Reynolds—Transport Theorem (RTT)

- A **system** is a quantity of matter of fixed identity. *No mass can cross a system boundary.*
- A **control volume** is a region in space chosen for study. Mass can cross a control surface.
- The fundamental conservation laws (conservation of mass, energy, and momentum) apply directly to systems.
- However, in most fluid mechanics problems, control volume analysis is preferred over system analysis (for the same reason that the Eulerian description is usually preferred over the Lagrangian description).
- Therefore, we need to transform the conservation laws from a system to a control volume. This is accomplished with the Reynolds transport theorem (RTT).

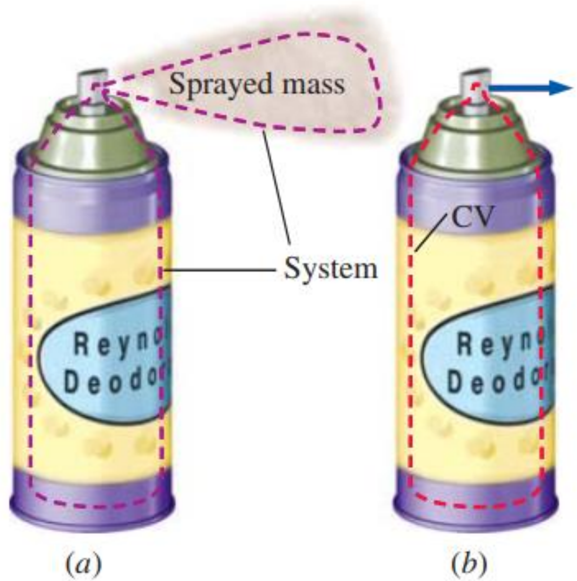
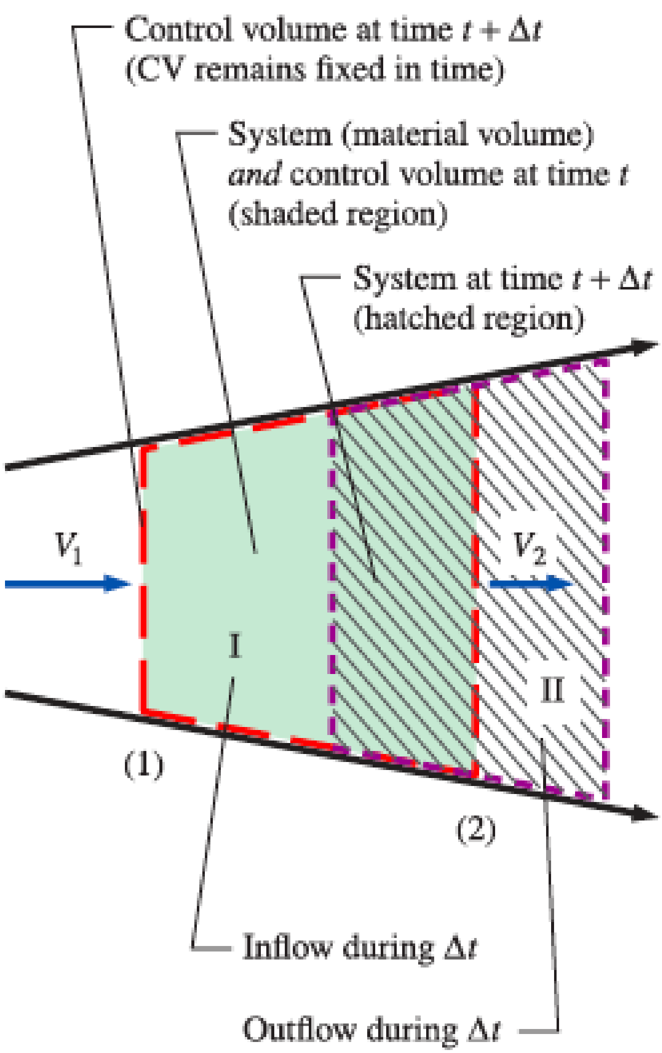


FIGURE 4–53

Two methods of analyzing the spraying of deodorant from a spray can: (*a*) We follow the fluid as it moves and deforms. This is the *system approach*—no mass crosses the boundary, and the total mass of the system remains fixed. (*b*) We consider a fixed interior volume of the can. This is the *control volume approach*—mass crosses the boundary.

