Redes complexas

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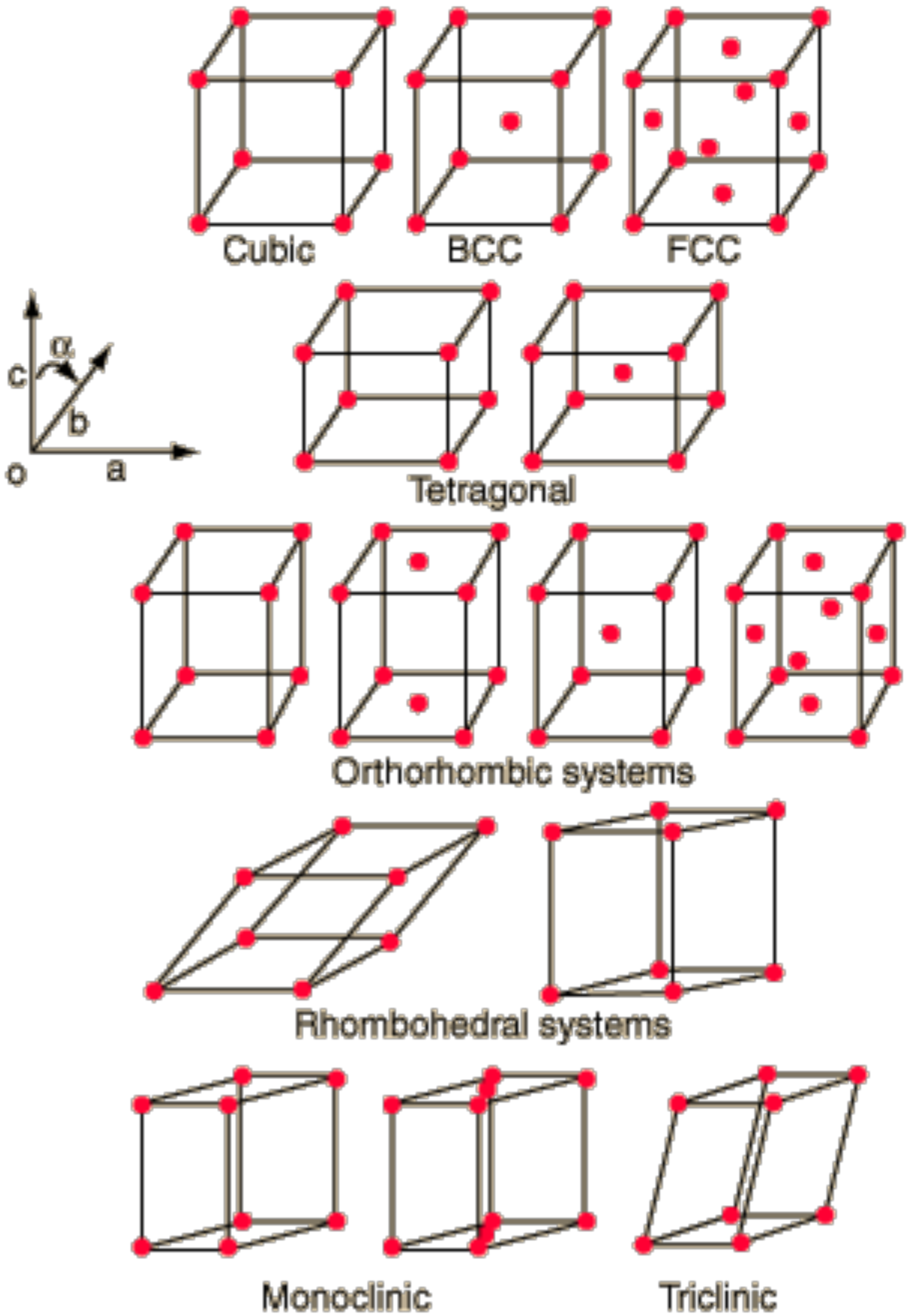
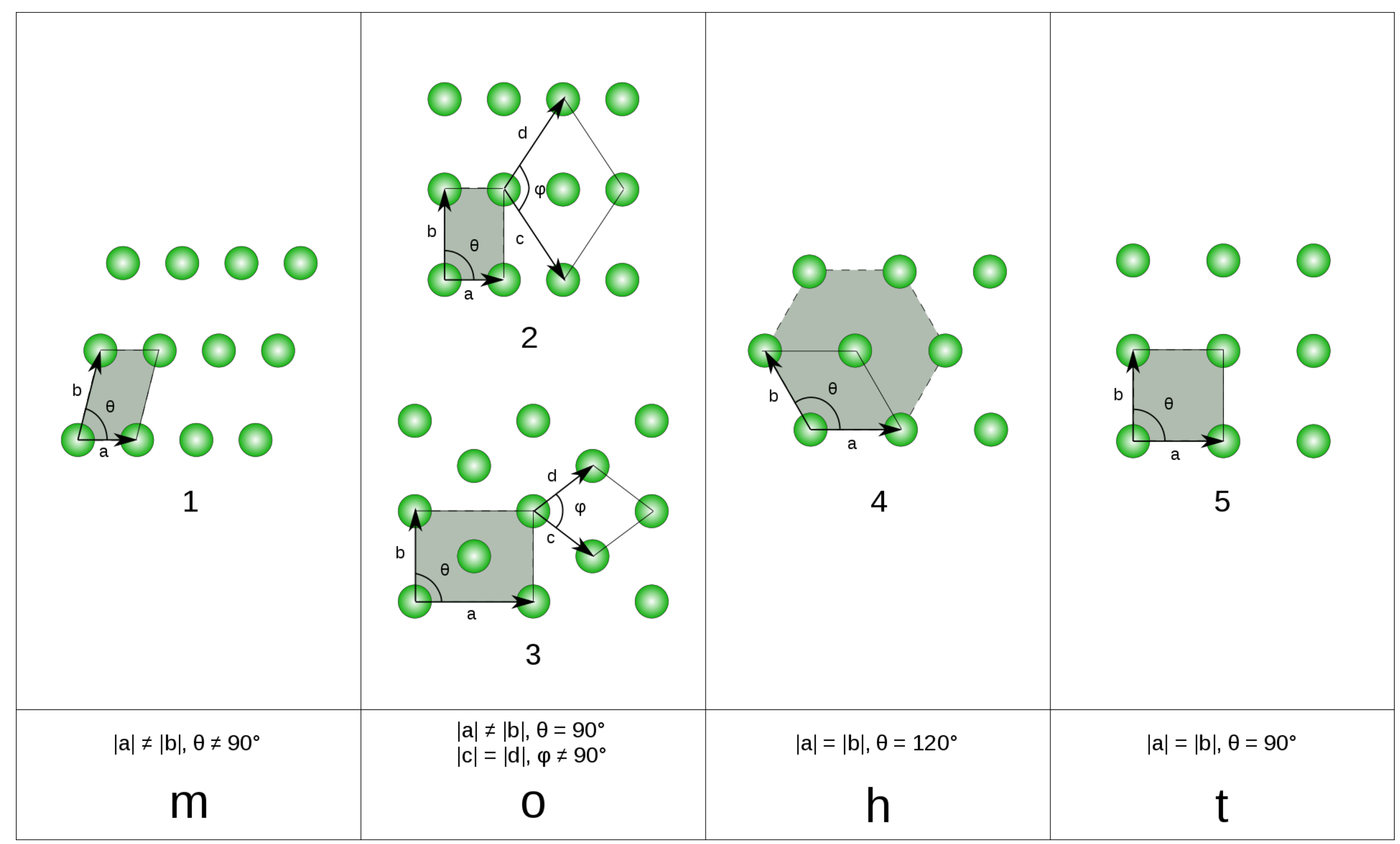
http://www.namaraujo.net

