Ciências ULisboa

Mecânica dos Meios Contínuos

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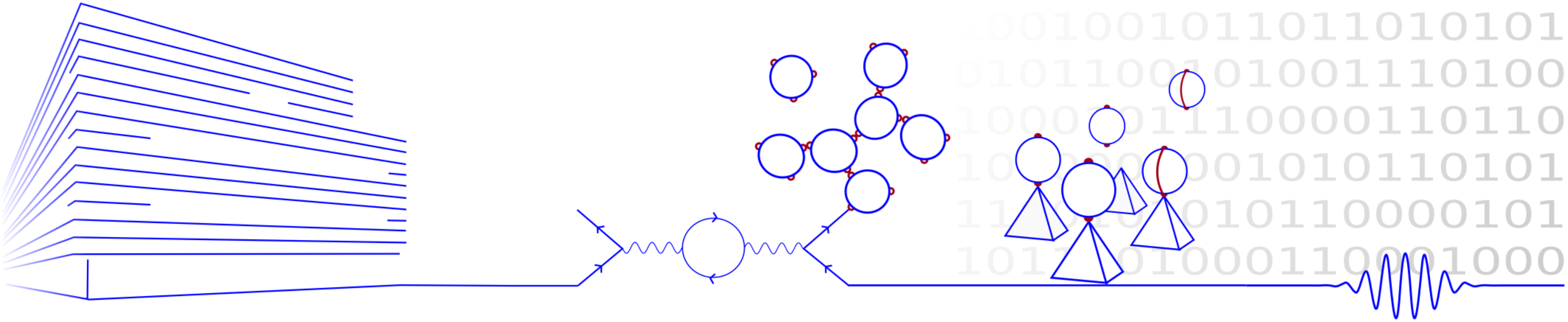
2021/22



Apresentação do curso

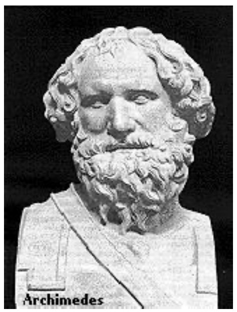
- Avaliação contínua
  - (50%) 6 exames curtos realizados durante as TPs a cada duas semanas dos quais 5 são obrigatórios;
  - (50%) exame final;
  - É obrigatória a entrega de pelo menos de 5 problemas e a presença em pelo menos 2/3 das aulas TPs. Os problemas tem pesos iguais.
- Avaliação não contínua: exame final (100%).
- Bibliografia
  - Fluid Dynamics for Physicists, by T E Faber, Cambridge University Press;
  - Elementary Fluid Dynamics (Oxford Applied Mathematics and Computing Science Series) by D J Acheson, Oxford University Press;
  - Fluid Mechanics: Fundamentals and Applications, by Çengel & Cimbala, McGraw-Hill series in mechanical engineering.

Programa

- 1. **Introdução** e visão geral da unidade curricular
- 2. **Cinemática**. Descrição de Lagrange e de Euler. Operador D/Dt. Visualização. O tensor da taxa de deformações. Vorticidade. O teorema de transporte de Reynolds. Conservação da massa e equação da continuidade. Dinâmica. O tensor das tensões. Equação do momento linear. Teorema de Pascal. Fluido ideal e **equação de Euler**. Aplicações: Equilíbrio hidrostático e o vórtice do ralo. O teorema e a equação de Bernoulli. Aparelhos para medir a velocidade e a taxa de escoamento.
- 3. **Escoamento potencial**. O teorema da circulação de Kelvin. Sobreposição. Fontes e sumidouros. Soluções da equação de Laplace em 2d e em 3d. Aplicações: Escoamento potencial à volta de uma esfera. Efeito de Magnus. Forças de elevação e de arrasto.
- 4. **Viscosidade e equação de Navier-Stokes**. Equação de Cauchy. Tensões de corte em fluidos Newtonianos. Viscosidade. O tensor das tensões. Escoamento laminar plano. Escoamento laminar cilíndrico. Equação de Navier-Stokes adimensional. Semelhança dinâmica. Equação de Stokes. Escoamento à volta de uma esfera e lei de Stokes.
- 5. **Vorticidade e camadas limite**. Linhas de vorticidade. Camadas limite. Separação das camadas limite e formação de turbilhões. Turbilhões estacionários na esteira de esferas e cilindros. Aplicações. Equações da camada limite.
- 6. **Instabilidades e Turbulência**. A instabilidade de Rayleigh-Taylor. A instabilidade de Saffman-Taylor. A instabilidade de Rayleigh-Plateau. Turbulência.

