Hydrology in Practice

Third edition

Elizabeth M.Shaw
Formerly of the Department of Civil Engineering
Imperial College of Science, Technology and Medicine

First published in 1983 by Chapman & Hall
This edition published in the Taylor & Francis e-Library, 2005.
"To purchase your own copy of this or any of Taylor & Francis or Routledge's collection of thousands of eBooks please go to http://www.ebookstore.tandf.co.uk/.

Second edition 1988
Third edition 1994

A catalogue record for this book is available from the British Library

ISBN 0-203-01325-5 Master e-book ISBN

ISBN 0 7487 4448 7 (Print Edition)

Page make-up by Colset Private Ltd

Contents

13
Rainfall-Runoff Relationships

The derivation of relationships between the rainfall over a catchment area and the resulting flow in a river is a fundamental problem for the hydrologist. In most countries, there are usually plenty of rainfall records, but the more elaborate and expensive streamflow measurements, which are what the engineer needs for the assessment of water resources or of damaging flood peaks, are often limited and are rarely available for a specific river under investigation. Evaluating river discharges from rainfall has stimulated the imagination and ingenuity of engineers for many years, and more recently has been the inspiration of many research workers.

To facilitate comparisons it is usual to express values for rainfall and river discharge in similar terms. The amount of precipitation (rain, snow, etc.) falling on a catchment area is normally expressed in millimetres (mm) depth, but may be converted into a total volume of water, cubic metres (m^3) falling on the catchment. Alternatively, the river discharge (flow rate), measured in cubic metres per second (m^3s^{-1} or cumecs) for a comparable time period may be converted into total volume (m^3) and expressed as an equivalent depth of water (in mm) over the catchment area. The discharge, often termed *runoff* for the defined period of time, is then easily compared with rainfall depths over the same time period.

Estimating runoff or discharge from rainfall measurements is very much dependent on the timescale being considered. For short durations (hours) the complex interrelationship between rainfall and runoff is not easily defined, but as the time period lengthens, the connection becomes simpler until, on an annual basis, a straight-line correlation may be obtained. The time interval used in the measurement of the two variables affects the derivation of any relationship, although with continuously recorded rainfall and stream discharge this constraint is removed and only the purpose of the study influences choice of time interval. Hence, relating a flood peak to a heavy storm requires continuous records, but determining water yield from a catchment can be accomplished satisfactorily using relationships between totals of monthly or annual rainfall and runoff.

Naturally, the size of the area being considered also affects the relationship. For very small areas of a homogeneous nature—a stretch of motorway, say—the derivation of the relationship could be fairly simple; for very large drainage basins on a national or even international scale (the River Danube, for example) and for long time periods, differences in catchment effects are smoothed out giving relatively simple rainfall-runoff relationships. However, in general and for short time periods, great complexities occur when spasmodic rainfall is unevenly distributed over an area of varied topography and geological composition. For catchments with many different surface characteristics affected by a single severe storm—say catchments up to about 200–300 km^2 in equable climates—the direct relationship between specific rainfalls and the resulting discharge or runoff is extremely complicated.

In the intermediate scale of both area and time, other physical and hydrological factors, such as evaporation, infiltration and groundwater flow are very significant, and thus any direct relationship between rainfall alone and runoff is not easily determined. The derivation of runoff from rainfall plus other measurable variables will be considered in the next chapter, but here the simpler methods relating rainfall alone to runoff will be given. Many of these methods have been derived by engineers for immediate practical use.

13.1 Rational Method

The first concern of a civil engineer engaged upon construction work on or near a river is to gain some idea of the flow regime of the river throughout the proposed life of the project. Briefly, this usually means that the designer wants to know the flood levels and the chance of occurrence of major flood discharges. Hence, the hydrologist may be called upon to estimate likely peak flows of the river at the point in question. In former times, the river engineer made all his own calculations using any local information he could lay hands on. If he could not find any records for a river gauging station, he looked for historical evidence of flood peaks and searched the records for rainfall observations.

The concept of the rational method for determining flood peak discharges from measurements of rainfall depths owes its origins to Mulvaney, an Irish engineer who was concerned with land drainage (Mulvaney, 1850). Some Americans attribute first mention of the formula to one of their engineers engaged upon sewer design (Kuichling, 1889). The use of the rational formula is sometimes referred to as the Lloyd-Davies method, since it was applied also to sewer design calculations in England (Lloyd-Davies, 1906). The formula to give the peak flow Q_p is:

$$Q_p=CiA \tag{13.1}$$

where C is the coefficient of runoff (dependent on catchment characteristics), i is the intensity of rainfall in time T_c and A is the area of catchment.

T_c is the *time of concentration,* the time required for rain falling at the farthest point of the catchment to flow to the measuring point of the river. Thus, after time T_c from the commencement of rain, the whole of the catchment is taken to be contributing to the flow. The value of i, the mean intensity, assumes that the rate of rainfall is constant during T_c, and that all the measured rainfall over the area contributes to the flow. The peak flow Q_p occurs after the period T_c.

The rational formula was devised in Imperial units. Thus, with i in in h^{-1} and A in acres, the value of Q_p is approximately in cusecs (ft^3s^{-1}). In metric units, with i in mm h^{-1} and A in km^2, the time conversion needs a factor of 0.278 to give the Q_p in cumecs (m^3s^{-1}). Thus:

$$Q_p(m^3s^{-1})=0.278C\, i(\text{mm h}^{-1})A(\text{km}^2) \tag{13.2}$$

Values of C vary from 0.05 for flat sandy areas to 0.95 for impervious urban surfaces, and considerable knowledge of the catchment is needed in order to estimate an acceptable value. The coefficient of runoff also varies for different storms on the same catchment, and thus, using an average value for *C*, only a crude estimate of Q_p is obtained, that may have wide margins of error. However, used with discretion, the formula can provide a rough first value, especially in small uniform urban areas. It has been used for many years as a basis for engineering design for small land drainage schemes and storm-water channels. However when the rational method leads to over design, modern engineers need to proceed to more precise and reliable methods in order to optimize designs and thereby reduce the construction costs of major schemes.

13.2 Time-Area Method

The time-area method of obtaining runoff or discharge from rainfall can be considered as an extension and improvement of the rational method. The

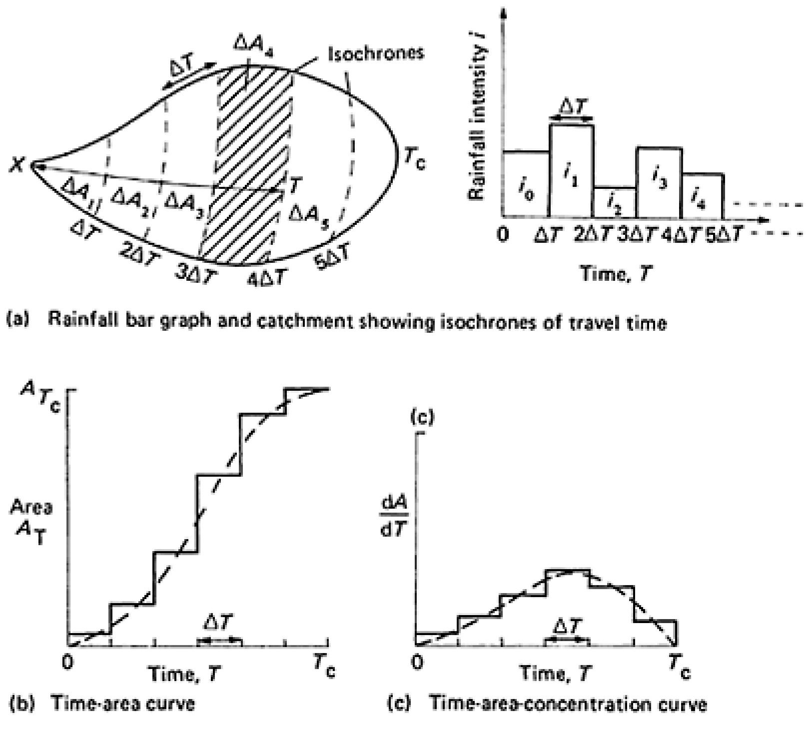

Fig. 13.1 Time-area method.

peak discharge Q_p is the sum of flow-contributions from subdivisions of the catchment defined by time contours (called *isochrones*), which are lines of equal flow-time to the river section where Q_p is required. The method is illustrated in Fig. 13.1(a). The flow from each contributing area bounded by two isochrones ($T-\Delta T, T$) is obtained from the product of the mean intensity of effective rainfall *(i)* from time $T-\Delta T$ to time T and the area (ΔA). Thus Q_4, the flow at X at time *4* h is given by:

$Q_4=i_3\Delta A_1+i_2\Delta A_2+i_1\Delta A_3+i_0\Delta A_4$

i.e:

$$Q_T = \sum_{k=1}^{T} i_{(T-k)}\Delta A_{(k)} \qquad (13.3)$$

As before, the whole catchment is taken to be contributing to the flow after T equals T_c.

Using the above nomenclature it is seen that the peak flow at X when the whole catchment is contributing to the flow, a period T_c after the commencement of rain, is:

$$Q_p = \sum_{k=1}^{n} i_{(n-k)}\Delta A_{(k)} \qquad (13.4)$$

where *n*, the number of incremental areas between successive isochrones, is given by $T_c/\Delta T$, and k is a counter.

The unrealistic assumption made in the rational method of uniform rainfall intensity over the whole catchment and during the whole of T_c is avoided in the time-area method, where the catchment contributions are subdivided in time. The varying intensities within a storm are averaged over discrete periods according to the isochrone time interval selected. Hence, in deriving a flood peak for design purposes, a design storm with a critical sequence of intensities can be used for the maximum intensities applied to the contributing areas of the catchment that have most rapid runoff. However, when such differences within a catchment are considered, there arises difficulty in determining T_c, the time after the commencement of the storm when, by definition, Q_p occurs.