Regular lattices

14 Bravais lattices in 3D



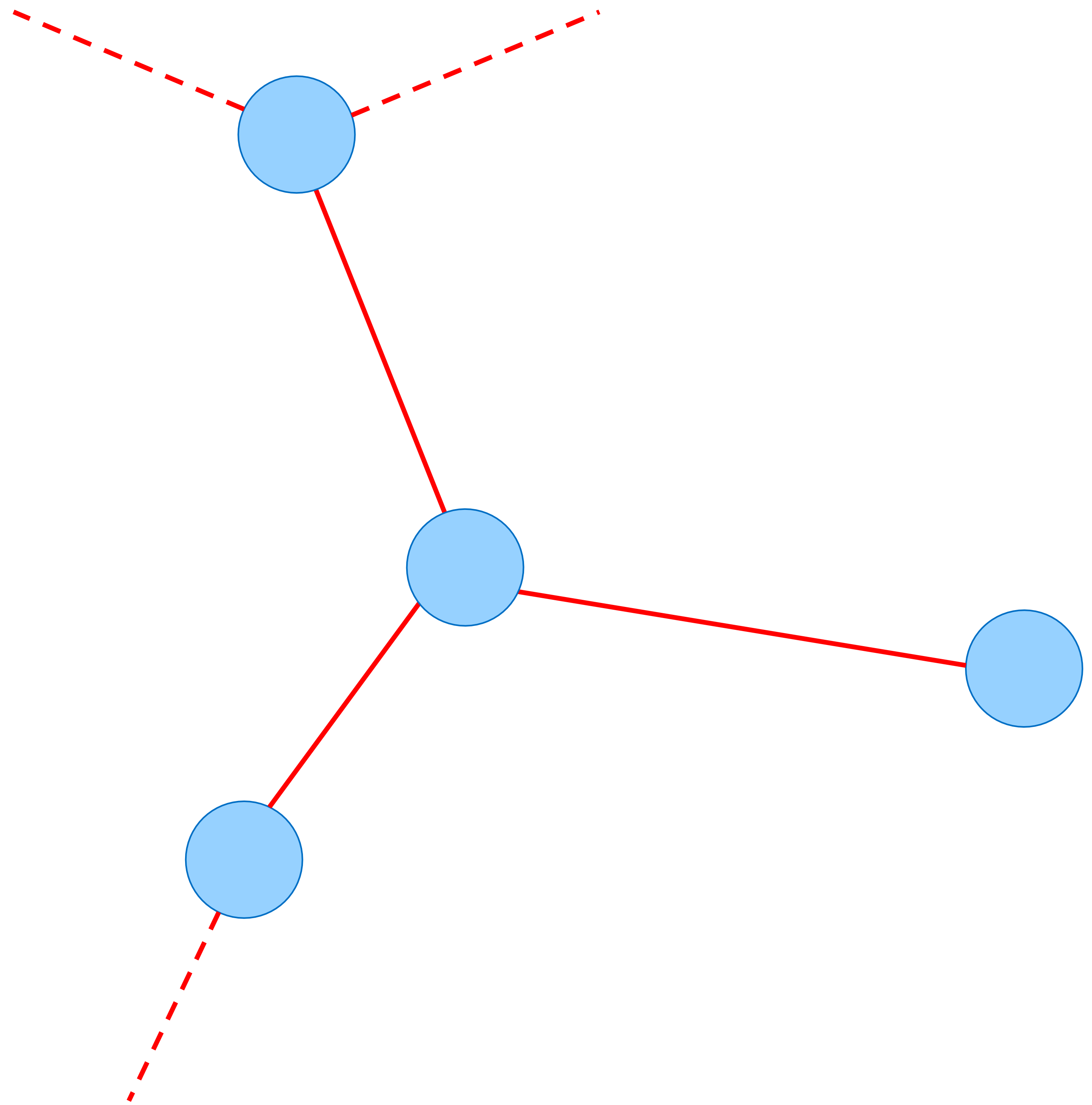
5 Bravais lattices in 2D



https://en.wikipedia.org/wiki/Bravais_lattice
http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/bravais.html

Networks: basic ingredients

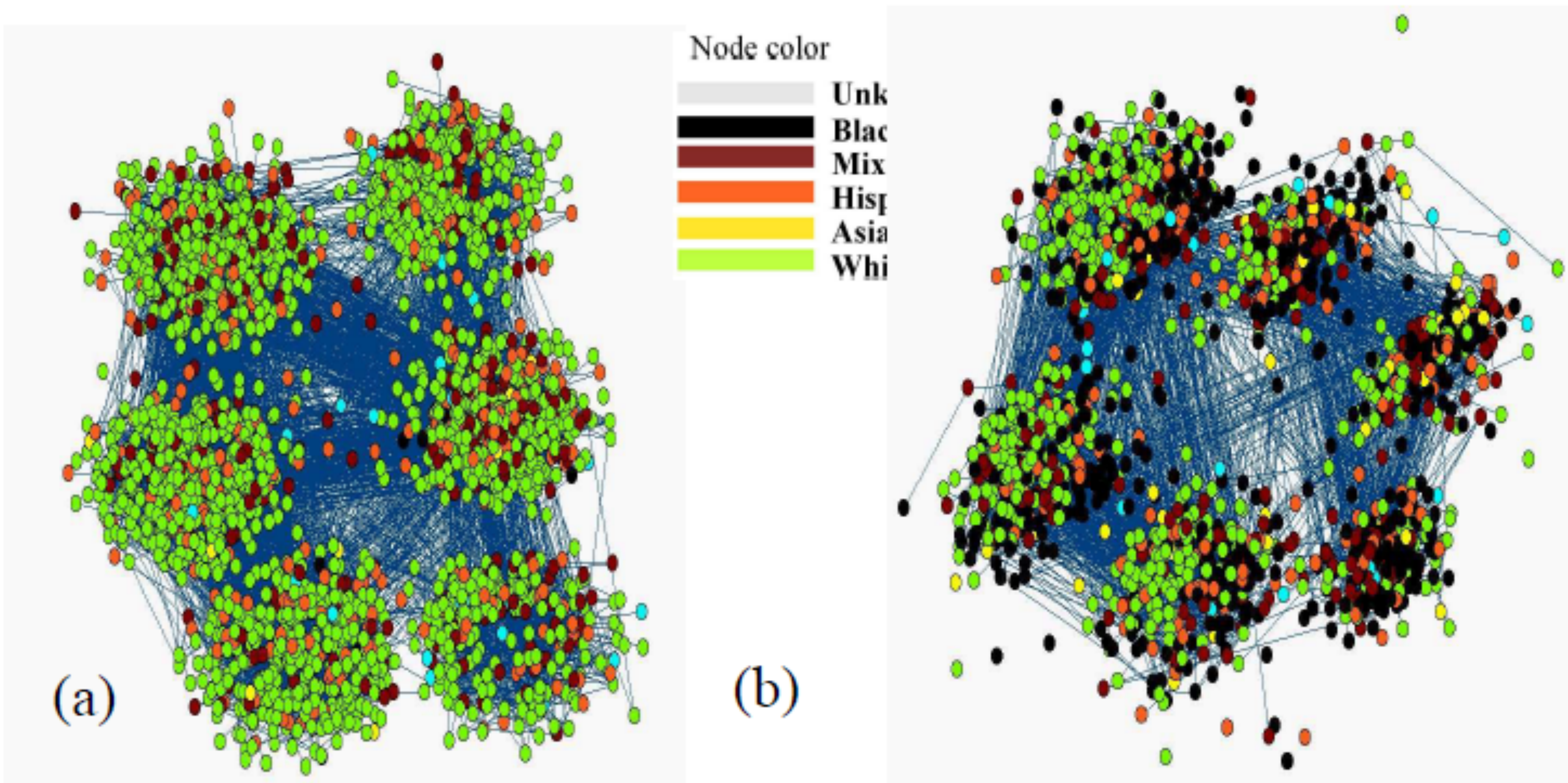
nodes and links



Networks: some examples

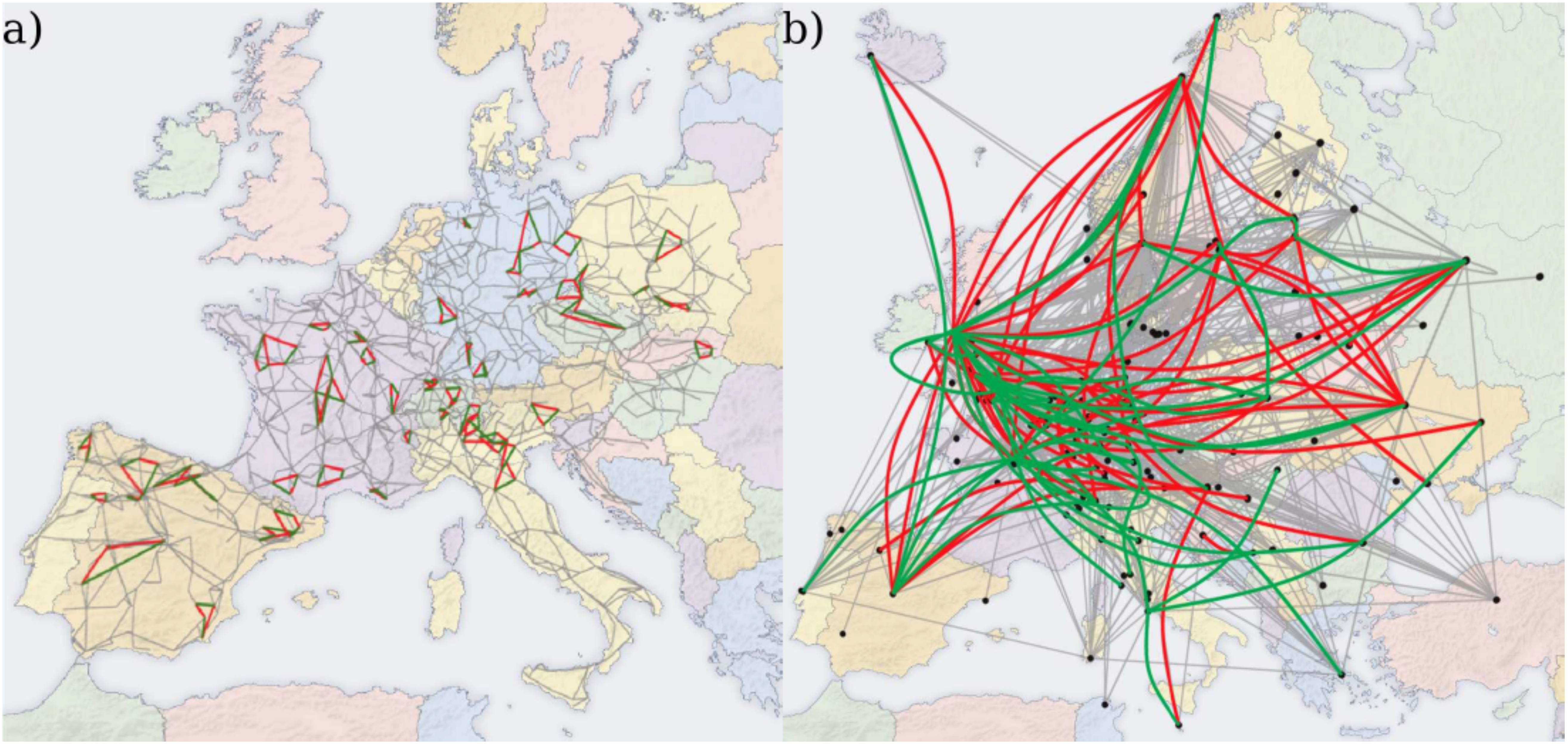
social…

Friendship networks



Physica A **379**, 307 (2007)

Networks: some examples
infrastructural...