History

Faces of Fluid Mechanics



Archimedes
(C. 287-212 BC)

Newton
(1642-1727)

Leibniz
(1646-1716)

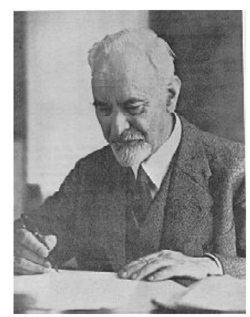
Bernoulli
(1667-1748)

Euler
(1707-1783)

Navier
(1785-1836)

Stokes
(1819-1903)

Reynolds
(1842-1912)



Prandtl
(1875-1953)

Taylor
(1886-1975)

Significance

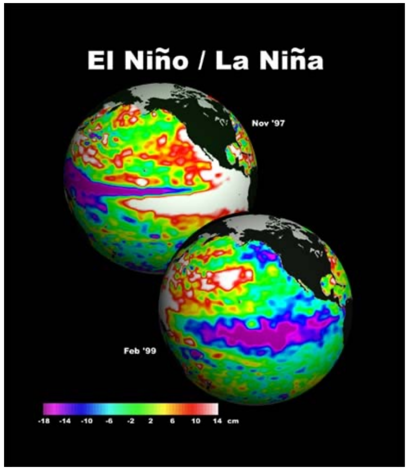
- Fluids everywhere
    - Weather & climate
    - Vehicles: automobiles, trains, ships, and planes, etc.
    - Environment
    - Physiology and medicine
    - Sports & recreation
    - Many other examples!

Weather & Climate

Tornadoes

Thunderstorm

Global Climate



Hurricanes

Vehicles

Aircraft

Surface ships

High-speed rail

Submarines

Environment

Air pollution

River hydraulics

Why do rivers curve?

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https://www.youtube.com/watch?v=8a3r-cG8Wic&feature=emb_title
https://physicstoday.scitation.org/doi/10.1063/PT.3.4523

Physiology and Medicine

Blood pump

A BVS blood pump

Ventricular assist device

Sports & Recreation

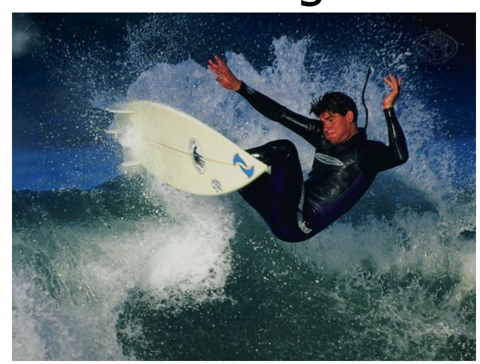
Water sports

Cycling

Offshore racing

Auto racing

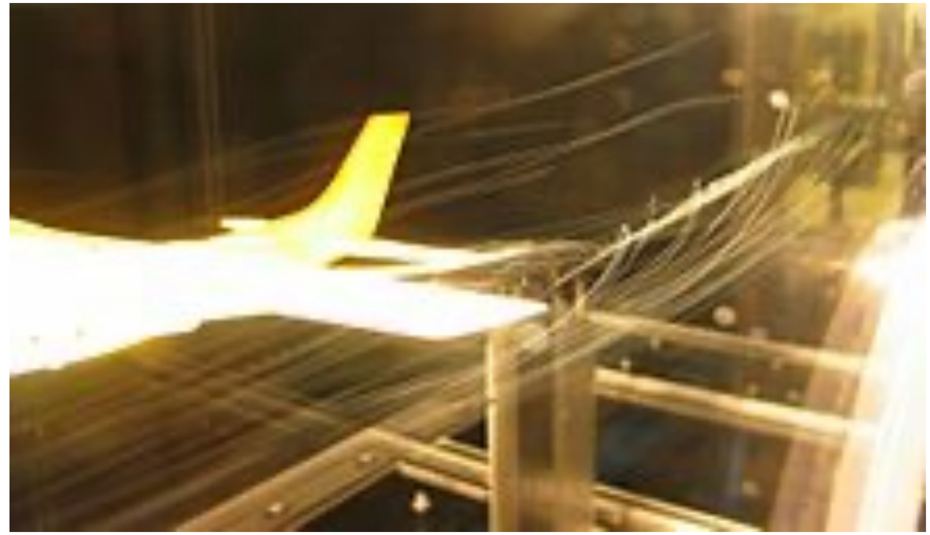
Surfing



Analytical Fluid Dynamics

- The theory of mathematical physics problem formulation
- Control volume & differential analysis (RTT)
- Exact solutions only exist for simple geometry and conditions
- Approximate solutions for practical applications
  - Linear
  - Empirical relations using EFD data