For small natural catchments, a formula derived from data published by Kirpich for agricultural areas could be used to give T_c in hours (Kirpich, 1940):

$$T_c = 0.00025\left(\frac{L}{\sqrt{S}}\right)^{0.80}$$

where L is the length of the catchment along the longest river channel (in m) and S is the overall catchment slope (in m m^{-1}).

There are many similar formulae for T_c, and further considerations of this critical measure will be given in Chapter 18 when urban catchments are discussed.

To fix the isochrones considerable knowledge of the catchment is required, so that the times of overland flow and flow in the river channels may be determined for wet or even saturated conditions, the worst conditions likely to cause a major flood flow. Isochrones for urban areas are more readily obtained by direct observation during storm periods and are, of course, more simply determined for small sewer-drained catchments.

The time-area method for calculating peak flows from rainfall has been elaborated and extended by many engineers and research workers since its inception in the 1920s. Many of the amendments have been related to specific catchment areas or conditions, and are available from the more advanced hydrological textbooks and original published papers (see Dooge, 1973). The method forms the basis for the Transport and Road Research

Laboratory (TRRL) technique for designing storm-water sewers, which will be described in detail in Chapter 18.

The simple discrete form of the time-area concept can be generalized by making ΔT very small and considering increases in the contributing area to be continuous with increasing time. Thus, in Fig. 13.1(b), the plot of catchment area against time is shown as a dashed line and this is known as the time-area curve. Its limits are the total area of the catchment and the time of concentration. For any value of *T*, the corresponding area *A* gives the maximum flow at the river outfall caused by a rainfall of duration *T*. The derivative of the time-area curve shown in Fig. 13.1(c) gives the rate of increase in contributing area with time, and is called the time-area-concentration curve, since the length of the time base is equal to the time of concentration of the catchment.

13.3 Hydrograph Analysis

Before trying to analyse a hydrograph, which describes the whole time history of the changing rate of flow from a catchment due to a rainfall event rather than just the peak flow, it is essential first to appreciate some of its simple components. In Fig. 13.2, rainfall intensity (i, in mm h^{-1}) is shown in discrete block intervals of time *(T)*. The lower continuous curve of discharge (Q, in $m^3 s^{-1}$) is the hydrograph resulting from the event. The discharge hydrograph is obtained from continuously recorded river stages and the stage-discharge relationship (Chapter 6) appropriate to the river gauging station.

The hydrograph of discharge against time has two main components, the area under the hump, labelled *surface runoff* (which is produced by a volume of water derived from the storm event), and the broad band near the time axis, representing *baseflow* contributed from groundwater.

At the beginning of the rainfall, the river level (and hence the discharge) is low and a period of time elapses before the river begins to rise. During this period the rainfall is being intercepted by vegetation or is soaking into the ground and making up soil-moisture deficits. The length of the delay before the river rises depends on the wetness of the catchment before the storm and on the intensity of the rainfall itself.

When the rainfall has made up catchment deficits and when surfaces and soil are saturated, the rain begins to contribute to the stream flow. The proportion of rainfall that finds its way into a river is known as the *effective*

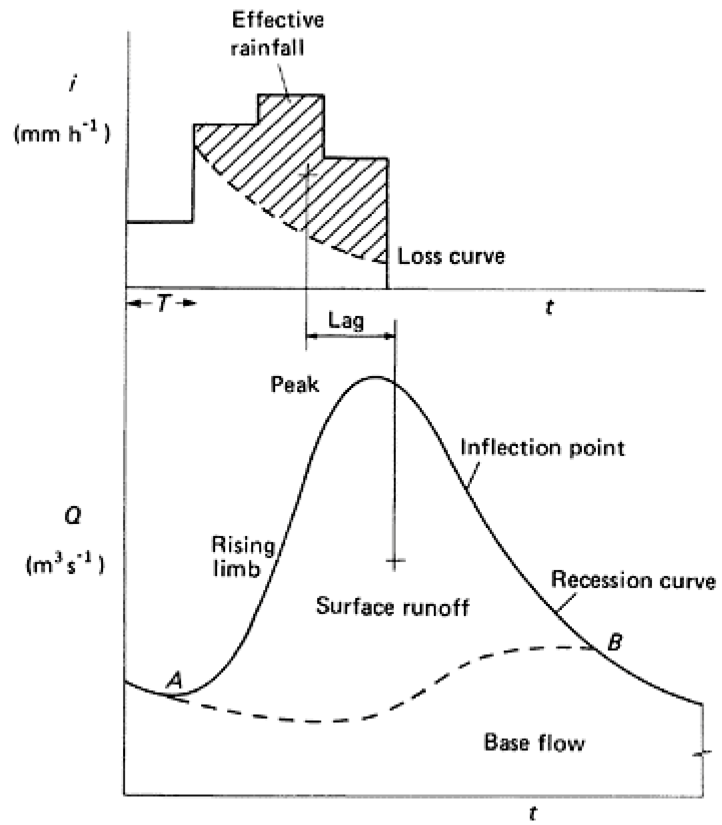

Fig. 13.2 Rainfall and a river hydrograph.

rainfall, the rest being lost (to quick runoff) in evaporation, detention on the surface or retention in the soil. As the storm proceeds, the proportion of effective rainfall increases and that of lost rainfall decreases as shown by the loss curve (Fig. 13.2).

The volume of surface runoff, represented by the area under the hydrograph minus the baseflow, can be considered in two main subdivisions to simplify the complex water movements over the surface and in the ground. The effective rainfall makes the immediate contribution to the rising limb from *A* to the peak of the hydrograph and, even when the rainfall ceases, continues to contribute until the *inflection point* (Fig. 13.2). Beyond this point, it is generally considered that the flow comes from the water temporarily stored in the soil. This so-called *interflow* continues to provide the flow of the *recession curve* until the water from the whole of the effective rainfall is completely depleted at *B* (Fig. 13.2). One final term, lag or lag time requires explanation. There are many definitions of lag, which is a measure of the catchment response time, but here it is taken from the centre of gravity of the effective rainfall to the centre of gravity of the direct surface runoff.

The boundary between surface runoff and baseflow is difficult to define and depends very much on the geological structure and composition of the catchment. Permeable aquifers, such as limestone and sandstone strata, sustain high baseflow contributions, but impervious clays and built-up areas provide little or no baseflow to a river. The baseflow levels are also affected by the general climatic state of the area: they tend to be high after periods of wet weather and can be very low after prolonged drought. During the course of an individual rainfall event, the baseflow component of the hydrograph continues to fall even after river levels have begun to rise, and only when the storm rainfall has had time to percolate down to the water table does the baseflow division curve (shown schematically in Fig. 13.2) begin to rise. The baseflow component usually finishes at a higher level at the end of the storm surface runoff than at the rise of the hydrograph and thus there is enhanced river flow from groundwater storage after a significant rainfall event. Groundwater provides the total flow of the general recession curve until the next period of wet weather.

The main aims of the engineering hydrologist are to quantify the various components of the hyetogram and the hydrograph, by analysing past events, in order to relate effective rainfall to surface runoff, and thereby to be able to estimate and design for future events. As a result of the complexity of the processes that create stream flow from rainfall, many simplifications and assumptions have to be made.

13.4 The Unit Hydrograph

A major step forward in hydrological analysis was the concept of the *unit hydrograph* introduced by the American engineer Sherman in 1932. He defined the unit hydrograph as the hydrograph of surface runoff resulting from effective rainfall falling in a unit of time such as 1 hour or 1 day and produced uniformly in space and time over the total catchment area (Sherman, 1942).

In practice, a T hour unit hydrograph is defined as resulting from a unit depth of effective rainfall falling in T h over the catchment. The magnitude chosen for T depends on the size of the catchment and the response time to major rainfall events. The standard depth of effective rainfall was taken by Sherman to be 1 in, but with metrication, 1 mm or sometimes 1 cm is used. The definition of this rainfall-runoff relationship is shown in Fig. 13.3(a), with 1 mm of uniform effective rainfall occurring over a time T producing the hydrograph labelled TUH. The units of the ordinates of the T-hour unit hydrograph are m^3s^{-1} per mm of rain. The volume of water in the surface runoff is given by the area under the hydrograph and is equivalent to the 1 mm depth of effective rainfall over the catchment area.

The unit hydrograph method makes several assumptions that give it simple properties assisting in its application.

(a) There is a direct proportional relationship between the effective rainfall and the surface runoff. Thus in Fig. 13.3(b) two units of effective rainfall falling in time T produce a surface runoff hydrograph that has its ordinates twice the TUH ordinates, and similarly for any proportional value. For example, if 6.5 mm of effective rainfall fall on a catchment area in T h, then the hydrograph resulting from that effective rainfall is obtained by multiplying the ordinates of the TUH by 6.5.

