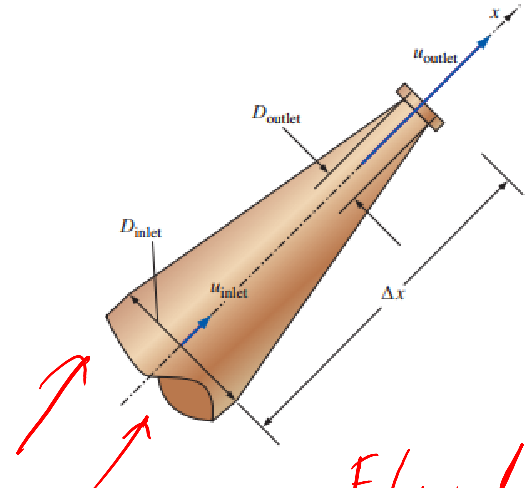
Acceleration Field

EXAMPLE 4–2 **Acceleration of a Fluid Particle through a Nozzle**

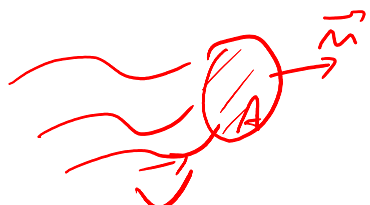
Nadeen is washing her car, using a nozzle similar to the one sketched in Fig. 4–8. The nozzle is 3.90 in (0.325 ft) long, with an inlet diameter of 0.420 in (0.0350 ft) and an outlet diameter of 0.182 in (see Fig. 4–9). The volume flow rate through the garden hose (and through the nozzle) is $\dot{V}$ = 0.841 gal/min (0.00187 ft³/s), and the flow is steady. Estimate the magnitude of the acceleration of a fluid particle moving down the centerline of the nozzle.



Fluido incompressível

Taxa de escoamento

$$Q = \int \vec{V} \cdot d\vec{A}$$



Escoamento estacionário: $\frac{\partial \vec{V}}{\partial t} = 0$

$$Q_{inlet} = Q_{outlet}$$

$$\boxed{V_{in} \cdot A_{in} = V_{out} \cdot A_{out}}$$

Acceleration Field

$$Q = V \cdot A \Rightarrow V_{in} = \frac{Q}{A_{in}} , \quad V_{out} = \frac{Q}{A_{out}}$$

$$\vec{a} = \cancel{\frac{\partial \vec{V}}{\partial t}}^{\,0} + \vec{V} \cdot \nabla \vec{V} = u \cdot \frac{\partial \vec{V}}{\partial x} + \underbrace{\cancel{v}}_{=0} \cdot \frac{\partial \vec{V}}{\partial y} + \underbrace{\cancel{w}}_{=0} \cdot \frac{\partial \vec{V}}{\partial z}$$

$$\frac{\partial \vec{V}}{\partial x} = u\hat{x}$$

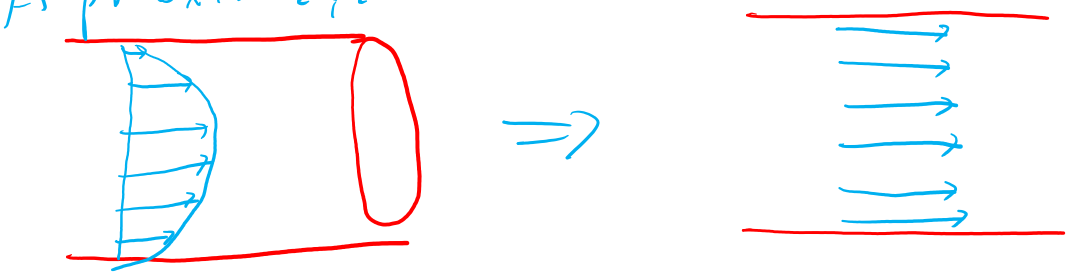
$$= u \frac{\partial u}{\partial x} \cong \langle u \rangle \cdot \frac{\Delta u}{\Delta x} =$$