Full and model scales: wind tunnel



- Scales: full-scale and model
- Selection of the model scale: governed by dimensional analysis and similarity

Computational Fluid Dynamics

- CFD is use of computational methods for solving fluid engineering systems, including modeling (mathematical & Physics) and numerical methods (solvers, finite differences, and grid generations, etc.).
- Rapid growth in CFD technology since advent of computer

ENIAC 1, 1946

IBM WorkStation

Purpose

- The objective of CFD is to model the continuous fluids with Partial Differential Equations (PDEs) and discretize PDEs into an algebra problem, solve it, validate it and achieve simulation based design instead of “build & test”

- Simulation of physical fluid phenomena that are difficult to be measured by experiments: scale simulations (full-scale ships, airplanes), hazards (explosions,radiations,pollution), physics (weather prediction, planetary boundary layer, stellar evolution).

Modeling

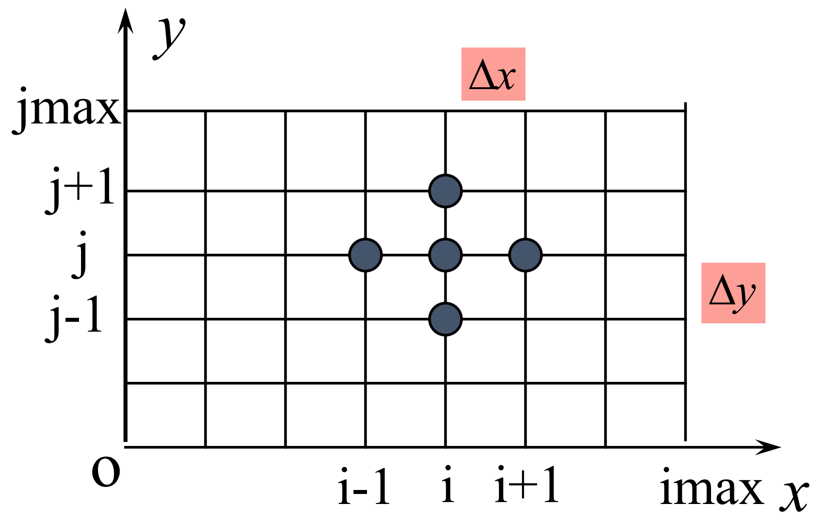
- Mathematical physics problem formulation of fluid engineering system
- Governing equations: Navier-Stokes equations (momentum), continuity equation, pressure Poisson equation, energy equation, ideal gas law, combustions (chemical reaction equation), multi-phase flows(e.g. Rayleigh equation), and turbulent models (RANS, LES, DES).
- Coordinates: Cartesian, cylindrical and spherical coordinates result in different form of governing equations
- Initial conditions(initial guess of the solution) and Boundary Conditions (no-slip wall, free-surface, zero-gradient, symmetry, velocity/pressure inlet/outlet)
- Flow conditions: Geometry approximation, domain, Reynolds Number, and Mach Number, etc.

Numerical methods

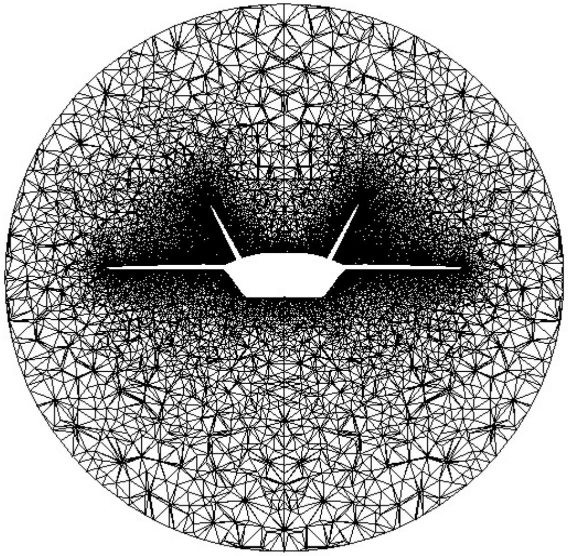
- **Finite difference methods**: using numerical scheme to approximate the exact derivatives in the PDEs