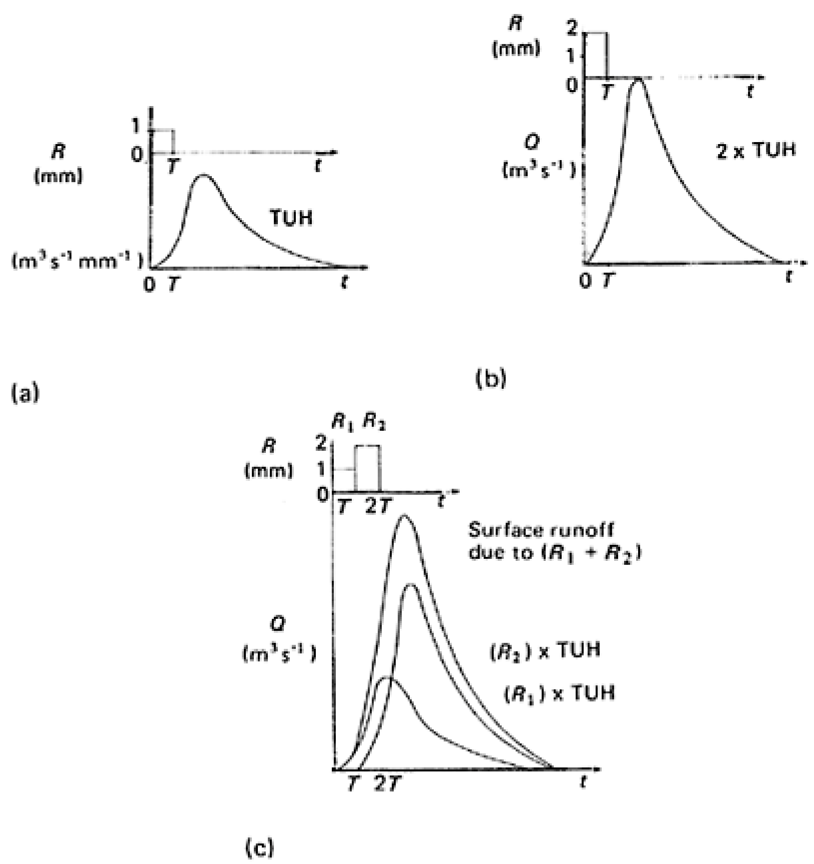

Fig. 13.3 The unit hydrograph.

(b) A second simple property, that of superposition, is demonstrated in Fig. 13.3(c). If two successive amounts of effective rainfall, R_1 and R_2 each fall in T h, then the surface runoff hydrograph produced is the sum of the component hydrographs due to R_1 and R_2 separately (the latter being lagged by T h on the former). This property extends to any number of effective rainfall blocks in succession. Once a TUH is available, it can be used to estimate design flood hydrographs from design storms.

(c) A third property of the TUH assumes that the effective rainfall-surface runoff relationship does not change with time, i.e. that the same TUH always occurs whenever the unit of effective rainfall in T h is applied. Using this assumption of invariance, once a TUH has been derived for a catchment area, it could be used to represent the response of the catchment whenever required.

The assumptions of the unit hydrograph method must be borne in mind when applying it to natural catchments. In relating effective rainfall to surface runoff, the amount of effective rainfall depends on the state of the catchment before the storm event. If the ground is saturated or the catchment is impervious, then a high proportion of the rain becomes effective. In absorbing the rainfall, unsaturated ground will have a certain capacity before releasing effective rainfall to contribute to the surface runoff. Only when the ground deficiencies have been made up and the rainfall becomes fully effective will extra rainfall in the same time period produce proportionally more runoff. The first assumption of proportionality of response to effective rainfall conflicts with the observed non-proportional behaviour of river flow. In a second period of effective rain, the response of a catchment will be dependent on the effects of the first input, although the second assumption (Fig. 13.3(c)) makes the two component contributions independent. The third assumption of time invariance implies that whatever the state of the catchment, a unit of effective rainfall in *T* h will always produce the same TUH. However, the response hydrograph of a catchment must vary according to the season: the same amount of effective rainfall will be longer in appearing as surface runoff in the summer season when vegetation is at its maximum development and the hydraulic behaviour of the catchment will be 'rougher'. In those countries with no marked seasonal rainfall or temperature differences and constant catchment conditions throughout the year, then the unit hydrograph would be a much more consistent tool to use in deriving surface runoff from effective rainfall.

Another weakness of the unit hydrograph method is the assumption that the effective rainfall is produced uniformly both in the time *T* and over the area of the catchment. The areal distribution of rainfall within a storm is very rarely uniform. For small or medium sized catchments (say up to 500 km^2), a significant rainfall event may extend over the whole area, and if the catchment is homogeneous in composition, a fairly even distribution of effective rainfall may be produced. More usually, storms causing large river discharges vary in intensity in space as well as in time, and the consequent response is often affected by storm movement over the catchment area. However, rainfall variations are damped by the integrating reactions of the catchment, so the assumption of uniformity of effective rainfall over a selected period *T* is less serious than might be supposed at first. The effect of variable rainfall intensities can be reduced by making *T* smaller, and where a catchment is affected by major storms of different origins, separate TUHs can be derived for each storm type.

However, the unit hydrograph method has the advantage of great simplicity. Once a unit hydrograph of specified duration *T* has been derived for a catchment area (and/or specific storm type) then for any sequence of effective rainfalls in periods of *T*, an estimate of the surface runoff can be obtained by adopting the assumptions and applying the simple properties outlined above. The technique has been adopted and used worldwide over many years.

Examples of unit hydrographs for differing values of *T* are shown for two catchment areas in the Thames Basin in Fig. 13.4. These demonstrate the effect of catchment size and of the selection of *T*. For large catchments (over 1000 km^2 for example) with longer response times, the unit hydrograph can be derived from weekly or even 10-day effective rainfalls, when the rainfall event causing the increased river flow constituted a widespread wet spell over the whole area (Andrews, 1962).

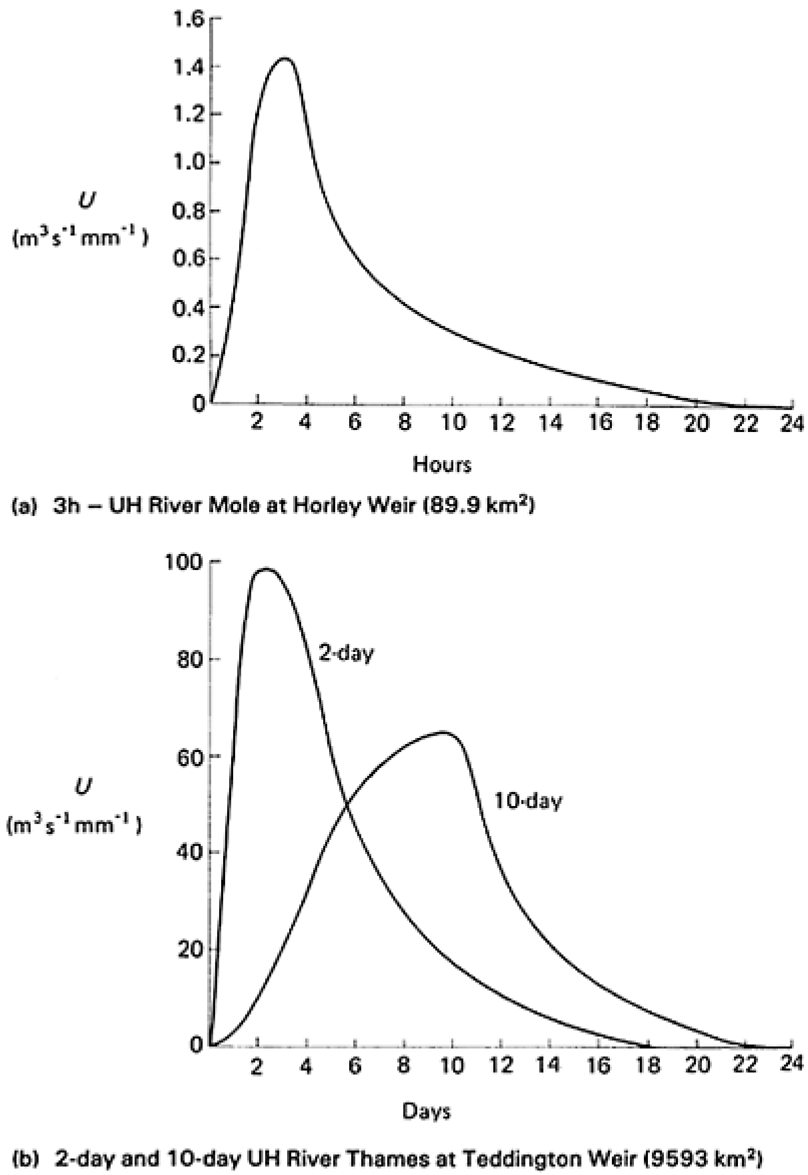

Fig. 13.4 Unit hydrographs from the Thames Basin.

The main problems in the derivation of a unit hydrograph are the assessment of the effective rainfall from the measured rainfall and the separation of the resulting surface runoff from the total hydrograph. How these difficulties are tackled is outlined in the following sections.