$$= \frac{u_{in} + u_{out}}{2} \cdot \frac{u_{out} - u_{in}}{\Delta x} = \boxed{\frac{u_{out}^2 - u_{in}^2}{2 \Delta x}}$$

$$= 160 \text{ ft/s}^2$$

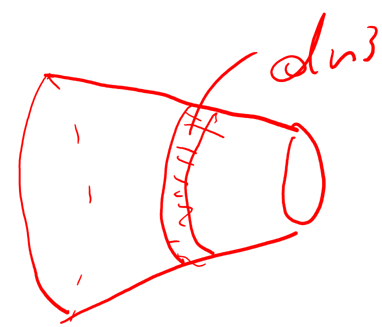
Acceleration Field

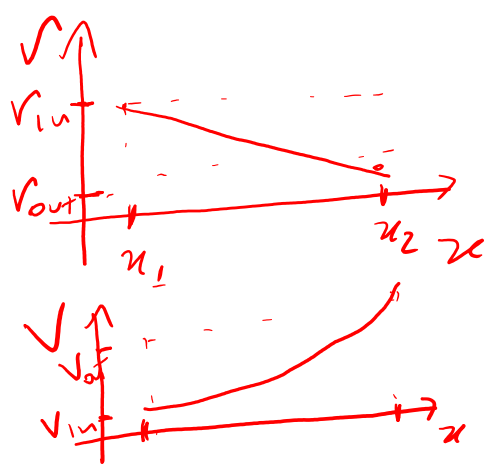
Aproximação 1



" 2

$$\vec{a} = \frac{1}{L^3}\int u(x)\cdot\frac{\partial u}{\partial x}(x)\, dr^3$$





$$r \sim \frac{1}{x}$$

$$V(x) = \frac{Q}{A(x)} = \frac{Q}{\pi r^2(x)} \sim \frac{1}{r^2} \sim x^2$$

Material Derivative

- The total derivative operator is called the **material derivative** and is often given special notation, D/Dt.

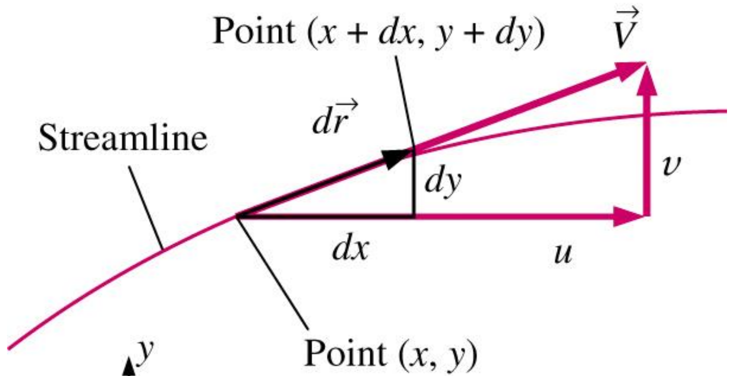
$$\boxed{\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \nabla}$$

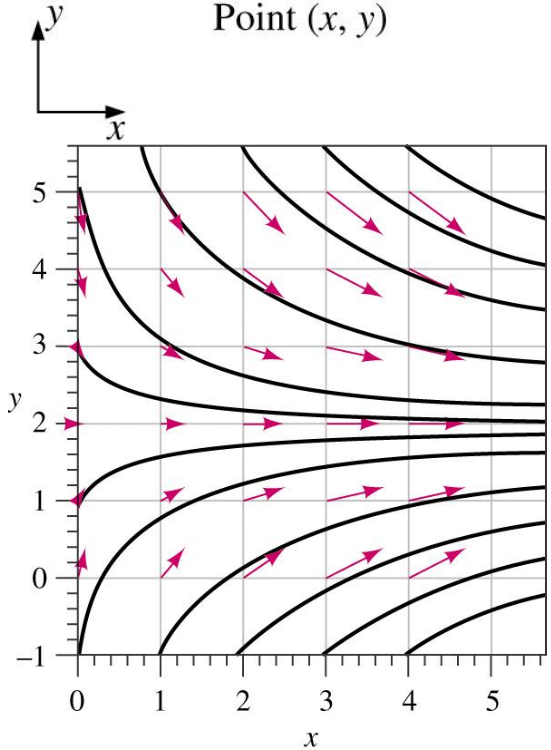
- Advective acceleration is nonlinear: source of many phenomena and primary challenge in solving fluid flow problems.
- Provides ``transformation'' between Lagrangian and Eulerian frames.
- Other names for the material derivative include: **total, particle,** and **substantial** derivative.