$$\frac{\partial^2 P}{\partial x^2} = \frac{P_{i+1} - 2P_i + P_{i-1}}{\Delta x^2}$$

$$\frac{\partial^2 P}{\partial y^2} = \frac{P_{j+1} - 2P_j + P_{j-1}}{\Delta y^2}$$



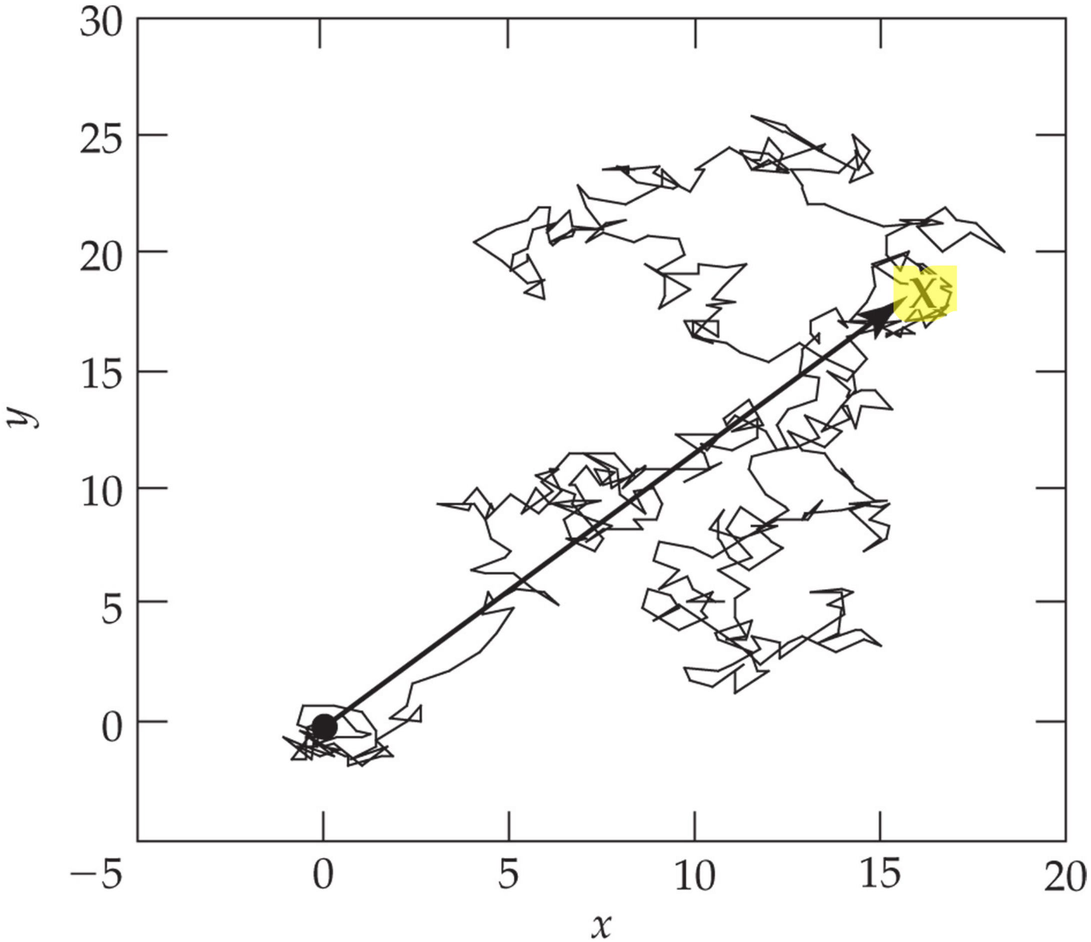
- **Finite volume methods**
- **Grid generation:** conformal mapping, algebraic methods and differential equation methods
- **Grid types**: structured, unstructured
- **Solvers**: direct methods (Cramer's rule, Gauss elimination, LU decomposition) and iterative methods (Jacobi, Gauss-Seidel, SOR)