13.4.1 Derivation of the Unit Hydrograph from Simple Storms

A selection of typical single-peaked hydrographs is taken from the continuous river gauge records and then the corresponding rainfall records, preferably from autographic rain gauges, are abstracted. If there is only one

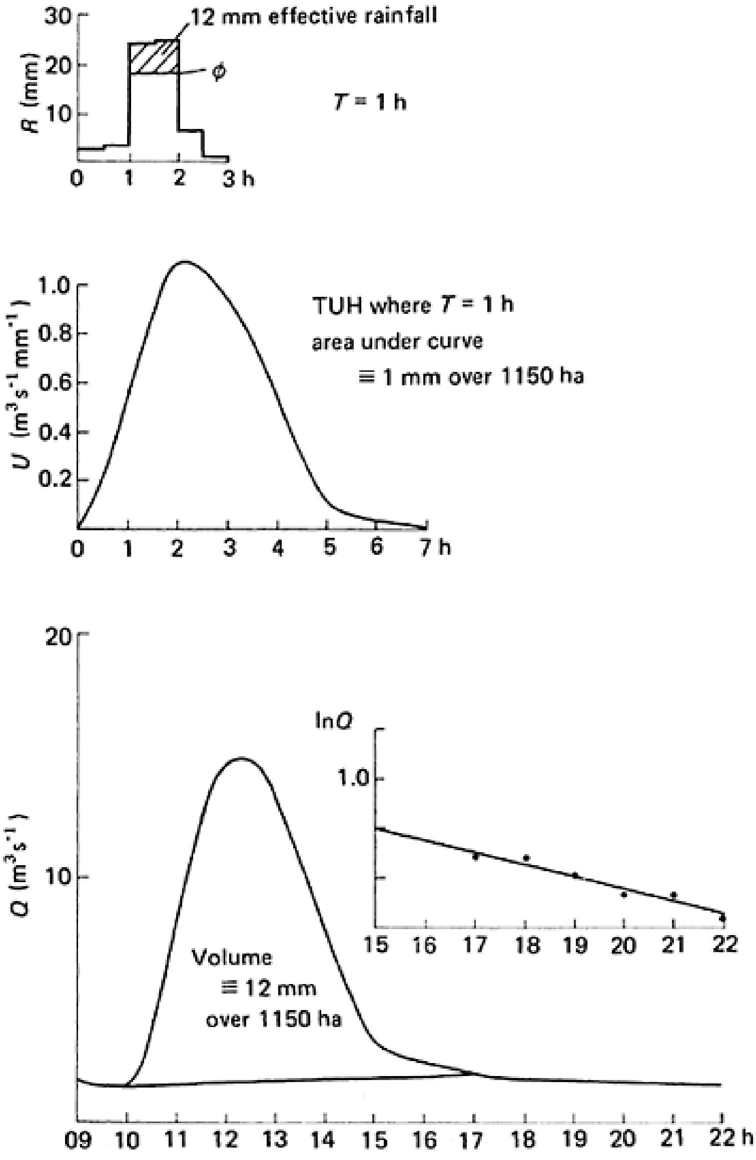

Fig. 13.5 Derivation of the unit hydrograph.

autographic rainfall record available along with some daily gauges, areal rainfall values for the storm in appropriate durations are obtained by proportioning daily totals (see Chapter 10). As an example, the areal rainfall *(R)* given in half-hour totals and the resulting hydrograph of discharge *(Q)* from a 1150 hectare catchment, are shown in Fig. 13.5. Such rainfall hyetograms with just single major blocks of rainfall are to be preferred.

The unit hydrograph derivation from one such storm event proceeds in the following stages:

(a) The first problem is the separation of the baseflow from the storm runoff. Fig. 13.6 shows several methods. The simplest separation would be given by a horizontal line to *a* from the start of the rise of the hydrograph. This would assume that the storm has no effect on or makes no contribution to groundwater. The more realistic separation is curve *c,* which shows a marked storm effect peaking some time after the peak of the stream flow. However, the drawing of this curve would be quite subjective, and various answers would be produced by different analysts. The straight line to point *b* on the recession curve can be singularly determined. After point *b,* the shape of the hydrograph recession is assumed to become exponential, and is fixed uniquely on the semi-logarithmic plot as shown. This is a satisfactory compromise and the method is straightforward and gives consistent results. It is seen applied in Fig. 13.5.

In their series of Low Flow Studies the Institute of Hydrology defined a Base Flow Index (BFI) as a catchment characteristic (IOH, 1980). The index is the ratio of the smoothed minimum mean daily flow to the mean daily flow of the total recorded hydrograph and can be considered as a measure of the proportion of the river runoff that derives from natural storage. It is recommended that the index be evaluated from at least a year's records of mean daily flows. It may also be derived from considerations of the catchment geology. In the present context, it is the smoothed minimum mean daily flow which is required to be separated from the total hydrograph. From the daily data, five-day minima are identified and turning points in this sequence are marked. These turning points define the smoothed minima giving the line of the baseflow.

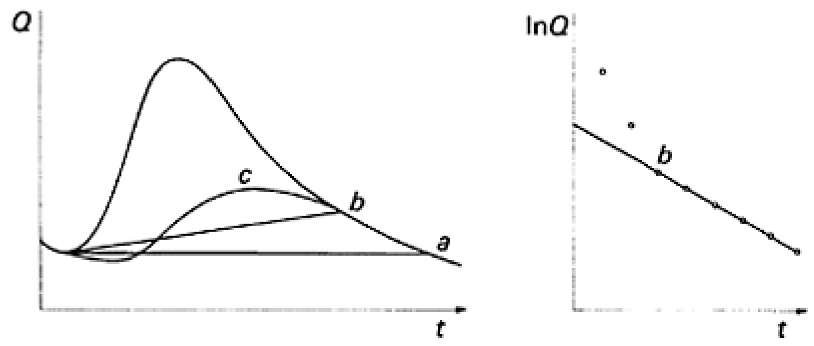

Fig. 13.6 Separation of base flow.

(b) The area above the baseflow separation line and enclosed by the hydrograph is then measured (by planimeter, or by using squared paper) to give the volume of surface runoff. The equivalent depth of runoff (mm) is evaluated for the catchment area. This is then by definition the effective rainfall from the storm (12 mm in Fig. 13.5).

(c) The determination of that part of the measured total rainfall that constitutes the effective rainfall is the next step. In Fig. 13.5, the effective rainfall (12 mm) is the hatched area deducted from the large 1-h central block of the areal rainfall bargraph. The duration, T, of the effective rainfall is then clearly indicated as 1 h in the example shown in Fig. 13.5.

(d) The time axis of the storm hydrograph is then usually subdivided into periods of duration T, beginning at the rise of the hydrograph, but any convenient interval can be chosen. The corresponding ordinates of surface runoff (Q_s) given by the total hydrograph discharge (Q) minus the corresponding baseflow, are then each divided by the effective rainfall to give the ordinates (U) of the unit hydrograph ($m^3 s^{-1} mm^{-1}$).

(e) The unit hydrograph for effective rainfall of duration T, the TUH, is then plotted, and the area under the curve is checked to see if the enclosed volume is equivalent to unit effective rainfall over the area of catchment.

When all the single-peaked storms have been analysed and a corresponding number of unit hydrographs obtained, it will be noted that no two are identical, though they will all have the same general shape. Features of the unit hydrographs derived from eight storms on the same catchment are given in Table 13.1.

One way that an average unit hydrograph may be constructed is by taking the arithmetic means of the peak flows (U_p) and the times to peak (T_p), plotting the average peak at the appropriate mean value of T_p, and drawing the hydrograph to match the general shapes of the individual unit hydrographs as illustrated in Fig. 13.7. The resulting average unit hydrograph is then checked to ensure that the enclosed volume of runoff is equivalent to a unit of effective rainfall.

For rainfall bargraphs of a complex pattern, a more sophisticated rainfall separation procedure is needed. In Fig. 13.2, an idealized separation is shown by a curved loss-rate line. At the beginning of a storm there could be considerable interception of the rainfall and initial wetting of surfaces before the rainfall becomes ‘effective’—that is, begins to form surface runoff.

Table 13.1 Unit Hydrograph Parameters from 8 Storms

Data for Storm 1 from Fig. 13.5.

Storm	1	2	3	4	5	6	7	8	Averages
U_p ($m^3s^{-1}mm^{-1}$)	1.10	1.16	0.93	0.98	1.12	1.05	1.01	1.20	1.07
T_p (h)	2.1	2.6	2.4	1.9	2.7	1.7	2.2	1.5	2.1

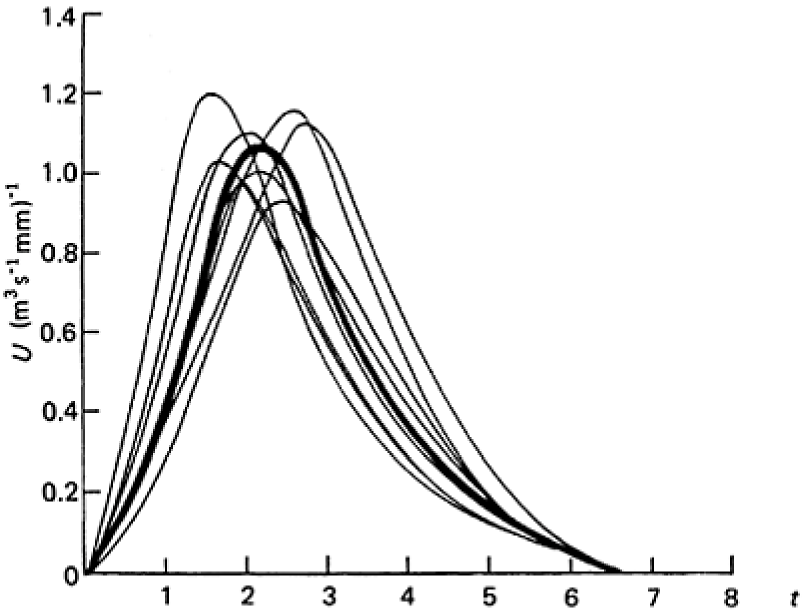

Fig. 13.7 Derivation of a unit hydrograph from eight storms.

The loss-rate is dependent on the state of the catchment before the storm and is difficult to assess quantitatively. Two simplified methods of determining the effective rainfall are given in Fig. 13.8. The Φindex method assumes a constant loss rate of Φmm from the beginning of the rainfall event: this amount accounts for interception, evaporation loss and surface detention in pools and hollows. It could rightly be considered, however, that there is a period of time after the commencement of the storm before *any* of the rainfall becomes effective. In Fig. 13.8(b), all the rainfall up to the time of rise of the hydrograph is considered lost, and there is a continuing loss-rate at some level afterwards. In both methods the rainfall separation line is positioned such that the hatched areas in Fig. 13.8 equate to the effective rainfall depth.