1254 generators
1811 power lines

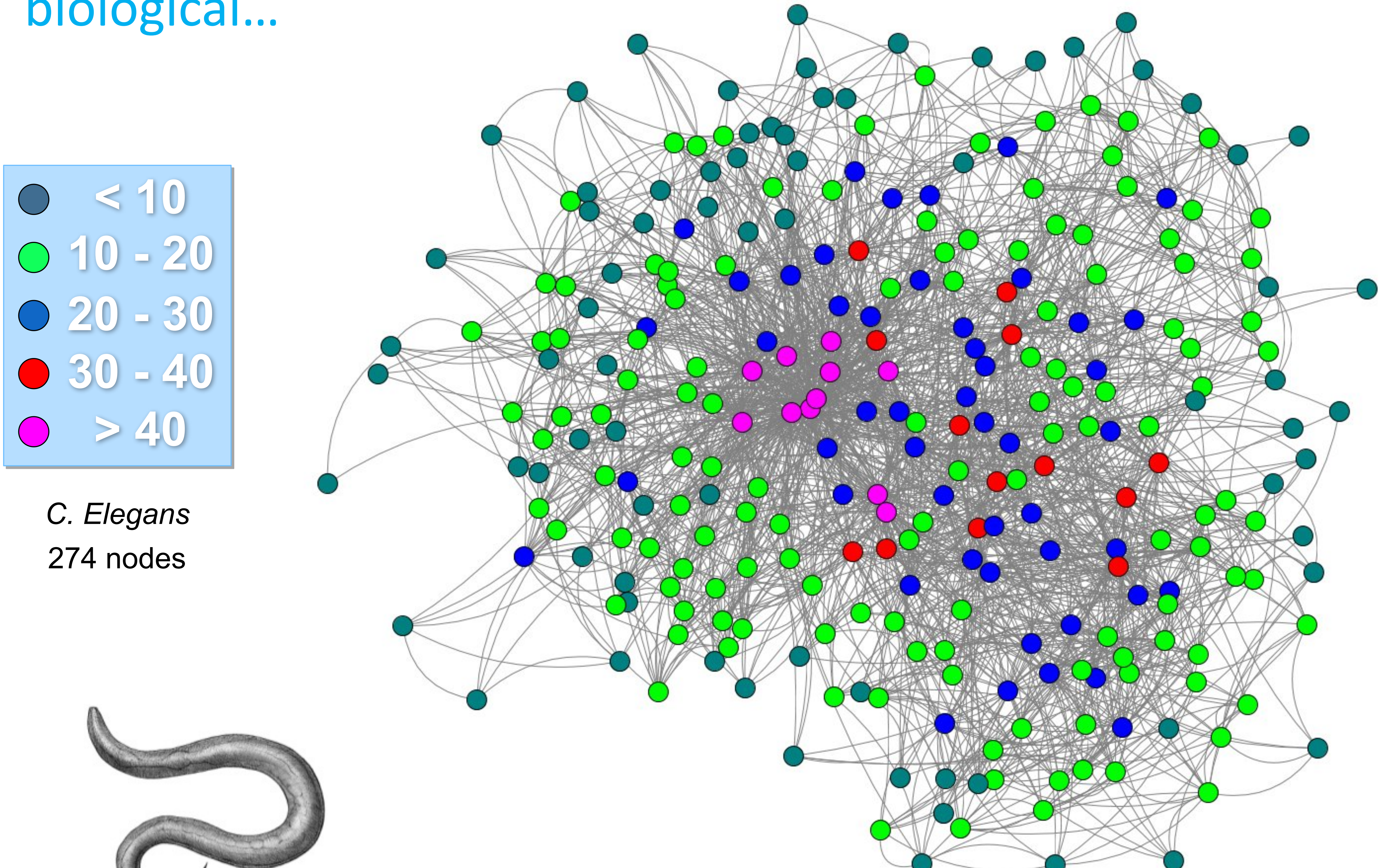
1098 service providers
6098 connections

IEEE Trans. Power Syst. **20**, 782 (2005)

http://vlado.fmf.uni-lj.si/pub/networks/pajek/

Networks: some examples

biological...



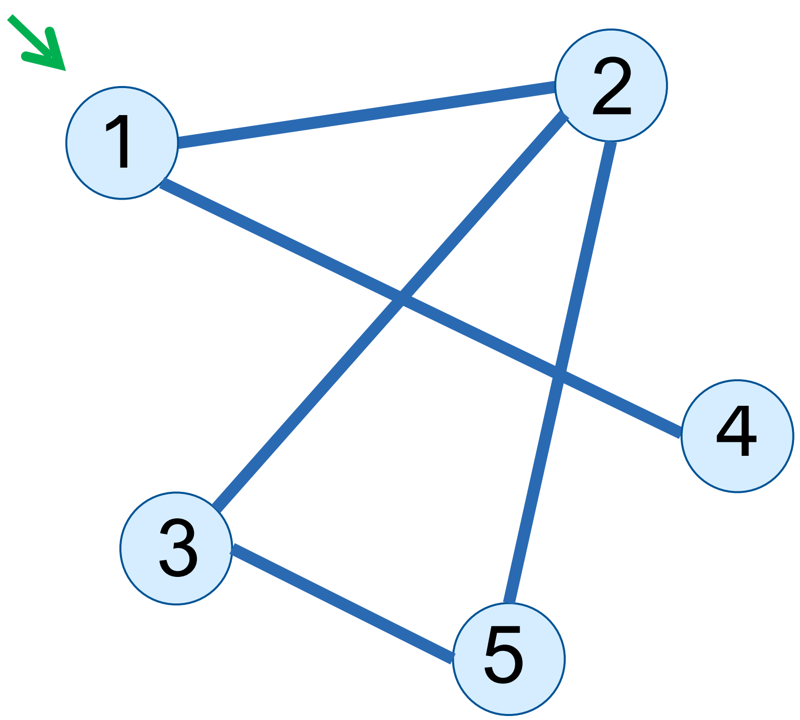
C. Elegans

274 nodes

PNAS **103**, 4723 (2006) *and* http://wormweb.org

Networks

adjacency matrix (A_{ij})



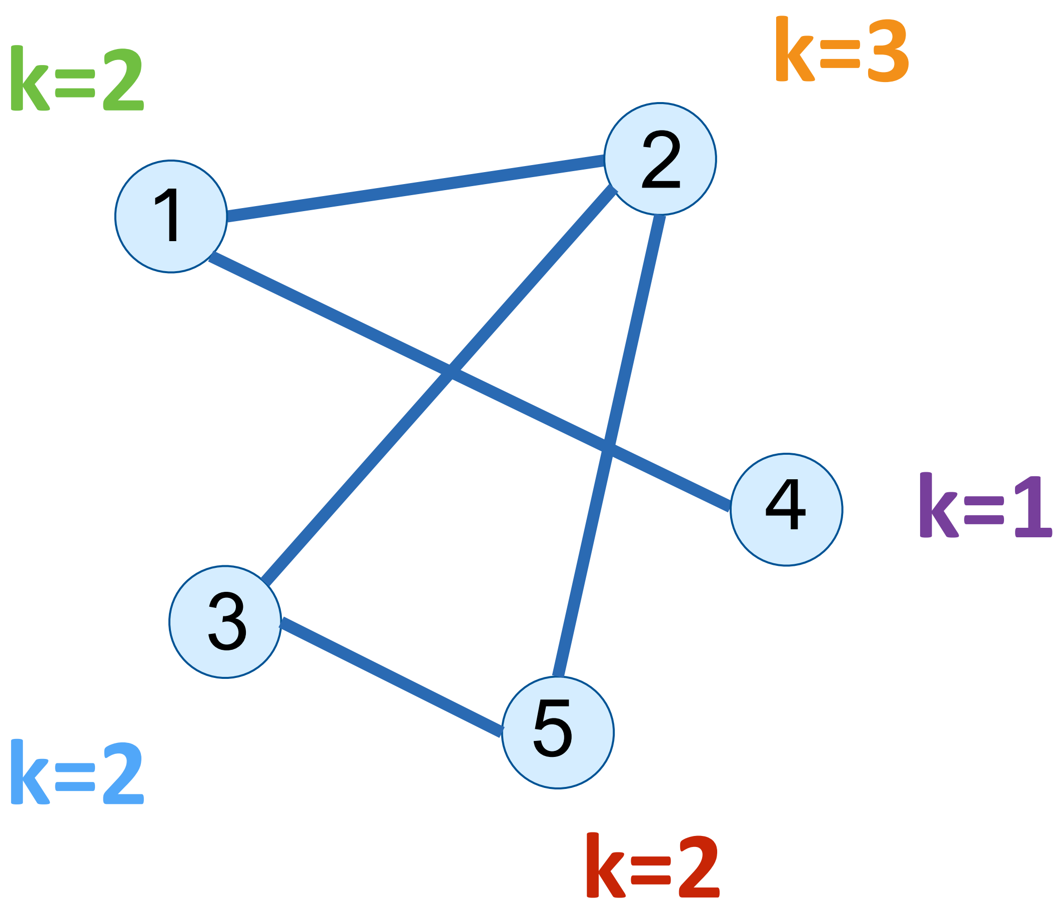
$$A_{ij} = \begin{cases} 1 \rightarrow \text{i and j are connected} \\ 0 \rightarrow \text{i and j are not connected} \end{cases}$$

$$A = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \end{array}$$

Networks

degree (k)

$$A_{ij} = \begin{cases} 1 \rightarrow \text{i and j are connected} \\ 0 \rightarrow \text{i and j are not connected} \end{cases}$$