Flow Visualization

- Flow visualization is the visual examination of flow-field features.
- Important for both physical experiments and numerical (CFD) solutions.
- Numerous methods
  - Streamlines and streamtubes
  - Pathlines
  - Streaklines
  - Timelines
  - Refractive techniques
  - Surface flow techniques

Streamlines





- A **Streamline** is a curve that is everywhere tangent to the *instantaneous* local velocity vector.
- Consider an arc length

$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

- $d\vec{r}$ must be parallel to the local velocity vector

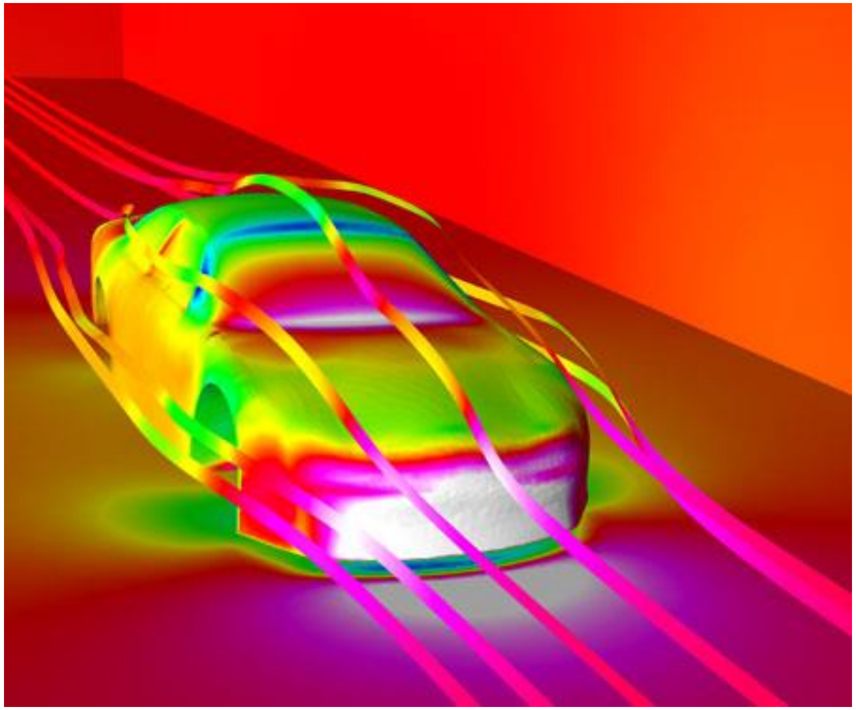
$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