A choice between the two methods depends on knowledge of the catchment but, as the timing of the extent of initial loss is arbitrary, the fixing of the beginning of effective rainfall at the beginning of runoff in the stream neglects any lag time in the drainage process and is thus somewhat unrealistic. A constant loss-rate, the Φindex, would therefore seem to be more readily applicable.

13.4.2 Derivation of the Unit Hydrograph from Composite Storms

Very often, particularly on larger catchments, it is difficult to find in the available records enough single-peaked storm events to provide a fair sample for analysis. Multi-peaked sequences of rainfalls and the resultant hydrographs must then be used in the unit hydrograph derivation. If signif-

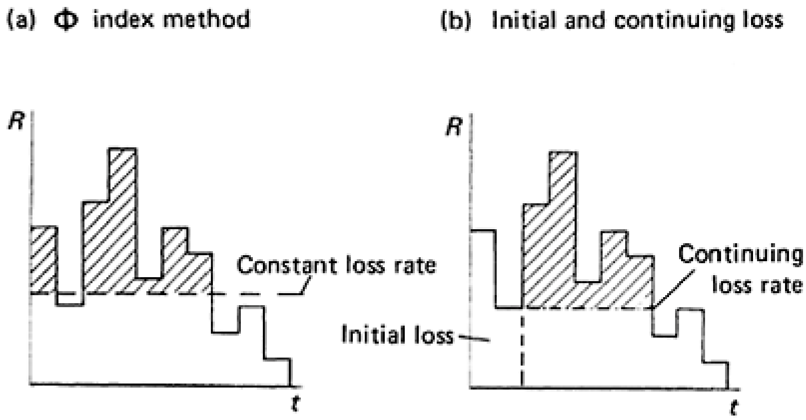

Fig. 13.8 Determination of effective rainfall.

icant peak flows can clearly be related to groups of higher rainfall values, it may be possible to separate the records into distinctive individual events to be treated as single storms. However, composite effects with overlapping storm hydrographs call for more complex treatment.

In Fig. 13.9 a sequence of *effective* rainfalls *(R)* has been plotted for durations of time *T* together with the resulting surface runoff hydrograph, with ordinates *(Q)* indicated also at intervals *T*. In the centre of the diagram is shown the TUH that commences under the rainfall block starting at time *iT* for the general case. Now from the unit hydrograph superimposition assumption shown in Fig. 13.3(c), each rainfall block produces a component part of the surface runoff at a later time *jT,* viz $R_i \times U_{j-i}$. Thus the surface runoff *Q* at time *jT* is given by summing all such components for all the rainfall blocks up to the block starting at time $(j-1)\,T$:

$$Q_j = \sum_{i=0}^{j-1} R_i U_{j-i} \qquad (13.5)$$

This shorthand notation represents a set of simultaneous equations for Q_j (j=1,2,3,…). If there are m blocks of effective rainfall (R_0, R_1, $R_2 \ldots R_{m-1}$) and n (non-zero) ordinates in the TUH (U_1, $U_2 \ldots U_n$) then there will be (m+n−1) such equations, and hence (m+n−1) non-zero surface runoff ordinates:

$$
\begin{array}{lllllllllll}
Q_1 & = R_0U_1 & + 0 & + 0 & + 0 \ldots & & & & & & + 0 \\
Q_2 & = R_1U_1 & + R_0U_2 & + 0 & + 0 \ldots & & & & & & + 0 \\
Q_3 & = R_2U_1 & + R_1U_2 & + R_0U_3 & + 0 \ldots & & & & & & + 0 \\
\vdots & & & & & & & & & & \\
Q_m & = R_{m-1}U_1 & + R_{m-2}U_2 & + R_{m-3}U_3 & + \ldots & + R_0U_m & + 0 & \ldots & & & + 0 \\
Q_{m+1} & = 0 & + R_{m-1}U_2 & + R_{m-2}U_3 & + \ldots & + R_1U_m & + R_0U_{m+1} & + \ldots & & & + 0 \\
\vdots & & & & & & & & & & \\
Q_{m+n-2} & = 0 & + 0 & + 0 & + \ldots & + 0 & + 0 & + \ldots & + R_{m-1}U_{n-1} & & + R_{m-2}U_n \\
Q_{m+n-1} & = 0 & + 0 & + 0 & + \ldots & + 0 & + 0 & + \ldots & + 0 & & + R_{m-1}U_n
\end{array}
$$

There are more equations than the n unknowns, U_1 to U_n. Expressing the equations first in matrix form followed by some matrix manipulation reduces the number of equations to n, as follows:

$$
\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ \vdots \\ Q_m \\ Q_{m+1} \\ \vdots \\ Q_{m+n-2} \\ Q_{m+n-1} \end{bmatrix}
=
\begin{bmatrix}
R_0 & 0 & & & \ldots & & & & 0 \\
R_1 & R_0 & 0 & & \ldots & & & & 0 \\
R_2 & R_1 & R_0 & 0 & \ldots & & & & 0 \\
\vdots & & & & & & & & \vdots \\
R_{m-1} & R_{m-2} & R_{m-3} & & \ldots & R_0 & 0 & \ldots & 0 \\
0 & R_{m-1} & R_{m-2} & & \ldots & R_1 & R_0 & 0 \ldots & 0 \\
\vdots & & & & & & & & \vdots \\
0 & 0 & & & \ldots & & & R_{m-1} & R_{m-2} \\
0 & 0 & & & \ldots & & & 0 & R_{m-1}
\end{bmatrix}
\times
\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ \\ \vdots \\ \\ \\ \\ U_n \end{bmatrix}
$$

Using conventional matrix notation, this becomes:

$$\underline{Q}=\underline{R}.\underline{U} \tag{13.6}$$

In order to obtain the elements of $\underline{U}$ from this relationship, it is first necessary to make the rectangular $\underline{R}$ matrix into a square matrix. To make this transformation, $\underline{R}$ must be pre-multiplied by its transpose $\underline{R^T}$ and so, premultiplying each side of the equation by $\underline{R^T}$, gives

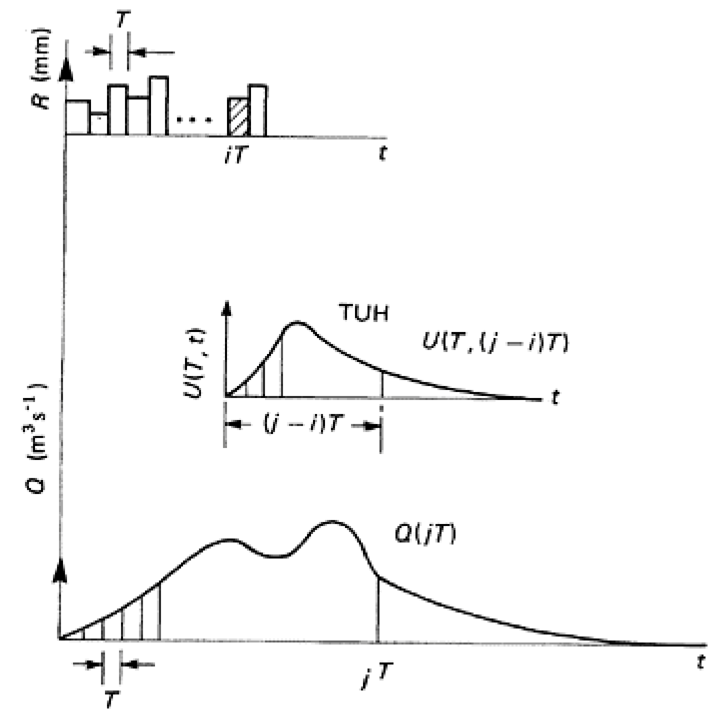

Fig. 13.9 The *T*-hour unit hydrograph (TUH). $Q(jT) = T\sum_{i=0}^{j} R_i U(T, (j-i)T)$

with j=1,2,3…or $Q(j) = \sum_{i=0}^{j} R_i U_{j-i}$
where R occurs in intervals of T h and $R_0=R(0\rightarrow 1T)$.

$$\underline{R}^T.\underline{Q}=\underline{R}^T.\underline{R}.\underline{U}$$

and hence

$$\underline{U}=(\underline{R}^T.\underline{R})^{-1}.\underline{R}^T.\underline{Q} \qquad (13.7)$$

Thus given a series of effective rainfalls and corresponding surface runoffs, the ordinates of the TUH can be obtained using standard computer programs for matrix manipulation. The method automatically gives a best solution for the TUH in the least-squares sense. The matrix procedure often yields unreal oscillatory irregular shapes for the TUH. The method may be repeated with revised estimates of the effective rainfall and surface runoff data, and a smoothing process on the derived TUH ordinates employed until a smooth form of the TUH is obtained.

13.4.3 Changing the T *of the TUH*

Although a TUH may have been derived for a catchment from a selection of simple storms having the same duration of effective rainfall, or from a series of composite storms for which some given value of *T* has been used in the analysis, there is often a need to find an estimated pattern of runoff for a different duration of effective rainfall. It may be necessary then to derive a unit hydrograph for a new value of *T*. A comprehensive method which can be used for making the new *T* greater or less than the original *T* involves the construction of (or a tabular representation of) a standard S-curve. The *S-curve* is given by the sequential accumulation of the ordinates of the TUH; that is, a graph of cumulative *U* plotted against time. In effect, the S-curve based on a TUH represents the surface runoff hydrograph caused by an effective rainfall of intensity $1/T$ mm h^{-1} applied indefinitely (see Fig. 13.10(a)).