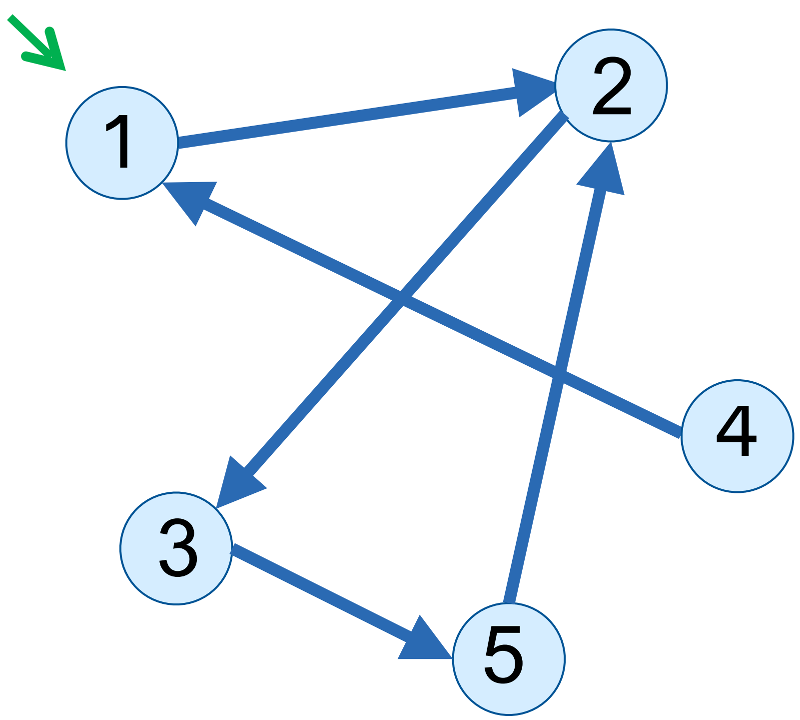


$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(directed) Networks

adjacency matrix (A_{ij})

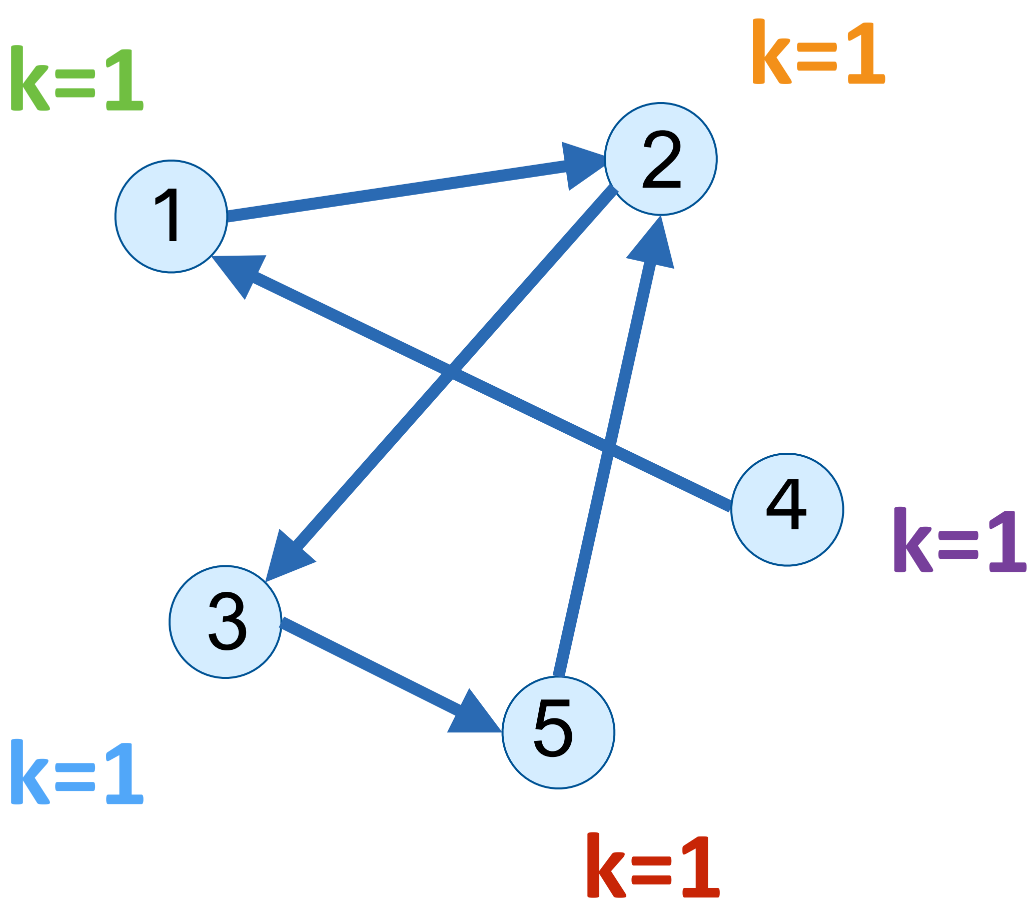
$$A_{ij} = \begin{cases} 1 & \rightarrow \text{ i and j are connected} \\ 0 & \rightarrow \text{ i and j are not connected} \end{cases}$$



$$A = \begin{array}{c c} \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \end{array}$$

(directed) Networks

out-degree (k_{out})

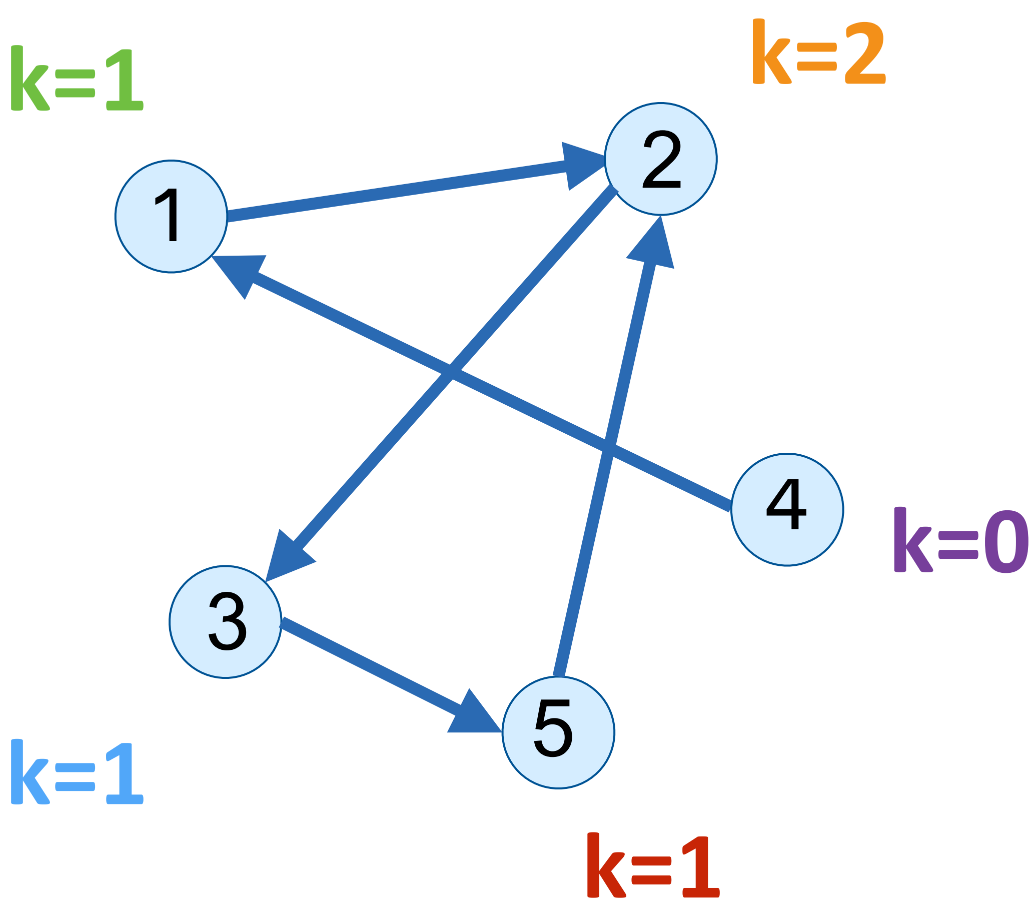


$$A_{ij} = \begin{cases} 1 & \rightarrow \text{i and j are connected} \\ 0 & \rightarrow \text{i and j are not connected} \end{cases}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(directed) Networks

in-degree (k_{in})