Slice of 3D mesh of a fighter aircraft

Diffusion & Convection

Diffusion: Random walk



$$t \approx \frac{x^2}{2D}$$

Range of Values for the Binary Diffusion Coefficient, D_{ij}, at Room Temperature

Diffusing quantity	Diffusion coefficients ($cm^2\ s^{-1}$)
Gases in gases	0.1 to 0.5
Gases in liquids	1×10^{-7} to 7×10^{-5}
Small molecules in liquids	1×10^{-5}
Proteins in liquids	1×10^{-7} to 7×10^{-7}
Proteins in tissues	1×10^{-7} to 7×10^{-10}
Lipids in lipid membranes	1×10^{-9}
Proteins in lipid membranes	1×10^{-10} to 1×10^{-12}

Table 1. Diffusion coefficient values for selected ions and small and large molecules.

Ion/Molecule	Atomic/Molecular Weight (g/mol)	Diffusion Coefficient (cm^2/s)
H^+	1.008	9.31×10^{-5}
Na^+	22.990	1.33×10^{-5}
K^+	39.098	1.96×10^{-5}
Ca^{2+}	40.078	0.79×10^{-5}
Cl^-	35.453	2.03×10^{-5}
Ammonia (NH_3)	17.031	1.51×10^{-5}
Oxygen (O_2)	31.999	2.10×10^{-5}
Carbon dioxide (CO_2)	44.01	1.97×10^{-5}
Urea	60.055	1.38×10^{-5}
Glucose	180.156	5×10^{-6}
Sucrose	342.296	5.23×10^{-6}
Hemoglobin	68,000	6.9×10^{-7}
DNA	≈ 6,000,000	1.3×10^{-8}

Note: The diffusion coefficient varies with temperature and is also a function of the medium in which diffusion occurs. The values shown are for diffusion in water (H_2O) at 25 °C.

Table 2. Time required for diffusion of O_2 over a range of distances.

Distance of Diffusion	Approximate Time Required
10 nm	23.8 ns
50 nm	595 ns
100 nm	2.38 µs
1 µm	238 µs
10 µm	23.8 ms
100 µm	2.38 s
1 mm	3.97 min
1 cm	6.61 hours
10 cm	27.56 days

In mammals, the circulatory system is such that no cell is more than approximately 10 µm from a capillary. This ensures proper nourishment and waste removal for all cells of the body.

Range of Values for Viscosity, Density, and Kinematic Viscosity at Room Temperature

	Viscosity, μ (g cm^{-1} s^{-1})	Density, ρ (g cm^{-3})	Kinematic viscosity, $\nu = \mu/\rho$ (cm^2 s^{-1})
Gases	10^{-4}	0.001	0.1
Liquids			
Water	0.01	1.0	0.01
Glycerol	10	1	10
Blood	0.03	1.2	0.025

Peclet number

Relative Importance of Diffusion and Convection

Molecule	MW (g mol^{-1})	D_{ij} (cm^2 s^{-1})	Diffusion time, L^2/D_{ij} (s)	$Pe = Lv/D_{ij}$
Oxygen	32	2×10^{-5}	5	0.05
Glucose	180	2×10^{-6}	50	0.50
Insulin	6,000	1×10^{-6}	100	1.0
Antibody	150,000	6×10^{-7}	167	1.67

Particle	Diameter	D_{ij} (cm^2 s^{-1})	Diffusion time (s)	Pe
Virus	0.1 μm	5×10^{-8}	2,000	20
Bacterium	1 μm	5×10^{-9}	20,000	200
Cell	10 μm	5×10^{-10}	200,000	2,000

Note: For $L = 100$ μm, and if $v = 1$ μm s^{-1}, the time for convection is always equal to $L/v = 100$ s for all molecules and particles.