In the next stage, if a T_2–UH is required, the same S-curve is plotted again (S_2) offset at a distance T_2 from the first S-curve (S_1), as shown graphically in Fig. 13.10(b). The difference between the two S-curves displaced by T_2 then represents surface runoff from $(1/T_1)\times T_2$ mm of effective rainfall in T_2 h. Thus the differences ΔS_t between the ordinates of the two S-curves form the surface runoff hydrograph produced by an effective rainfall of T_2/T_1 mm in a T_2-*h* period. Therefore the new T_2-hour unit hydrograph ordinates will be given by

$\Delta S_t \div (T_2/T_1)$, i.e. $(T_1/T_2)\times\Delta S_t$

The differences between the accumulated values of two S-curves are needed from the graphs at regular time intervals, but the operation is easier and more accurate if the working is done by means of a table. In Fig. 13.10(c), unit hydrographs are shown for two values of T_2, $T_2 = \frac{1}{2}$ h and 2 h, derived from the 1 h-UH. The computations are carried out in Table 13.2. It will be noted that with $T_2<T_1$ the new shorter period unit hydrograph peaks earlier and is an irregular shape because of the exaggeration of discrepancies in the data and the basic assumptions made in the method. For $T_2>T_1$, variations in catchment response are damped down and the unit

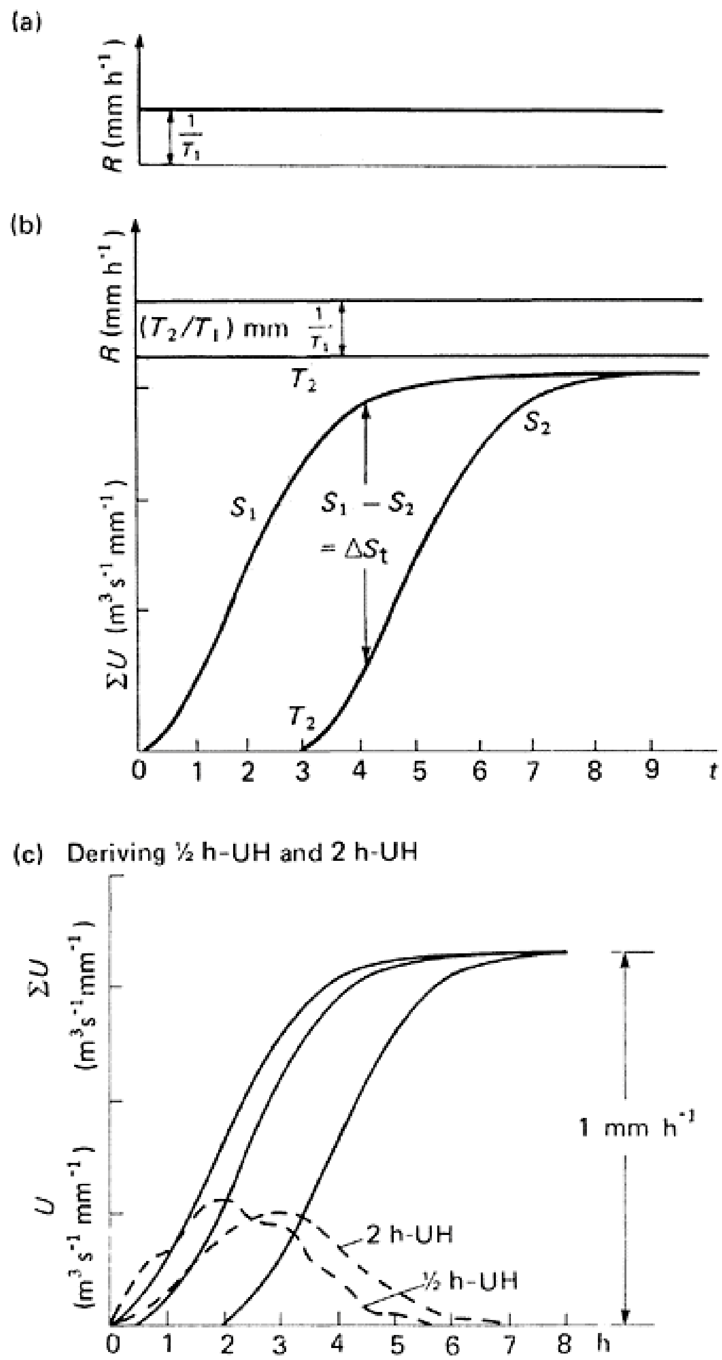

Fig. 13.10 Changing the *T* of the TUH.

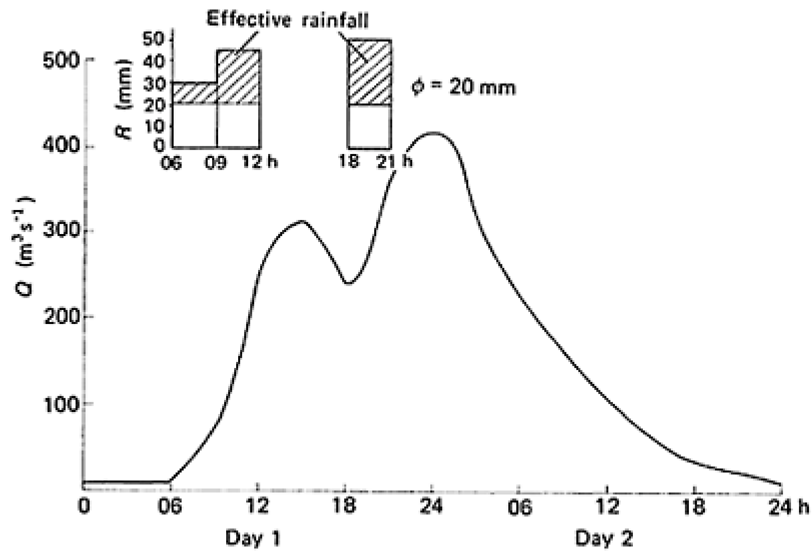

Fig. 13.11 A storm hydrograph from measured rainfall.

hydrograph is more generalized and therefore smoother. The new unit hydrographs should be checked to see that the enclosed area is equivalent to the unit of surface runoff and the final plotted curves amended if necessary. A 1-h UH and its S-curve are applicable to small and medium sized catchments that have response times measured in hours and for which the unit hydrograph method of analysis is realistic. For large catchments with slower responses and on which storms do not tend to be uniform in time or space, a longer standard period of 1 day could be used for changing T values. This latter operation would be rarely needed.

In carrying out analysis of effective rainfall and surface runoff records to obtain a unit hydrograph, it must be realized that some raw catchment data may not produce very good results, since sweeping assumptions are made in the theory. The simple linear properties of proportionality and superimposition do not hold for complex natural processes taking place in catchments of variable composition. There are catchments for which the method is not at all suitable, particularly those areas where the groundwater contribution to the flow may be high and where it is very difficult to carry out the base-flow separation. Many features of the flow within a catchment have non-linear characteristics and thus are not well modelled by the linear unit hydrograph procedure. Large catchments with varied geological structure and those experiencing variable storms over the area should be subdivided and more homogeneous subcatchments taken separately for the study of rainfall-runoff relationships.

Table 13.2 Changing the *T* of the TUH

Time (h)	1 h UH	S-curve	S-curve	S-curve difference	
Deriving the ½ h UH			(offset ½ h)		(½ h UH)
0	0	0		0	0
0.5		0.25*	0	0.25	0.50
1	0.58	0.58	0.25	0.33	0.66
1.5		1.10	0.58	0.52	1.04
2	1.09	1.67	1.10	0.57	1.14
2.5		2.15	1.67	0.48	0.96
3	0.94	2.61	2.15	0.46	0.92
3.5		2.90	2.61	0.29	0.58
4	0.51	3.12	2.90	0.22	0.44
4.5		3.18	3.12	0.06	0.12
5	0.12	3.24	3.18	0.06	0.12
5.5		3.27	3.24	0.03	0.06
6	0.05	3.29	3.27	0.02	0.04
6.5		3.29	3.29	0	0
7	0	3.29	3.29	0	0
Deriving the 2 h UH			(offset 2 h)		(2 h UH)
0	0	0		0	0
1	0.58	0.58		0.58	0.29
2	1.09	1.67	0	1.67	0.83
3	0.94	2.61	0.58	2.03	1.02
4	0.51	3.12	1.67	1.45	0.72
5	0.12	3.24	2.61	0.63	0.32
6	0.05	3.29	3.12	0.17	0.08
7	0	3.29	3.24	0.05	0.03
8			3.29	0	0

* ½ h values of S-curve taken from graph, Fig. 13.10(c).

13.4.4 Application of the TUH

Once a TUH has been determined satisfactorily for a catchment area, it can be used as a tool to obtain the storm hydrograph from measured rainfall amounts. Most often the interest of the engineer is concentrated on knowing the peak flow, but in some design projects it is necessary to know the total volume of flow or for how long a high flow rate over a critical level is likely to be sustained. The unit hydrograph method has the advantage of providing a continuous simulated record of discharge.