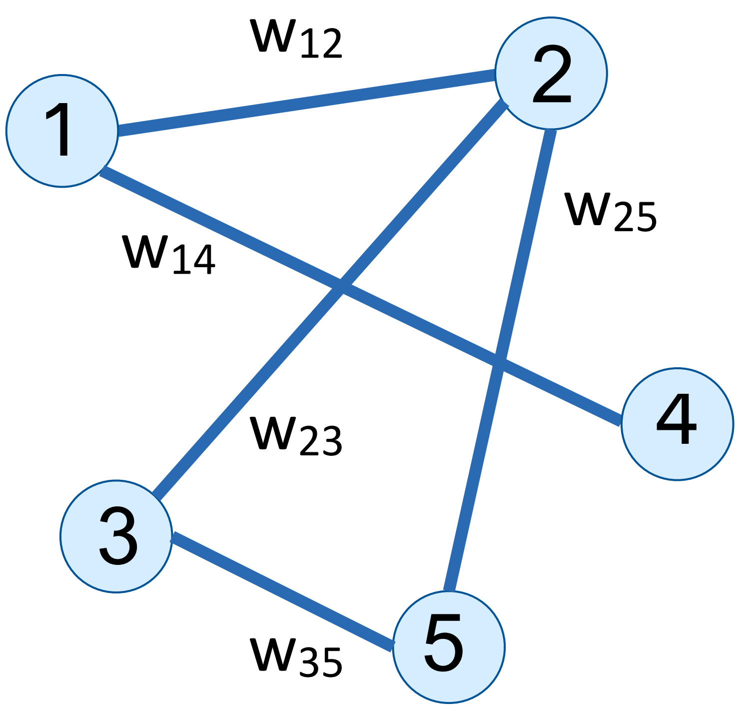
$$A_{ij} = \begin{cases} 1 & \rightarrow \text{ i and j are connected} \\ 0 & \rightarrow \text{ i and j are not connected} \end{cases}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(weighted) Networks

adjacency matrix (A_{ij})

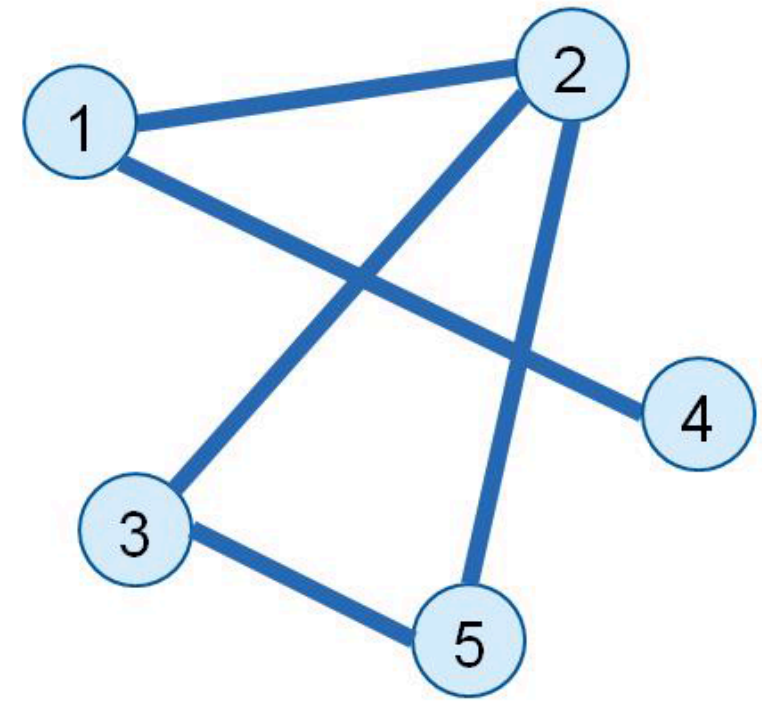
$$A_{ij} = \begin{cases} w_{ij} & \rightarrow \text{ i and j are connected} \\ 0 & \rightarrow \text{ i and j are not connected} \end{cases}$$



$$A = \begin{bmatrix} 0 & w_{12} & 0 & w_{14} & 0 \\ w_{12} & 0 & w_{23} & 0 & w_{25} \\ 0 & w_{23} & 0 & 0 & w_{35} \\ w_{14} & 0 & 0 & 0 & 0 \\ 0 & w_{25} & w_{35} & 0 & 0 \end{bmatrix}$$

Networks

list of neighbors



1st neighbor

2nd neighbor

3rd neighbor

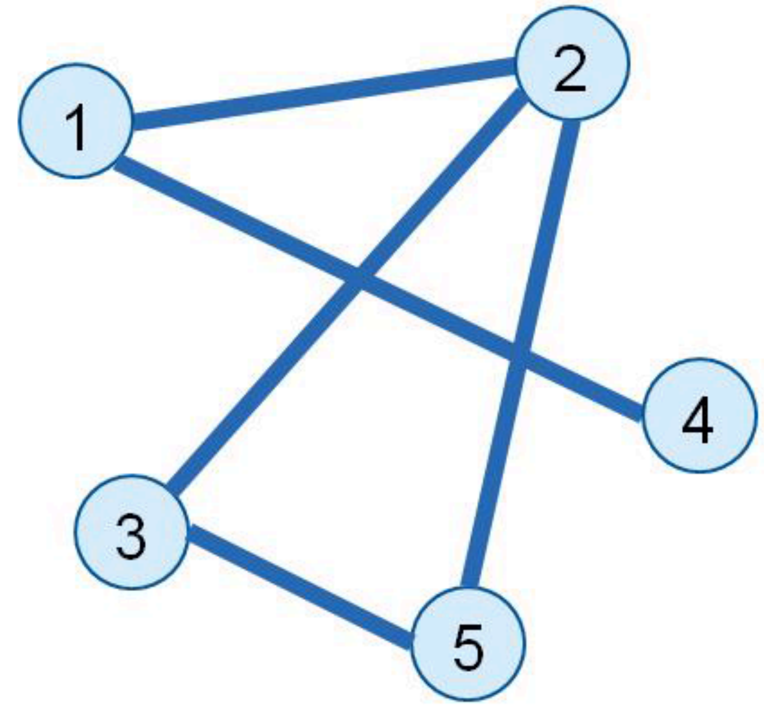
1	2	2	4	
2	3	1	3	5
3	2	2	5	
4	1	1		
5	2	2	3	

Number of neighbors

$$
A = \begin{array}{c c}
\begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} & \\
\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}
\end{array}
$$

Networks

random walk (transport properties)



	Number of neighbors	1st neighbor	2nd neighbor	3rd neighbor
1	2	2	4	
2	3	1	3	5
3	2	2	5	
4	1	1		
5	2	2	3	

Iteratively:

- Generate a random number between 1 and the number of neighbors;
- Move along the corresponding link.

Networks

shortest path

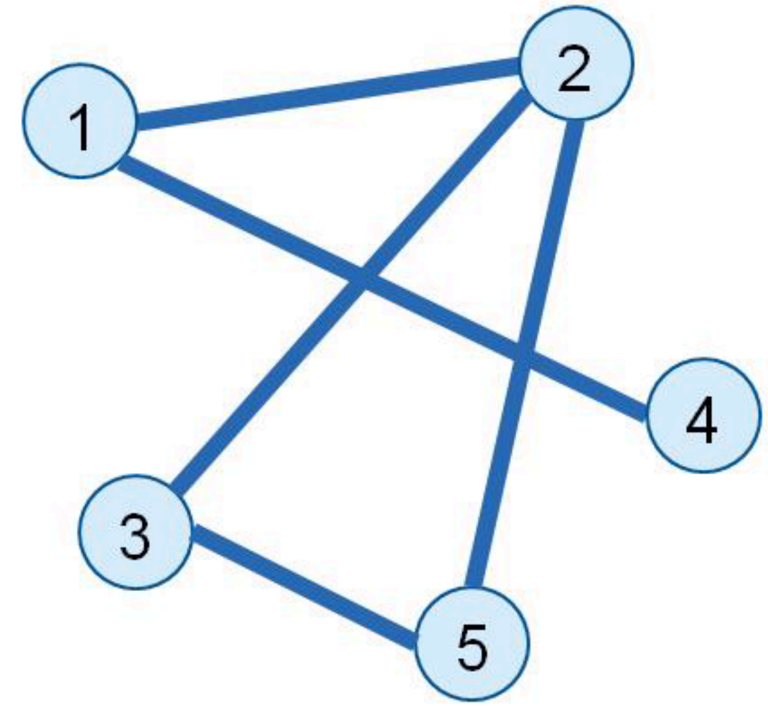
1st neighbor

2nd neighbor

3rd neighbor

1	2	2	4	
2	3	1	3	5
3	2	2	5	
4	1	1		
5	2	2	3	

Number of neighbors



Iteratively:

- Burning method following the links.