- Geometric arguments results in the equation for a streamline

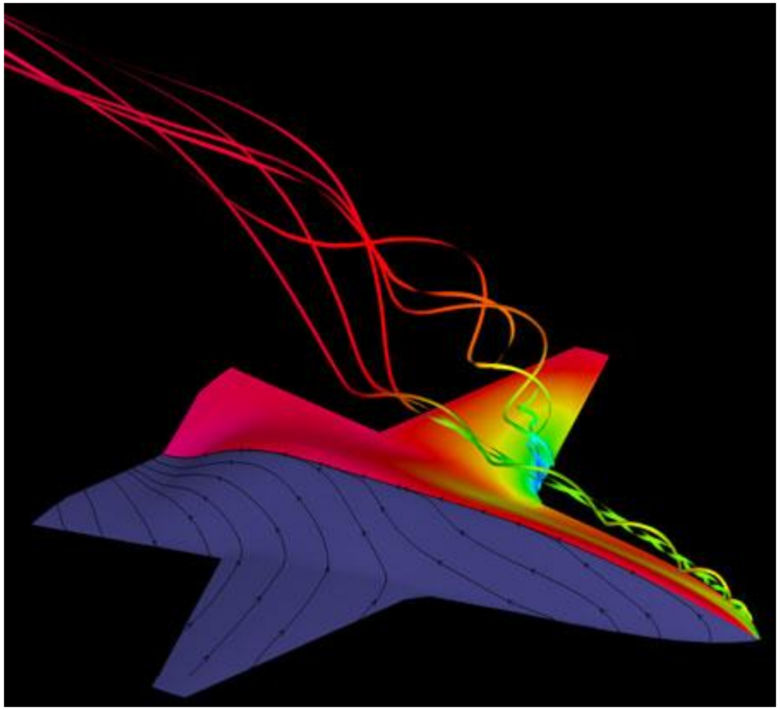
$$\boxed{\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

Streamlines

NASCAR surface pressure contours and streamlines



Airplane surface pressure contours, volume streamlines, and surface streamlines



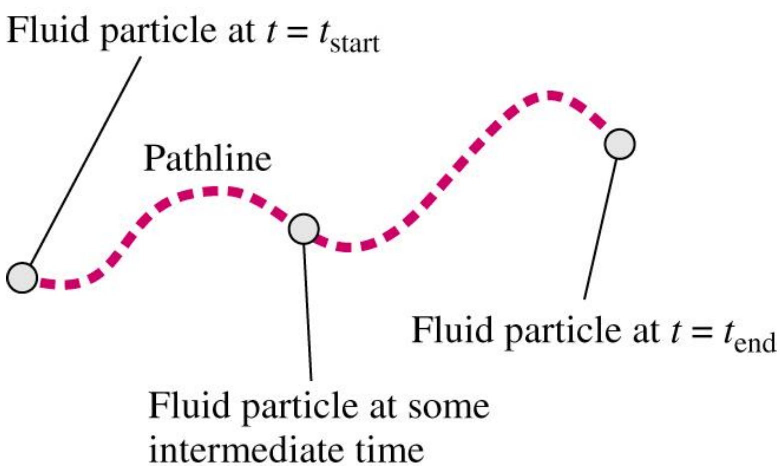
Calculate the **streamlines** for the following velocity field: $v_x = sin(t)\ and\ v_y = 1$

$$\frac{dx}{u} = \frac{dy}{v} \iff \frac{dx}{\sin t} = \frac{dy}{1}$$

$$\int_{x_0}^{x} dx = \int_{y_0}^{y} \sin t \, dy \iff x - x_0 = (y - y_0) \sin t$$

$$\boxed{x = x_0 + (y - y_0) \sin t}$$

Pathlines



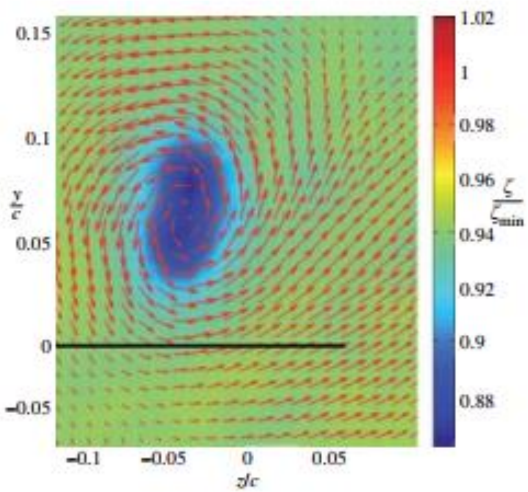


Photo by Michael H. Krane, ARL-Penn State.