The computation of a storm hydrograph is demonstrated in Table 13.3. The 3 h UH is given at 3-h intervals with the U values in $m^3s^{-1}mm^{-1}$, and two periods of heavy rain occur, 75 mm between 0600-h and 1200 h and 50 mm between 1800 h and 2100 h (Fig. 13.11). A constant loss-rate of 20 mm is

Table 13.3 Calculation of Discharge (m^3s^{-1}) from Effective Rainfall (mm)

			Effective rainfall×UH				
Hours	3 h UH	Time	10×UH	25×UH	30×UH	Base flow	Total flow
0	0	0600	0			10	10
3	6.0	0900	60	0		10	70
6	9.4	1200	94	150		9	253
9	7.1	1500	71	235		8	314
12	5.4	1800	54	177.5	0	8	239.5
15	4.0	2100	40	135	180	9	364
18	2.9	2400	29	100	282	10	421
21	1.8	0300	18	72.5	213	10	313.5
24	1.0	0600	10	45	162	11	228
27	0.4	0900	4	25	120	11	160
30	0	1200	0	10	87	12	109
		1500		0	54	12	66
		1800			30	12	42
		2100			12	12	24
		2400			0	12	12

assumed. The three 3-h blocks of effective rainfall (10, 25 and 30 mm) are applied to the ordinates of the TUH at their appropriate starting times. An estimated series of baseflow values related to the initial value of flow before the storm is added. The contributions made by the effective rainfalls and the baseflow are summed along each row to provide the total flows in the last column. These are plotted on Fig. 13.11. When evaluating flood peaks, extreme rainfall intensities are used and the catchment is assumed to be saturated initially.

As explained in the previous section, an estimate of the state of wetness of the catchment before a storm is required in order to assess the amount of effective rainfall that will form direct surface runoff. A general guide to the degree of moisture in the ground can be obtained from the *antecedent precipitation index (API)*. This is calculated on a daily basis and assumes that soil moisture declines exponentially when there is no rainfall. Thus:

$$\mathrm{API}_t = k^t.\mathrm{API}_0$$

i.e.

$$\mathrm{API}_t = k.\mathrm{API}_{t-1} \qquad (13.8)$$

where API_t is the index t days after the first day's API_0. The value of k is dependent on the potential loss of moisture (mainly through evapotranspiration) and varies seasonally usually between 0.85 and 0.98. If there is rainfall then this is added to the index and so a typical daily evaluation for the sixth day, for example, would be

$API_6 = k.API_5 + R_5$

where R_5 is the rainfall on the 5th day. At the beginning, an arbitrary value, e.g. 20 mm, may be assumed for API_0. As t increases, API_t becomes dominated by the recent rainfalls and the transient effect of API_0 disappears after 20 days or so. API_t thus gives a fair representation of the state of wetness of the catchment. The API can be related to the ϕ index by comparison with known events, and thus a good first estimate of the rainfall loss can be made.

13.5 The Instantaneous Unit Hydrograph

The general equation shown on Fig. 13.9:

$$Q_j = \sum_{i=0}^{j-1} R_i U_{j-i} \qquad j = 1,2,3, \ldots \tag{13.5}$$

where R_0 is the effective rainfall in the first time interval T, describes the discrete summation operation, i.e. using depths of effective rainfall R, over finite intervals, T, to give individual Q ordinates also at discrete intervals T. If R is represented as a continuous rainfall intensity function $R(t)$ rather than as discrete depths R_i over finite intervals, then the rainfall term in the summation Equation 13.5 may be replaced by $\{R(t).\mathrm{d}t\}$ for an infinitesimal time interval dt (Fig. 13.12). Considering the corresponding changes in the form of the TUH, as T becomes the infinitesimal dt, the unit depth of effective rainfall must be thought of as occurring at greater and greater intensity over shorter and shorter intervals, to the limiting case of being deposited instantaneously on the catchment. The resulting limiting form of the TUH is called the *instantaneous unit hydrograph* (IUH). Thus the ordinates U_{j-i} of the TUH at intervals T are replaced by $H(t-\tau)$ describing the continuous function of the IUH. The summation of the discrete Equation 13.5 becomes an integration of the product of the effective rainfall function and IUH to give a continuous function of surface runoff, $Q(t)$, instead of the discrete ordinates Q_j. Thus:

$$Q(t) = \int_0^t \{R(\tau).\mathrm{d}\tau\}.H(t-\tau) = \int_0^t R(\tau).H(t-\tau).\mathrm{d}\tau$$

with τ the dummy time variable of integration.

The relationship of a continuous rainfall with the corresponding surface runoff hydrograph via the IUH is shown in Fig. 13.12. The IUH is the impulse response of the catchment to an instantaneous unit input of effective rainfall. This theoretical concept, though physically impossible, has played a major role in UH theory and its development and has provided a fertile field for many research workers in hydrology (O'Donnell, 1966; Dooge, 1973).

A wide range of mathematical techniques surrounds the determination of *H(t),* but the complexity of the methods has deterred practising engineers from applying them in day-to-day problems. For catchments that have no

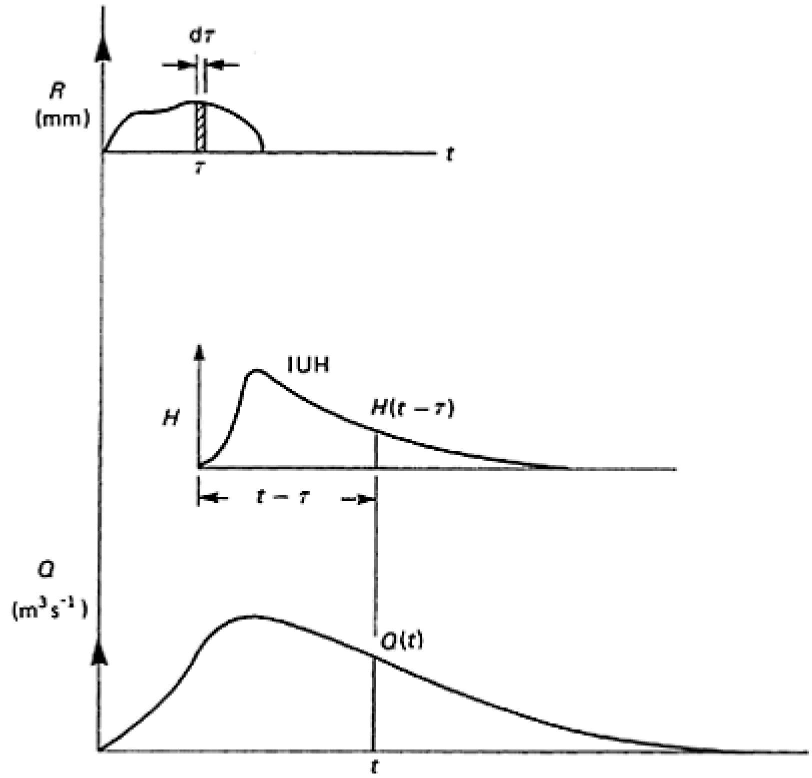

Fig. 13.12 Instantaneous unit hydrograph (IUH).

$$Q(t) = \int_0^t R(\tau).H(t-\tau).d\tau = \int_0^t R(t-\tau).H(\tau)d\tau$$ (convolution integral) or *Q(t)=R(t)*H(t).*

hydrometric records, techniques incorporating catchment parameters have had their attractions, e.g. FSSR 16 gives the time to peak of the instantaneous unit hydrograph as follows

$$T_p(0)=283.0\,S1085^{-0.33}(1+URBAN)^{-2.2}\,SAAR^{-0.54}\,MSL^{0.23}$$

(Section 17.3.2) (NERC, 1985). However, these methods, though based on unit hydrograph theory, are more akin to mathematical models, and will be considered in the next chapter.

13.6 Rainfall-Runoff Relationships over Longer Periods

Water resources engineers are primarily concerned with catchment yields and usually study hydrometric records on a monthly basis. Most rainfall measurements are made daily, and a day's rain may be part of a continuous storm that can be related to resulting stream flow by means of the unit hydrograph method. With the summation of daily values to give a monthly total, periods of both wet and dry days are included, and the relationship of the rainfall with stream flow is indirect. Monthly mean discharges from a catchment area are converted to volumes of water produced and then to equivalent depths over the catchment area. Rainfall and runoff in the same units can then be compared.

The nature of the relationship of rainfall to runoff over longer periods again depends primarily on the structure and composition of the catchment area, but it can also be affected by the climate of the area. In the British Isles, rainfall occurs all year round and seasonal differences are small. Taking average monthly values over several years, regional variations in the rainfalls of the wettest and driest months can be identified. However, the range of variation of rainfall in any one month can be considerable, and in most places in the country any month can be the wettest or driest in any calendar year. Resultant effects on runoff are dependent on the time factor; the rainfall-runoff relationship is distinctly modified by the occurrence of sequences of wet or dry months.

The mean monthly rainfall pattern is shown in Fig. 13.13 for the catchment area of the River Severn rising in the mountains of mid-Wales and flowing down to the river gauging station at Bewdley. The data are taken from the River Severn Basin Hydrological Survey (Ministry of Housing and Local Government, 1960).

The monthly runoff has been plotted on the same bar graph, and it must be noted that the 12 months are not shown in the ordinary sequence of a calendar year. In the UK the *water year* is usually taken to begin on October 1st at the end of the evaporation season when the groundwater storages are, on average, at a similar level each year. The bottom part of the diagram shows a value of loss given by the following relationship:

Loss=Rainfall−Runoff

Thus, given monthly rainfall totals, the monthly runoff should be obtained from:

Runoff=Rainfall−Loss

However, as can be seen from the plotted values of potential evaporation (MAFF, 1967) the water loss must have other components. In the months March to July, the loss is all due to evaporation, but where potential evaporation is less than the loss (from September round to February), the extra losses are making up groundwater deficiencies.