Networks

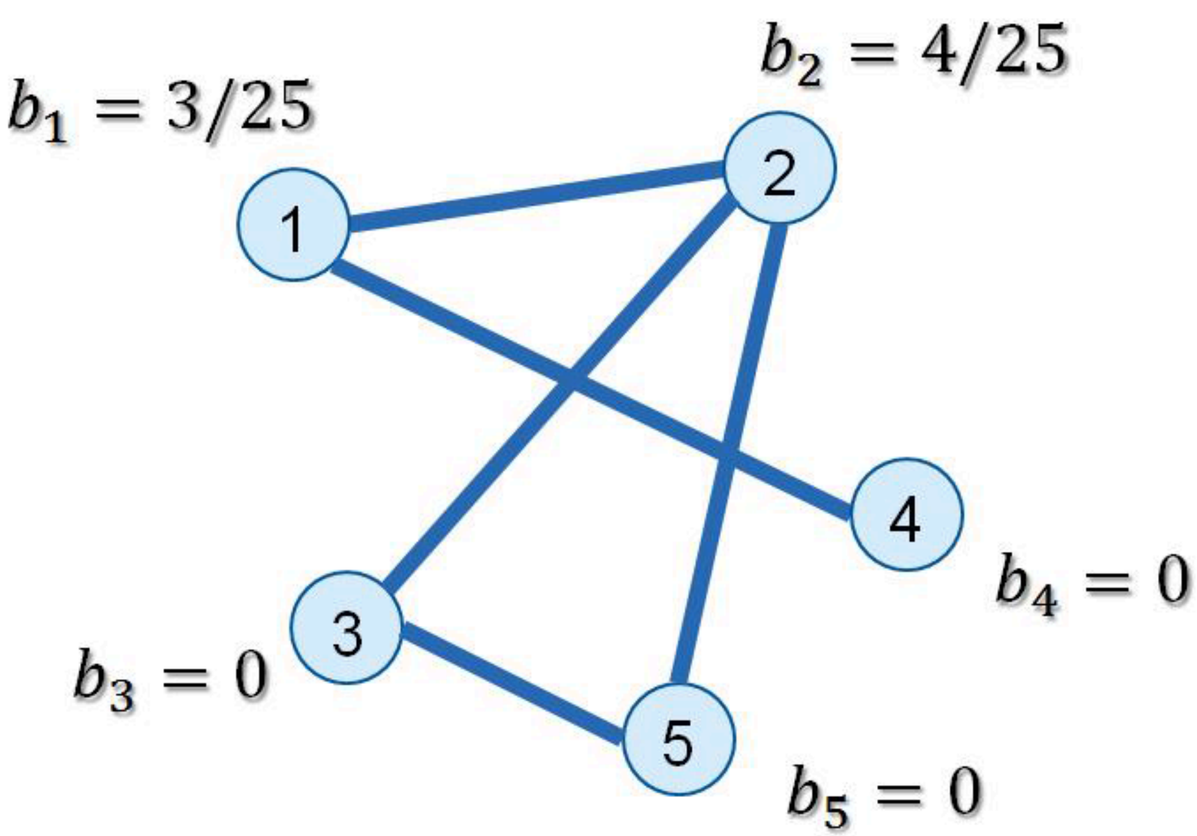
betweenness centrality (b_i)

$$b_i = \frac{1}{N^2} \sum_{s \neq i \neq t} \frac{n_{st}(i)}{n_{st}}$$

Fraction of *shortest paths* from all vertices to all others that include the node.

n_{st}: number of shortest paths between *s* and *t*.

$n_{st}(i)$: number of shortest paths between *s* and *t* going through *i*



Iteratively for each pair (s,t):

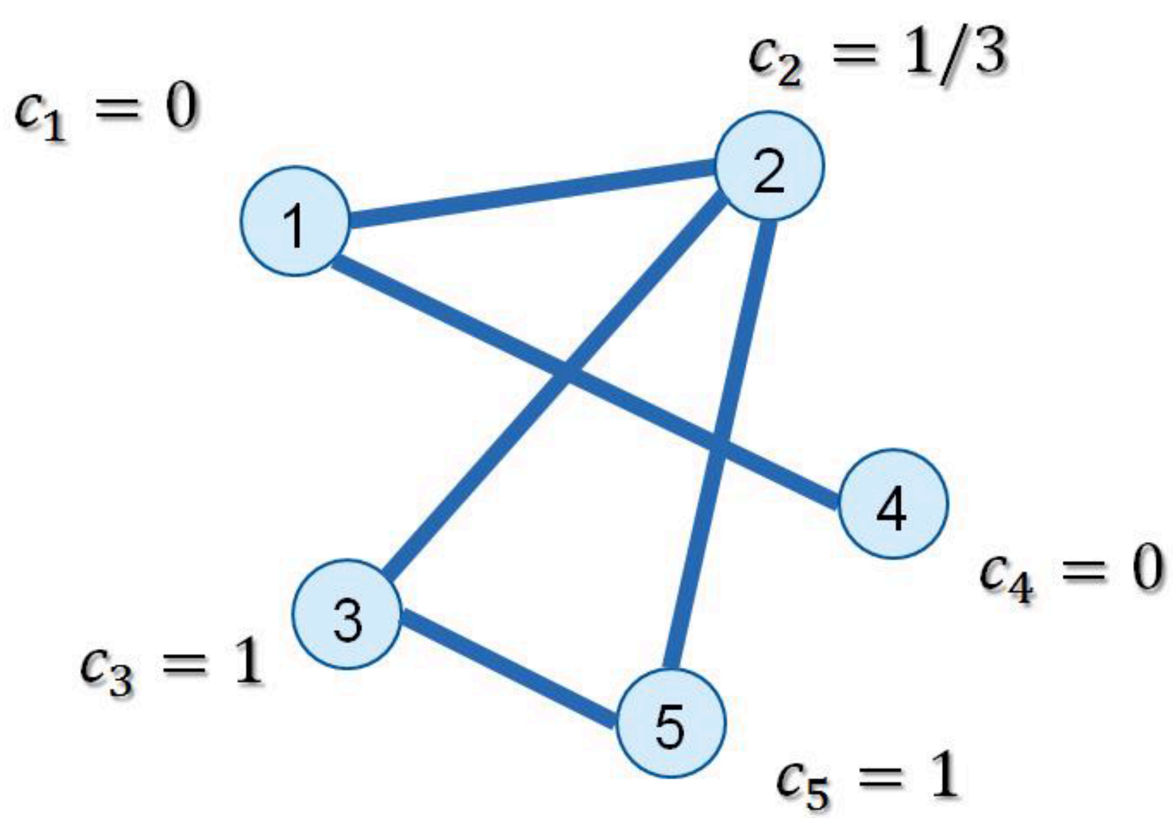
- Identify the shortest paths;
- Count $n_{st}(i)$ and n_{st}.

Networks

cluster coefficient (c_i)

$$C_i = \frac{2n_{jk}}{k_i(k_i - 1)}$$

n_{jk}: number of pairs of neighbors of *i* that are connected.

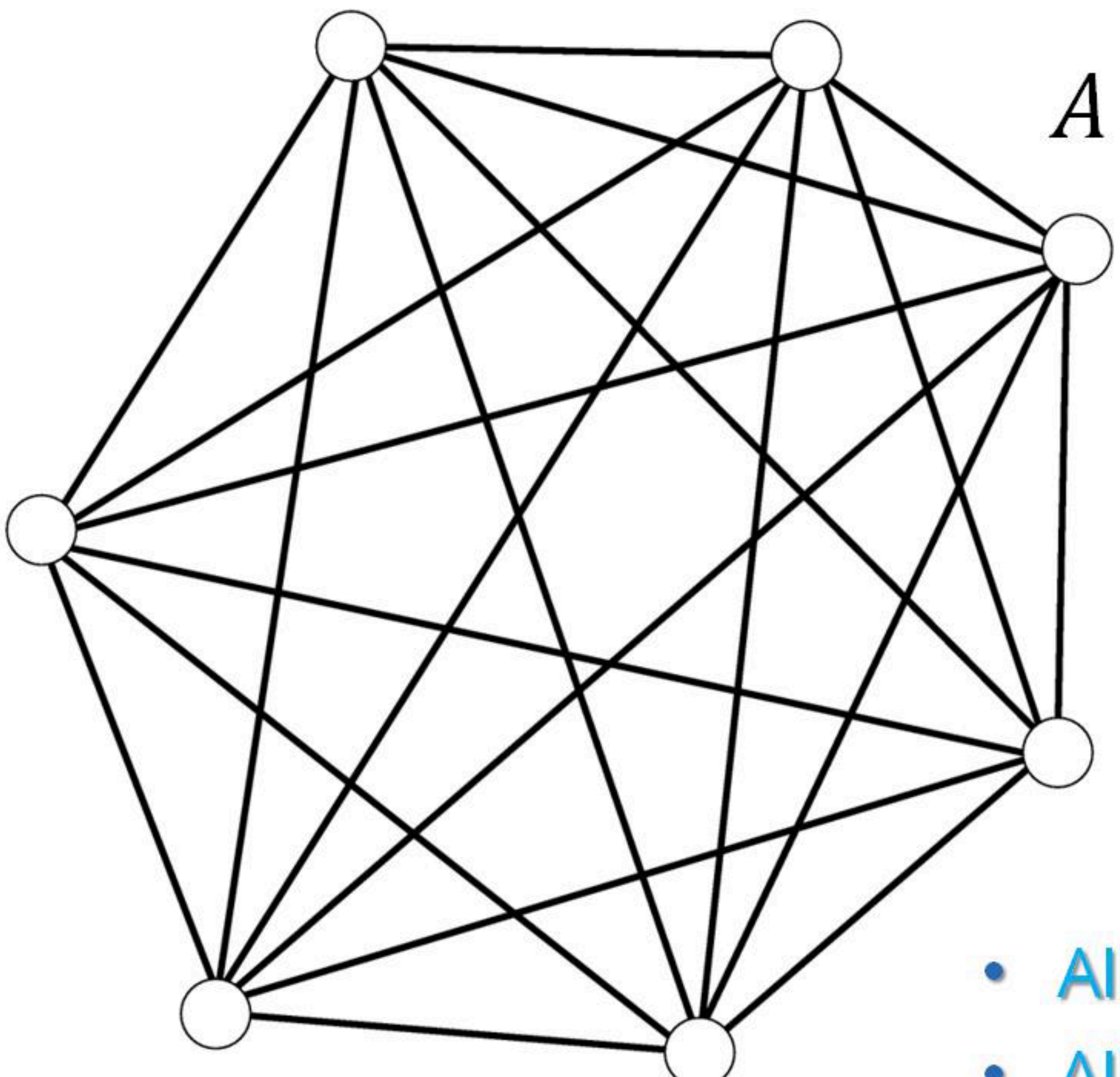


Iteratively for each site i:

- Compute the degree;
- Compute n_{jk}.