Peclet number

The Peclet number is the ratio of the rate of advection of a physical quantity by the flow to the rate of diffusion of the same quantity driven by an appropriate gradient.

$$Pe = \frac{VL}{D}$$

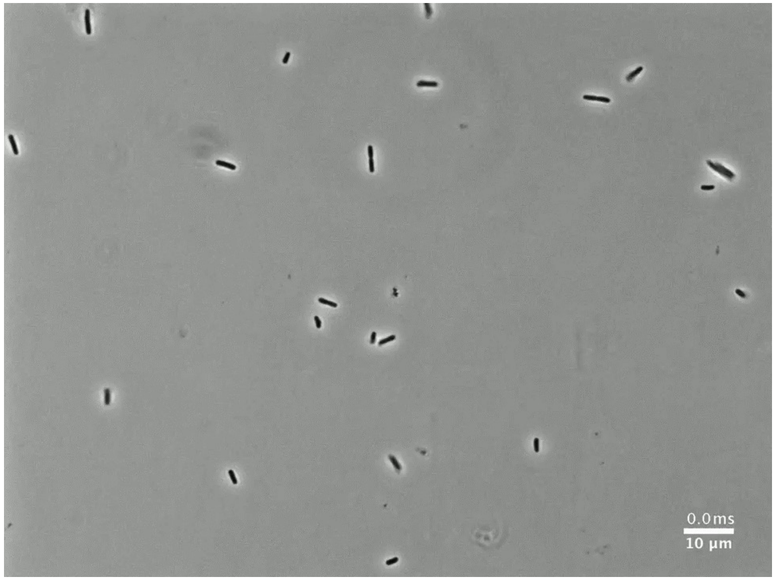
Reynolds number

The Reynolds number is the ratio of inertial forces to viscous forces within a fluid which is subjected to relative internal movement due to different fluid velocities.

