Applications, by Çengel & Cimbala, McGraw-Hill series in mechanical engineering.

What is a fluid ?

Tension (or stress): Force per unit area

- Normal tension: perpendicular to the surface
- Shear tension: parallel to the surface

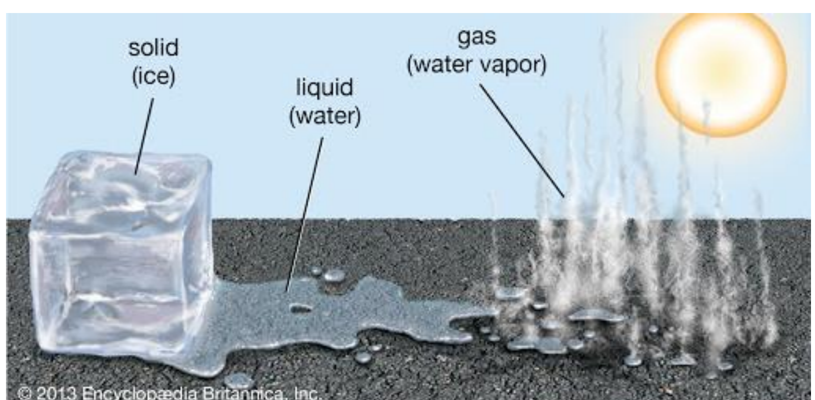
cisalhamento ou corte

Materials respond differently to shear stresses:

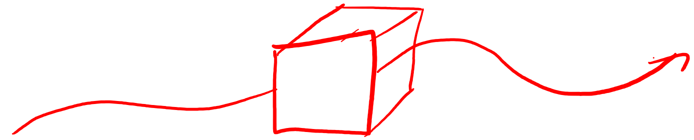
- Solids deform non-permanently
- Plastics deform permanently
- Fluids do not resist: they flow

In a fluid at mechanical equilibrium the shear stresses are ZERO.

A fluid may be a gas or a liquid



Lagrangian Description



- Lagrangian description of fluid flow tracks the position and velocity of individual particles.

- Based upon Newton's laws of motion.

- Difficult to use for practical flow analysis.
  - Fluids are composed of *billions* of molecules.
  - Interaction between molecules hard to describe/model.

- However, useful for specialized applications
  - Sprays, particles, bubble dynamics, rarefied gases.
  - Coupled Eulerian-Lagrangian methods.

- Named after Italian mathematician Joseph Louis Lagrange (1736-1813).

Eulerian Description

- Eulerian description of fluid flow: a **flow domain** or **control volume** is defined by which fluid flows in and out.

$\rho(t)$, $\vec{v}(t)$

$\vec{x} = c_{tr}$

- We define **field variables** which are functions of space and time.
    - Pressure field, P=P(x,y,z,t)
    - Velocity field, $\vec{V} = \vec{V}(x,y,z,t)$

$$\vec{V} = u(x,y,z,t)\vec{i} + v(x,y,z,t)\vec{j} + w(x,y,z,t)\vec{k}$$

$$\left\{\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}\right.$$

    - Acceleration field,

$$\vec{a} = \vec{a}(x,y,z,t)$$

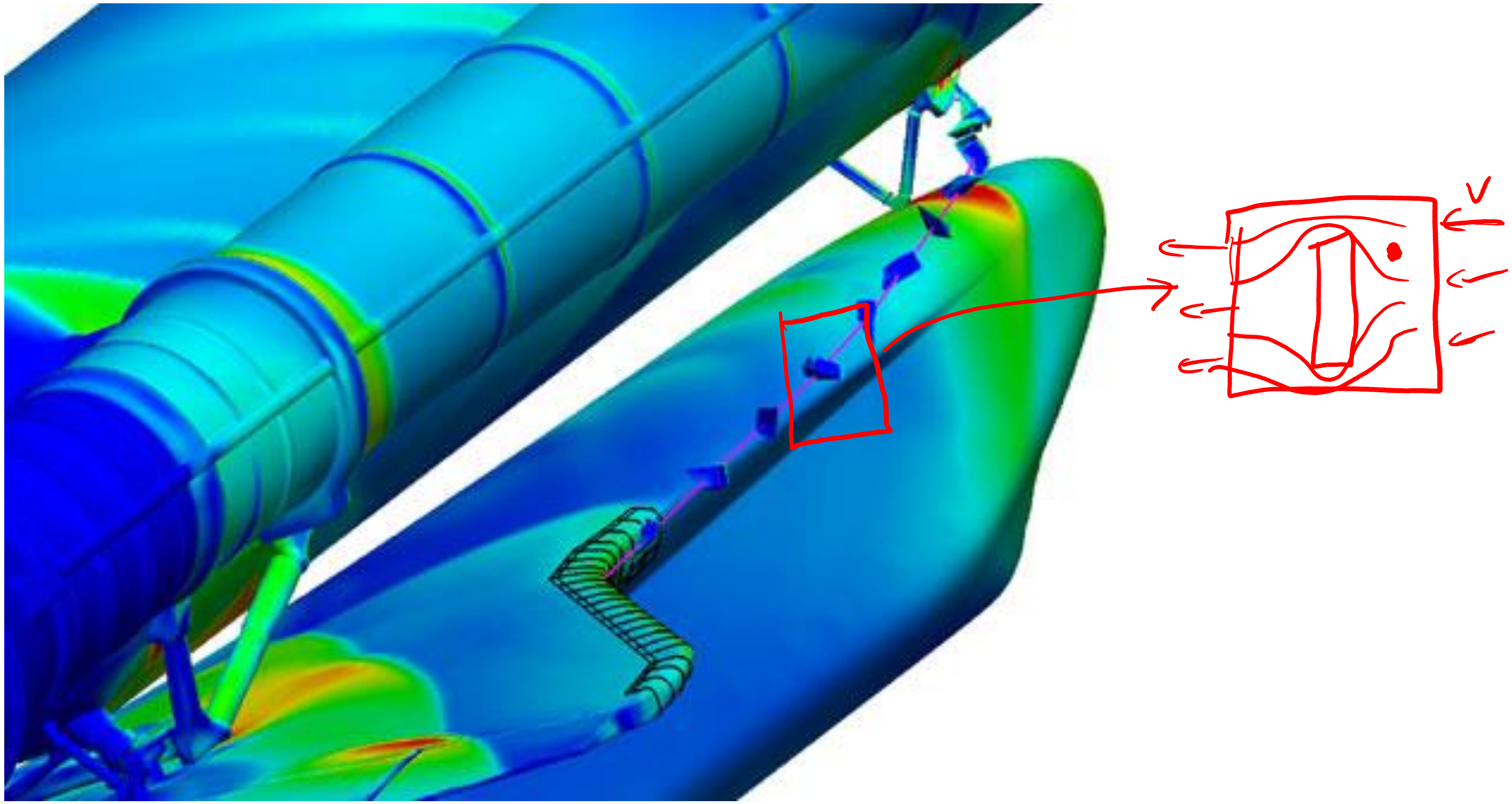
$$\vec{a} = a_x(x,y,z,t)\vec{i} + a_y(x,y,z,t)\vec{j} + a_z(x,y,z,t)\vec{k}$$