Network models

fully connected graph



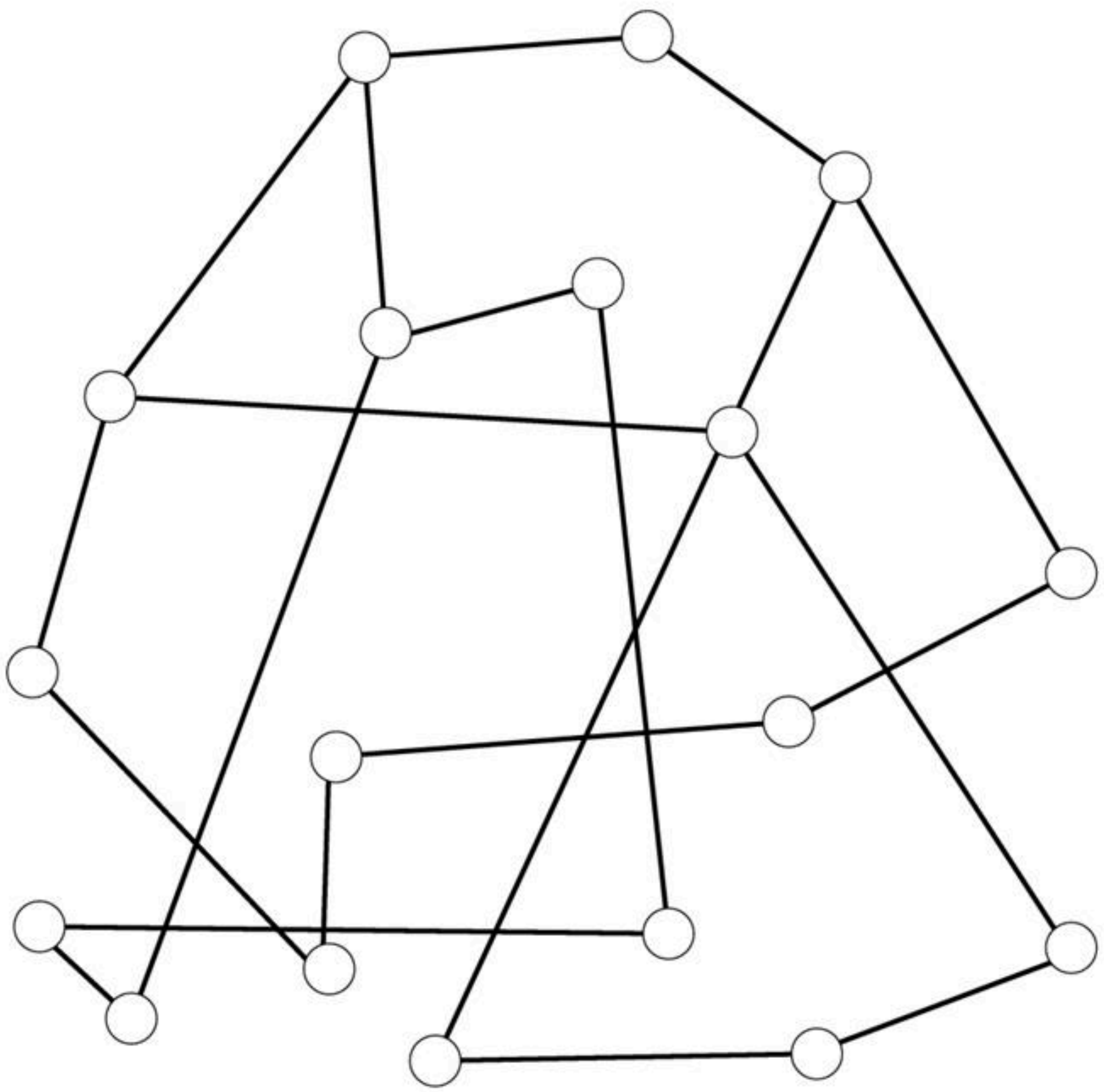
$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

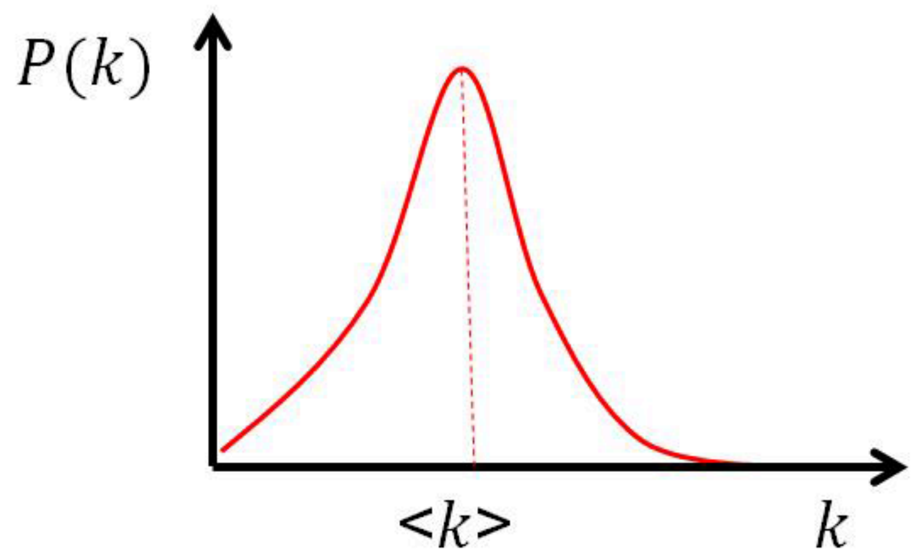
- All nodes interconnected;
- All nodes have the same degree (N-1);
- The shortest path between pairs is one.