- A **Pathline** is the actual path traveled by an individual fluid particle over some time period.

- Same as the fluid particle's material position vector

$$\left(x_{particle}(t), y_{particle}(t), z_{particle}(t)\right)$$

- Particle location at time t:

$$\vec{x} = \vec{x}_{start} + \int_{t_{start}}^{t} \vec{V} dt$$

- Particle Image Velocimetry (PIV) is a modern experimental technique to measure velocity field over a plane in the flow field.

Calculate the **pathlines** for the following velocity field: $v_x = sin(t)\ and\ v_y = 1$

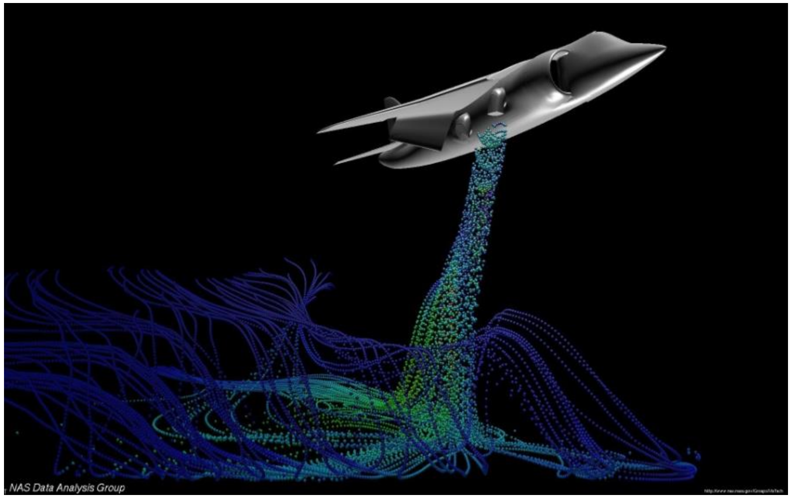
$$v_x = \frac{dx}{dt} = \sin t \Rightarrow \int_{x_0}^{x} dx = \int_0^t \sin t'\, dt'$$

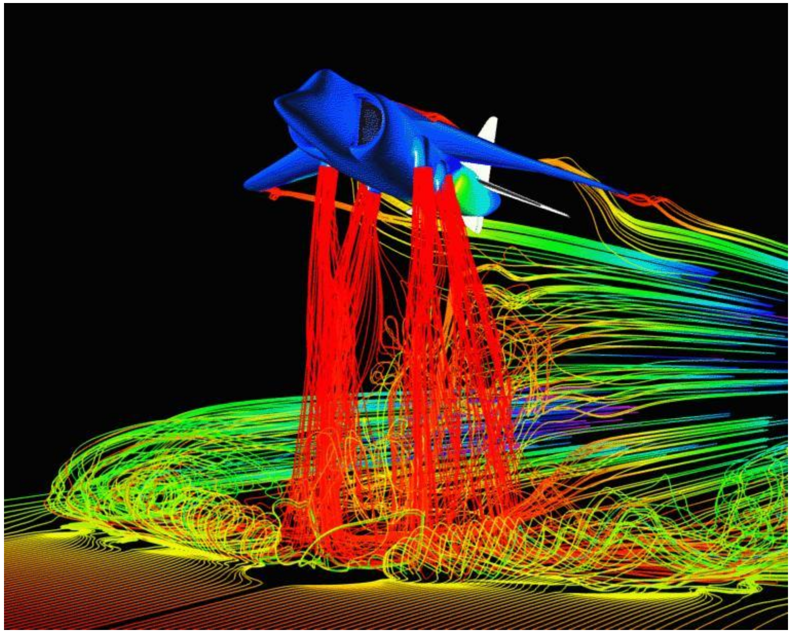
$$x = x_0 - \cos t' \Big|_0^t = x_0 + 1 - \cos t$$