    - These (and other) field variables define the **flow field**.

- Well suited for formulation of initial boundary-value problems (PDE's).

- Named after Swiss mathematician Leonhard Euler (1707-1783).

Example: Coupled Eulerian-Lagrangian Method



Forensic analysis of Columbia accident: simulation of shuttle debris trajectory using Eulerian CFD for flow field and Lagrangian method for the debris.

Acceleration Field

- Consider a fluid particle and Newton's second law,

$$\vec{F}_{particle} = m_{particle}\vec{a}_{particle}$$

- The acceleration of the particle is the time derivative of the particle's velocity.

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt}$$

- However, particle velocity at a point is the same as the fluid velocity,

$$\vec{V}_{particle} = \vec{V}\left(x_{particle}(t), y_{particle}(t), z_{particle}(t)\right\}, t)$$

- To take the time derivative of $V_{particle}$ the chain rule must be used.

$$a = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t}\overbrace{\frac{dt}{dt}}^{=1} + \frac{\partial \vec{V}}{\partial x}\overbrace{\frac{dx}{dt}}^{=u} + \frac{\partial \vec{V}}{\partial y}\overbrace{\frac{dy}{dt}}^{=v} + \frac{\partial \vec{V}}{\partial z}\overbrace{\frac{dz}{dt}}^{=w}$$

Acceleration Field

$$a = \frac{d\vec{V}}{dt} = \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{local}} + \underbrace{\vec{V} \cdot \nabla \vec{V}}_{\text{Termo convectivo}}$$

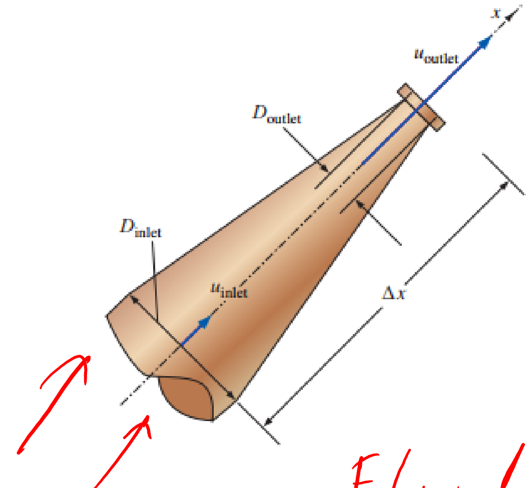
Derivada Material

$$\boxed{\frac{D}{Dt} = \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \nabla}$$

Acceleration Field

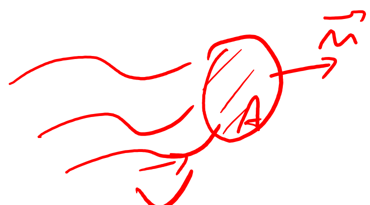
EXAMPLE 4–2 Acceleration of a Fluid Particle through a Nozzle

Nadeen is washing her car, using a nozzle similar to the one sketched in Fig. 4–8. The nozzle is 3.90 in (0.325 ft) long, with an inlet diameter of 0.420 in (0.0350 ft) and an outlet diameter of 0.182 in (see Fig. 4–9). The volume flow rate through the garden hose (and through the nozzle) is $\dot{V}$ = 0.841 gal/min (0.00187 ft^3/s), and the flow is steady. Estimate the magnitude of the acceleration of a fluid particle moving down the centerline of the nozzle.



Taxa de escoamento

$$Q = \int \vec{V} \cdot d\vec{A}$$



Fluido incompressível

Escoamento estacionário: $\frac{\partial \vec{V}}{\partial t} = 0$

$$Q_{inlet} = Q_{outlet}$$

$$\boxed{V_{in} \cdot A_{in} = V_{out} \cdot A_{out}}$$