The principle addition to a water balance equation on a monthly basis is the change in storage, and hence it is necessary to write:

$$RO=R-E-\Delta S$$

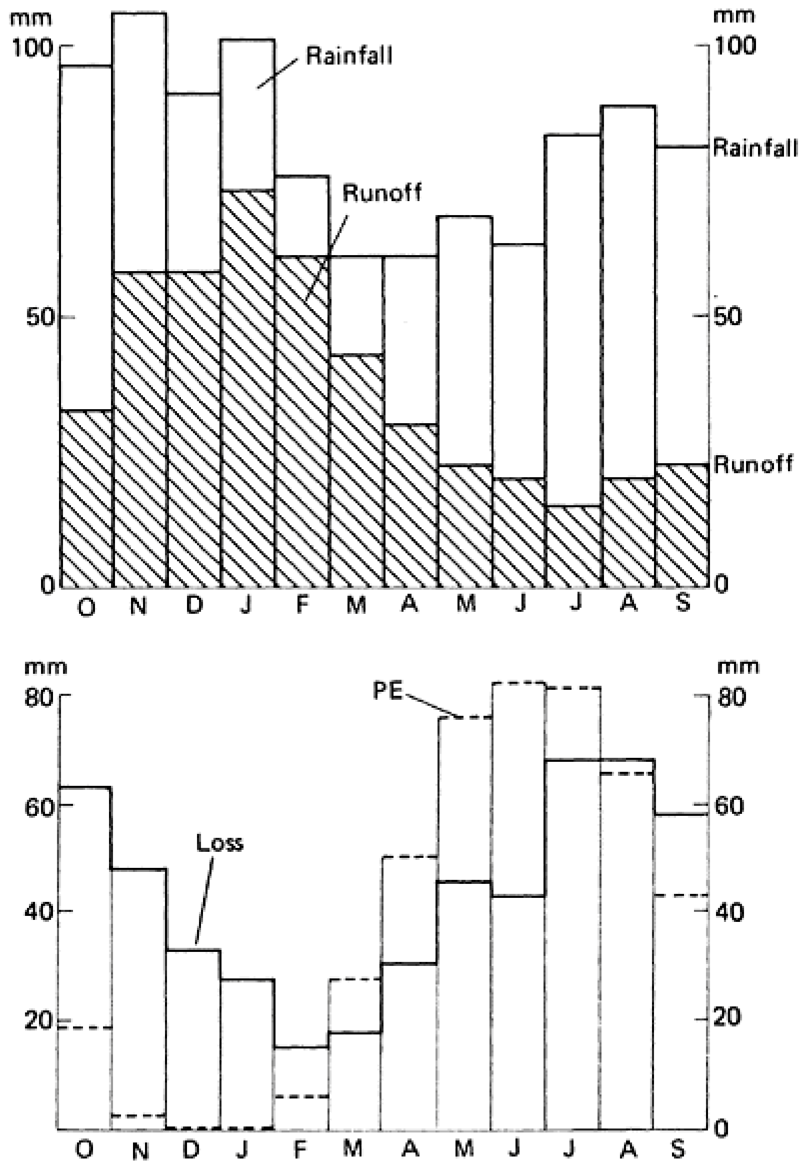

Fig. 13.13 Monthly rainfall and runoff. River Severn at Bewdley (1921–57). 4330 km^2. Loss=Rainfall−Runoff. PE=Average potential evaporation for Radnor.

in which evaporation E plus ΔS (the change in storage), constitute the loss (the difference between rainfall and runoff). Thus, even over a period of one month, when the variables have been lumped together, there are still complexities in the rainfall-runoff relationship, and other factors must be considered. To obtain the runoff from rainfall for one month, a statistical appraisal can be made of the relationship for a specific month (say June), if many years of records for June can be analysed. Such an analysis generally needs to take account of the seasonal changes from month to month, and the study then becomes a problem in time series analysis. This will be considered in Chapter 15.

The problem is further complicated in those regions of the world that have distinctive rainy and dry seasons, and in such cases it is advisable to analyse the rainfall and runoff for the combined months of the wet season or seasons.

When the winter and summer seasons are considered for the British Isles by combining the winter months October to March and the summer months April to September, significant relationships between rainfall and runoff can be obtained. In Fig. 13.14, 42 years of winter and summer rainfall and runoff for the River Derwent catchment are shown. The data are from Law

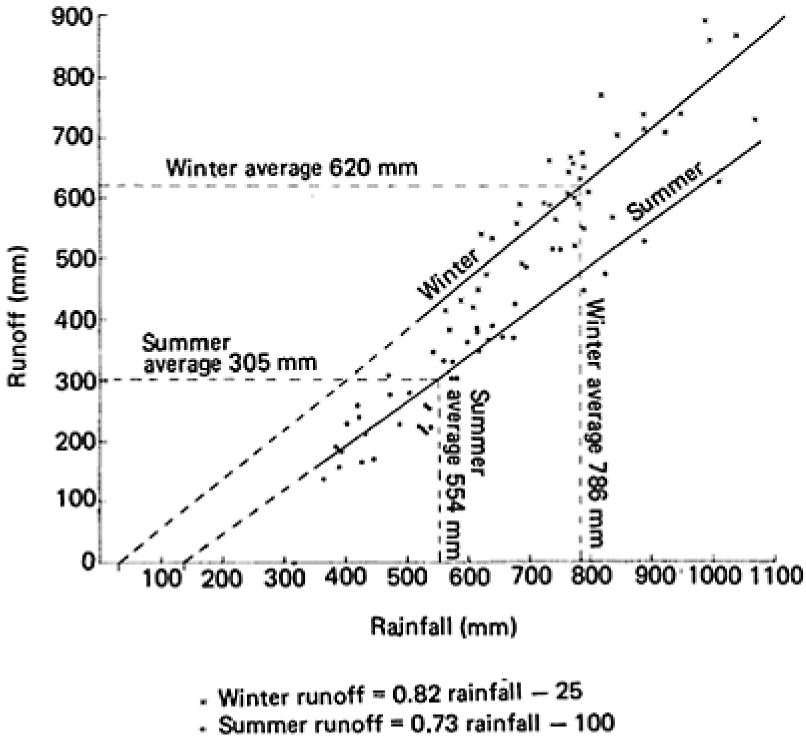

Fig. 13.14 Summer and winter rainfall and runoff. River Derwent catchment (Trent) 1906–47.

(1953), who showed that statistical correlation of the two variables could be used to estimate the reliable yield of a catchment. The regression lines have been fitted by 'least squares' to the scatter of points and the resulting equations give the best estimate of winter or summer runoff from a known or measured winter or summer rainfall. Further statistical properties of the estimates may be calculated if considered necessary. However, the correlation coefficients between runoff and rainfall are 0.87 for the winter period and 0.91 for the summer period, which indicate a satisfactory linear relationship. It is interesting to note that if the regression lines are extrapolated to intersect the rainfall axis, it could be inferred that 30 mm of rainfall are needed in the winter season before runoff occurs and 137 mm are needed in the summer season before there is any runoff. Although these figures, representing seasonal losses, are of the correct order, absolute values from such regression analyses should be treated with caution, since many other factors may also be operating.

In taking annual values for rainfall and runoff comparisons, seasonal effects are removed and consideration of changes in groundwater storage can be neglected. Then, the straightforward linear relationship of Runoff= Rainfall−Loss (mainly evaporation) holds with even less scattering of points round the regression line.

In deriving relationships between rainfall and runoff for different time periods from hours up to years, straight-line approximations have been used with greater confidence as the time spans increase. It has been emphasized that the catchment response to inputs of rainfall is by no means linear, the superposition process and the time invariant property of the unit hydrograph method being a greatly simplified representation of the complex temporal interrelationship of rainfall and runoff in a drainage basin. More realistic non-linear relationships will be explored when considering catchment modelling.

References

Andrews, F.M. (1962). 'Some aspects of the hydrology of the Thames Basin.' *Proc. Inst. Civ, Eng.*, **21,** 55–90.

Dooge, J.C.I. (1973). *Linear Theory of Hydrologic Systems.* Tech. Bull. No. 1468, USDA Agriculture Research Service.

Institute of Hydrology (IOH) (1980). *Low Flow Studies.* Research Report No. 3.

Kirpich, Z.P. (1940). 'Time of concentration of small agricultural water-sheds.' *Civ. Eng.,* **10**(6), 362.

Kuichling, E. (1889). 'The relation between the rainfall and the discharge of sewers in populous districts.' *ASCE Trans.*, **20,** 1–56.

Lloyd-Davies, D.E. (1906). 'The elimination of storm water from sewerage systems.' *Proc. Inst. Civ. Eng.*, **164,** 41–67.

Law, F. (1953). 'The estimation of the reliable yield of a catchment by correlation of rainfall and runoff.' *J. Inst. Water Eng.,* **7,** 273–293.

O'Donnell, T. (1966). 'Methods of computation in hydrograph analysis and synthesis.' *Recent Trends in Hydrograph Synthesis Paper III.* TNO Proc. No. 13. The Hague.

Ministry of Agriculture, Fisheries & Food (MAFF). (1967). *Potential Transpiration.* Tec. Bull. No. 16. HMSO.

Ministry of Housing and Local Government (1960). *River Severn Basin.* Hydrological Survey, HMSO.

Mulvaney, T.J. (1850). 'On the use of self-registering rain and flood gauges.' *Trans. Inst. Civ. Eng. Ireland,* **4**(2), 1–8.

Natural Environment Research Council (NERC) (1985). *Flood Studies.* Supplementary Report No. 16, Institute of Hydrology.

Sherman, L.K. (1932). 'Streamflow from rainfall by the unit-graph method.' *Eng. News Record,* **108,** 501–505.

Sherman, L.K. (1942). 'The unit hydrograph method,' Chapter X1E of *Hydrology,* ed. O.E.Meinzer, pp. 514–525.