$$v_y = \frac{dy}{dt} = 1 \Rightarrow y = y_0 + t \Rightarrow t = y - y_0$$

$$\boxed{x = x_0 + 1 - \cos(y - y_0)}$$

Streaklines

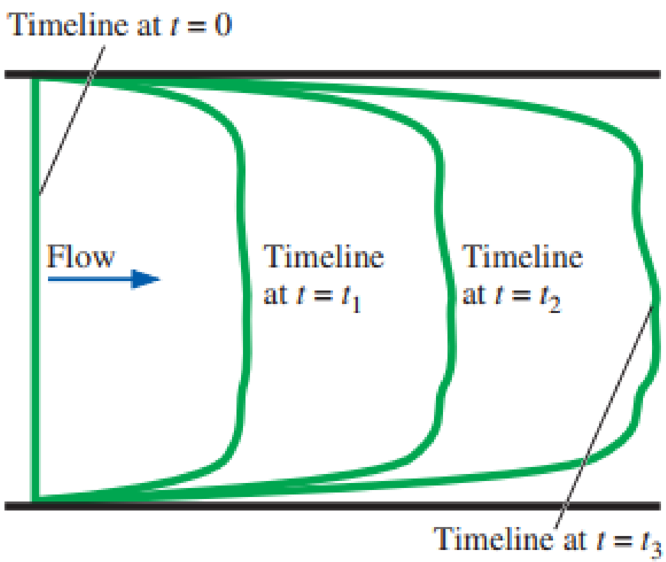


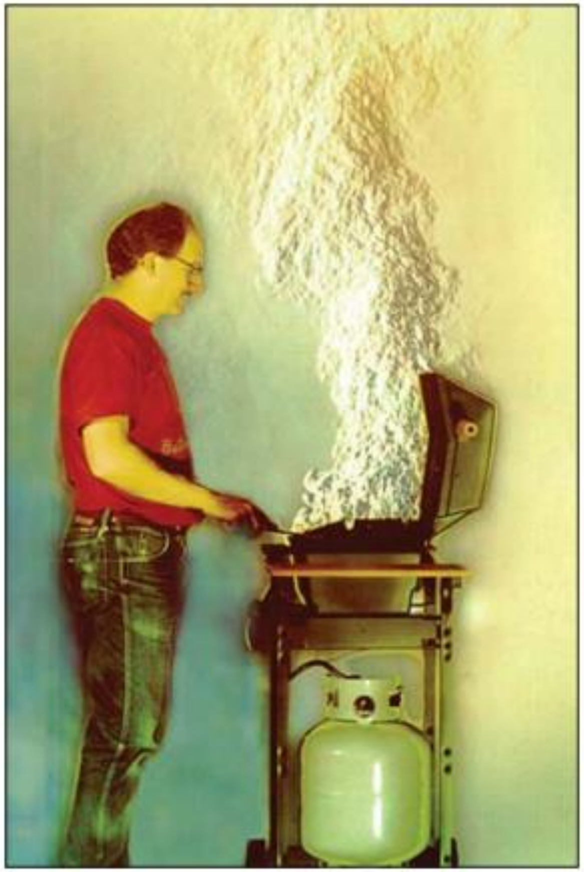


- A **Streakline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.
- Easy to generate in experiments: dye in a water flow, or smoke in an airflow.

Refractive Flow Visualization Techniques

Timelines: A timeline is a set of adjacent fluid particles that were marked at the same (earlier) instant in time.





Comparisons

- For steady flow, streamlines, pathlines, and streaklines are identical.

- For unsteady flow, they can be very different.
  - Streamlines are an instantaneous picture of the flow field.
  - Pathlines and Streaklines are flow patterns that have a time history associated with them.
  - Streakline: instantaneous snapshot of a time-integrated flow pattern.
  - Pathline: time-exposed flow path of an individual particle.

Flow rate

- The **volumetric flow rate** is the volume of fluid which passes per unit time; usually it is represented by the symbol *Q.*

$$Q = \int \vec{V} \cdot \vec{n} dA$$