$$Re = \frac{\rho V L}{\eta}$$

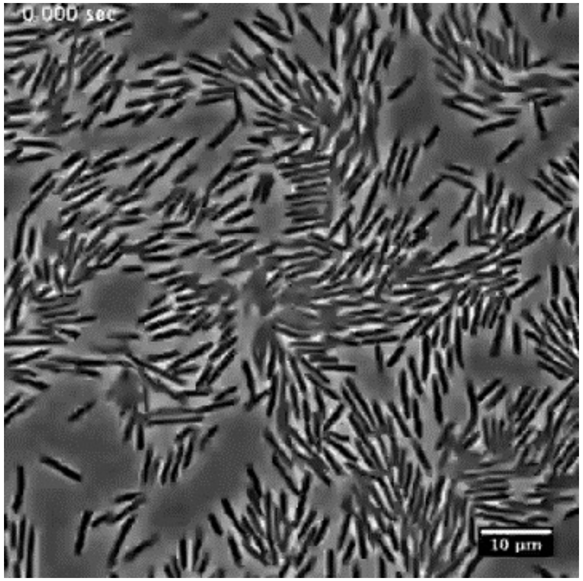
Collective behavior

Swimming E. coli

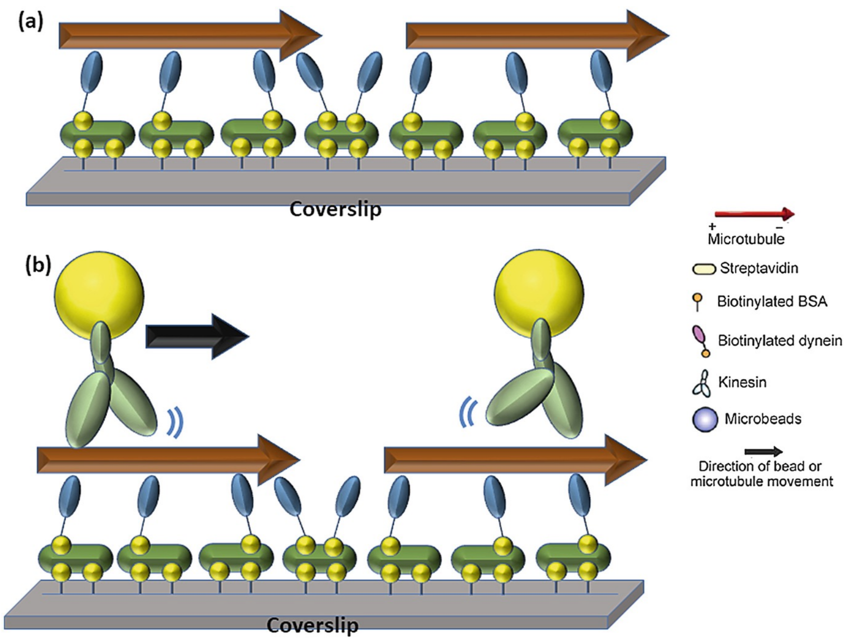


Individual motion

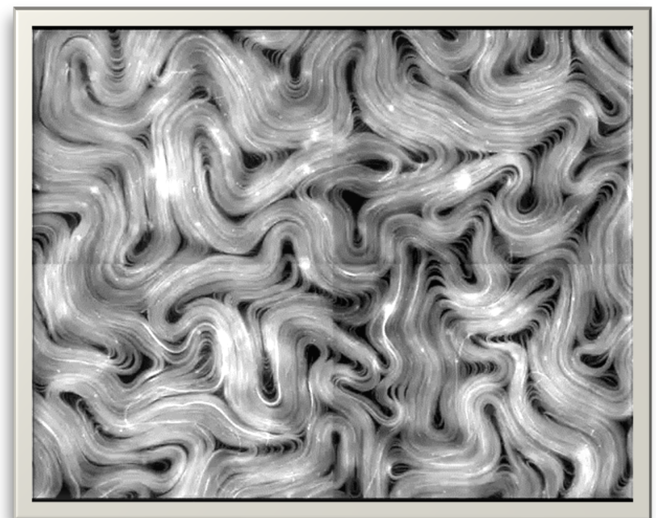
Swarming E. coli



Collective motion

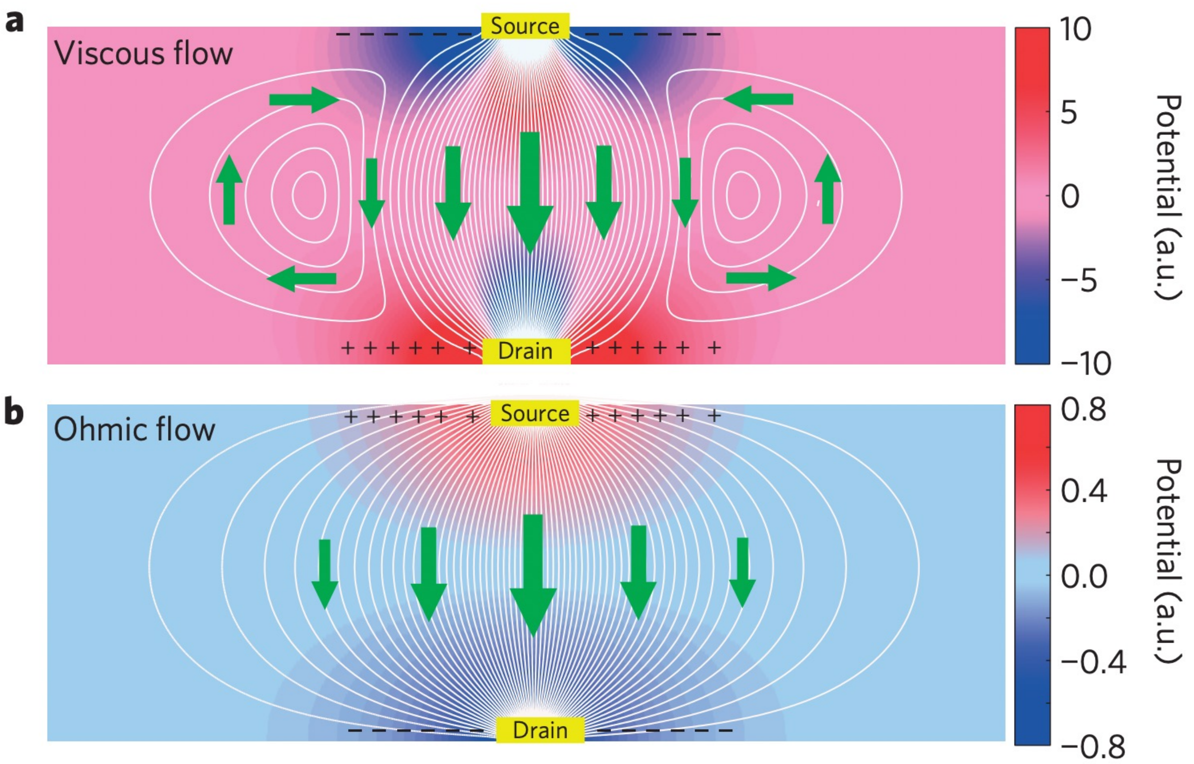


Schematic diagram of Molecular motors



Sanchez et. Al., Nature **491**, 431–434 (2012)

Electrons in graphene



Review on hydrodynamics of electrons: https://arxiv.org/pdf/1710.08425.pdf