System and control volume (simple geometry)

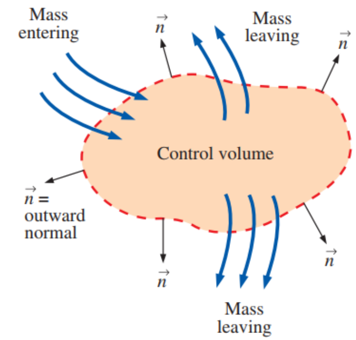


$$\frac{dB_{\text{sys}}}{dt} = \frac{dB_{\text{CV}}}{dt} - \dot{B}_{\text{in}} + \dot{B}_{\text{out}}$$

Reynolds—Transport Theorem (RTT)

- Material derivative (differential analysis):

$$\frac{Db}{Dt} = \frac{\partial b}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right) b$$



$$\dot{B}_{net} = \dot{B}_{out} - \dot{B}_{in} = \int_{CS} \rho b \vec{V} \cdot \vec{n}\, dA$$

- General RTT (integral analysis):

$$\boxed{\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\rho b)\, dV + \int_{CS} \rho b \vec{V} \cdot \vec{n}\, dA}$$

control volume

control surface

$b = \frac{B}{m}$

- Interpretation of the RTT:
  - Time rate of change of the property B of the system is equal to (Term 1) + (Term 2)
  - Term 1: the time rate of change of B of the control volume
  - Term 2: the net flux of B out of the control volume by mass crossing the control surface
  - b=B/m (intensive property)

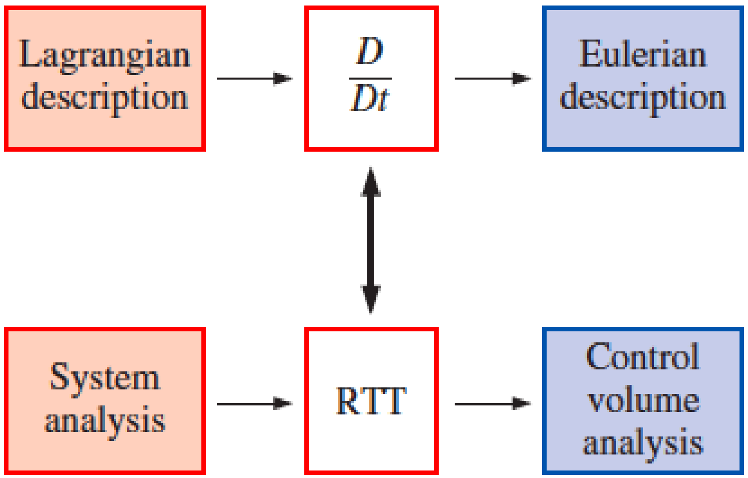
RTT Special Cases

For **moving** and/or **deforming** control volumes,

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dV + \int_{CS} \rho b \vec{V}_r \cdot \vec{n} \, dA$$

- Where the absolute velocity *V* in the second term is replaced by the **relative velocity** $\boxed{V_r = V - V_{CS}}$
- V_r is the fluid velocity expressed relative to a coordinate system moving **with** the control volume.

Reynolds—Transport Theorem (RTT)



There is an analogy between the transformation from Lagrangian to Eulerian descriptions (for differential analysis using infinitesimally small fluid elements) and that from systems to control volumes (for integral analysis using finite flow fields).

Conservation of mass (continuity equation)

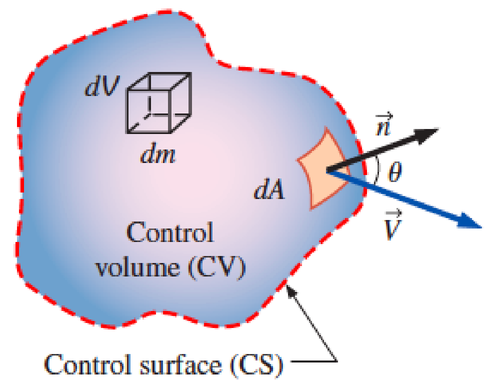
- Integral form

$b = 1$

$$\frac{\partial}{\partial t}\int_{\mathrm{CV}} \rho \, d\forall + \int_{\mathrm{CS}} \underbrace{\rho\vec{V}}_{\vec{J} = \rho\vec{V}} \cdot d\vec{A} = 0$$

$$\frac{\partial}{\partial t}\int_{\mathrm{CV}} \rho \, d\forall = -\int_{\mathrm{CS}} \rho\vec{V} \cdot d\vec{A}$$

Rate of increase of mass in CV = Net influx of mass



General conservation of mass:

$$\frac{d}{dt}\int_{\mathrm{CV}} \rho \, dV + \underbrace{\int_{\mathrm{CS}} \rho(\vec{V}\cdot\vec{n}) \, dA}_{\int_{\mathrm{CV}} \nabla\cdot(\rho\vec{V})\, d\forall} = 0$$

$$\Rightarrow \quad \frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0$$

Conservation of mass (continuity equation)

- Differential form

- Use Stokes Theorem to transform the surface integral into a volume integral and equate the integrands,

$$\boxed{\nabla \cdot (\rho \vec{V}) = -\frac{\partial \rho}{\partial t}}$$

$\rho = C_{te} \Rightarrow \boxed{\nabla \cdot \vec{V} = 0}$

- For an incompressible fluid (constant density) the continuity equation reduces to

$$\nabla \cdot \vec{V} = 0$$

- The velocity field has ZERO divergence.