Network models

random graph

$$P(k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$



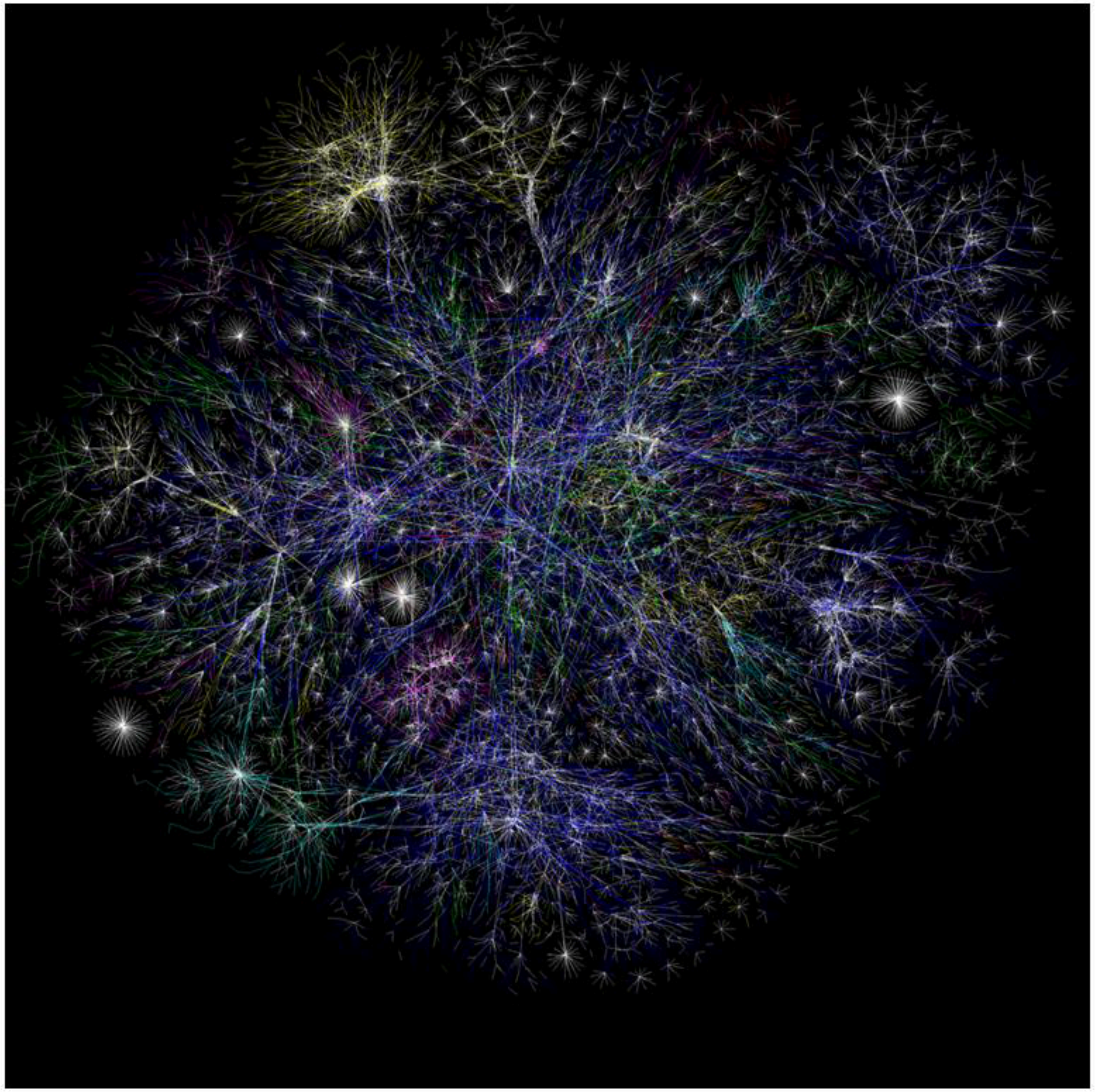


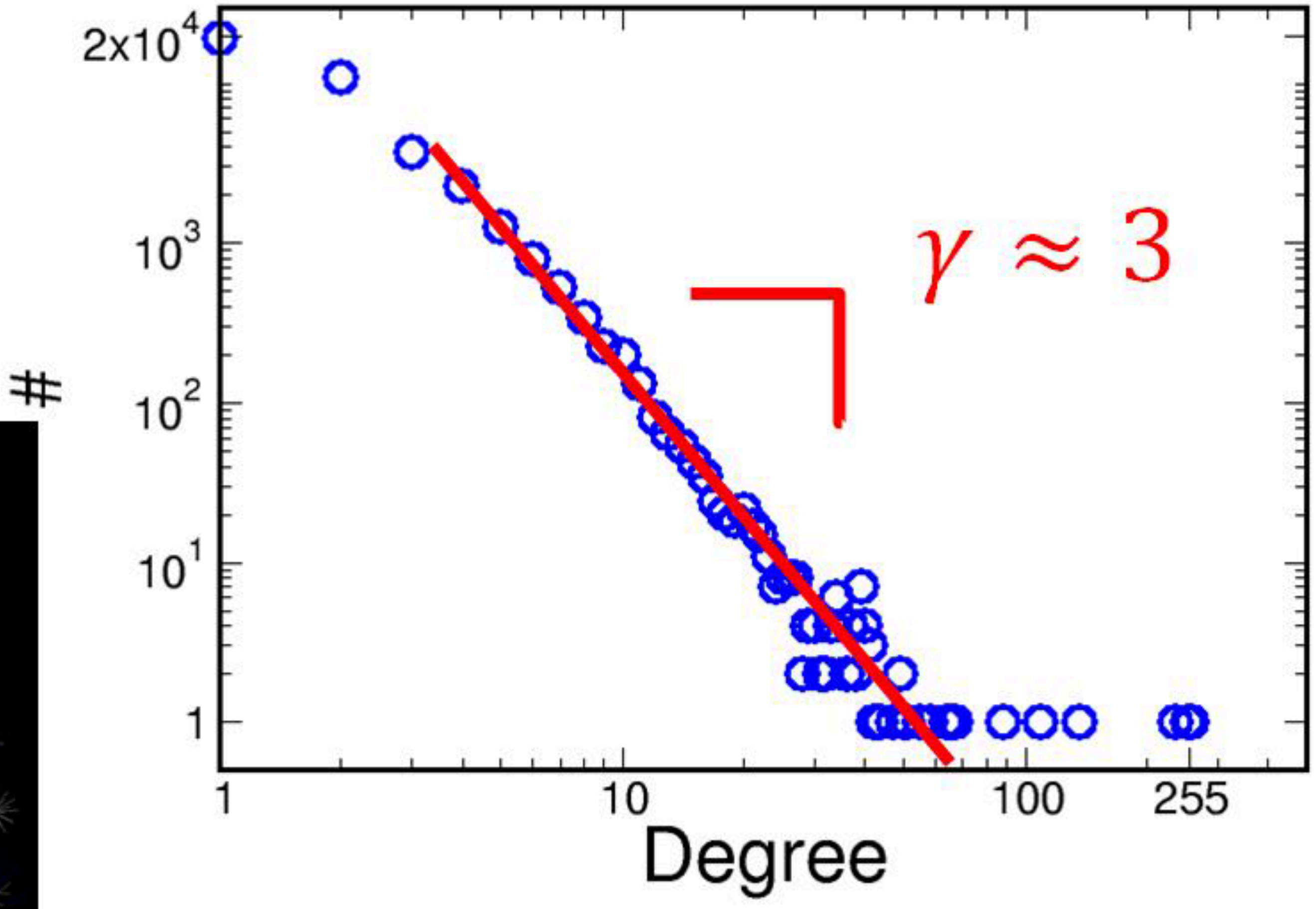
$$A_{ij} = \begin{cases} 1 & \rightarrow \text{with probability } p \\ 0 & \rightarrow \text{with probability } 1-p \end{cases}$$

- Nodes are connected with probability p;
- Average degree $<k>=(N-1)p$.

Network models

Internet (?)

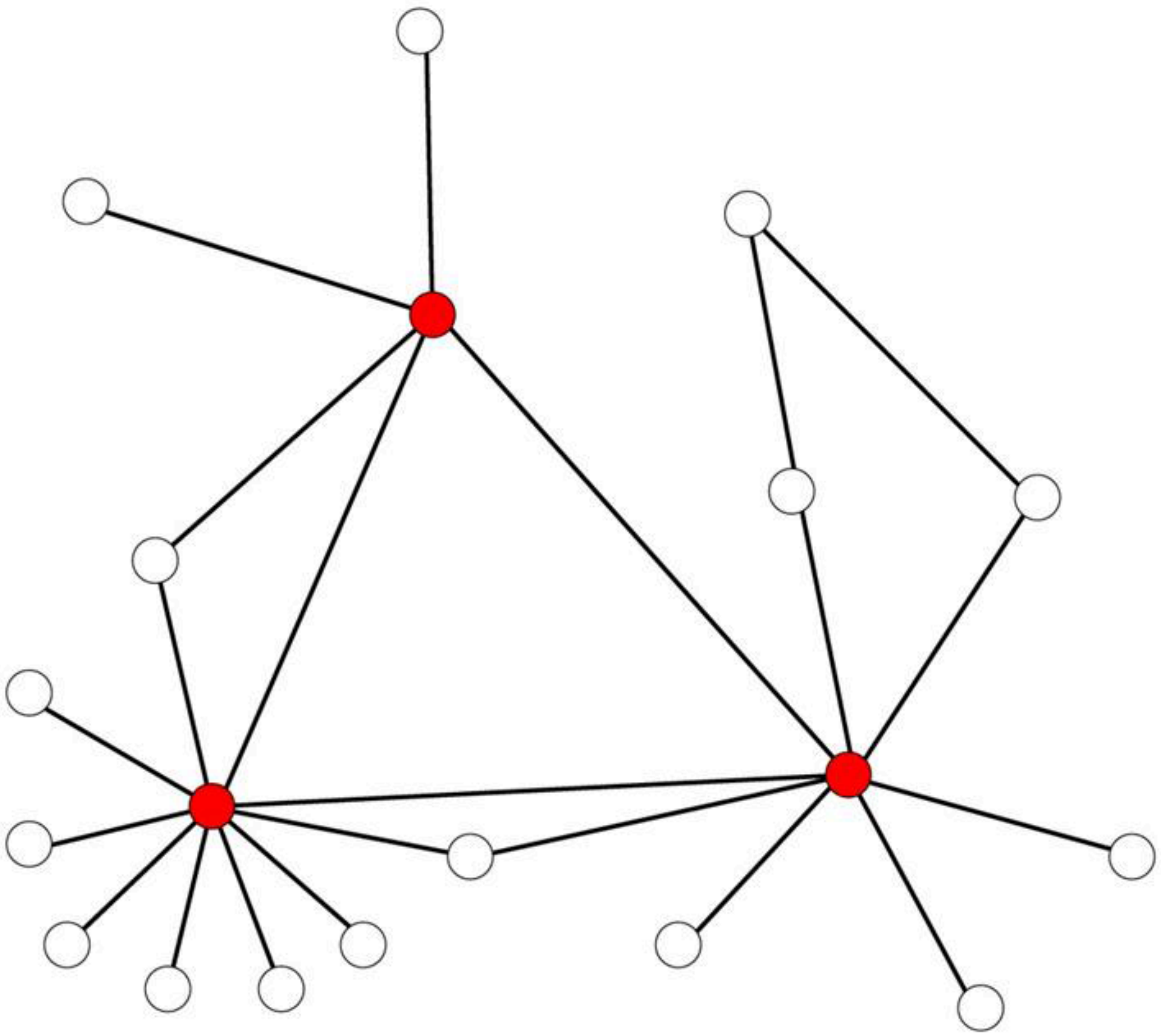




$$P(k) \sim k^{-\gamma}$$

Network models

preferential attachment (BA)



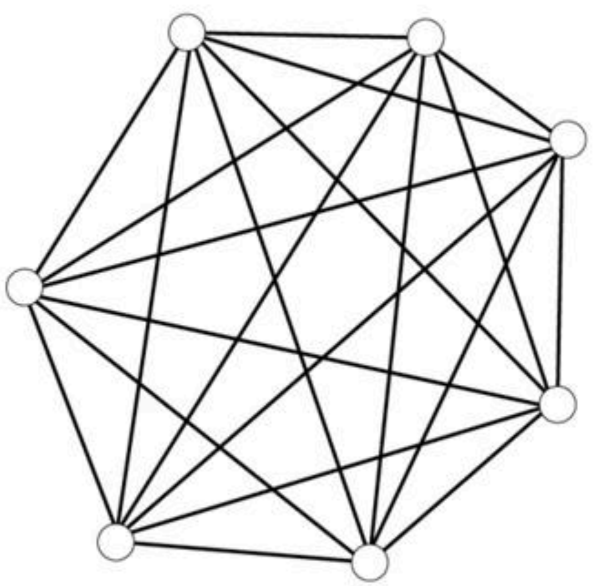
- Starts with a small core fully connected;
- Add new nodes with m links connected to old nodes;
- Each new link is connected to a old node i with a probability proportional to the degree ki.

- Connection probability proportional to the degree;
- Average degree <k>=2m;
- Presence of hubs;

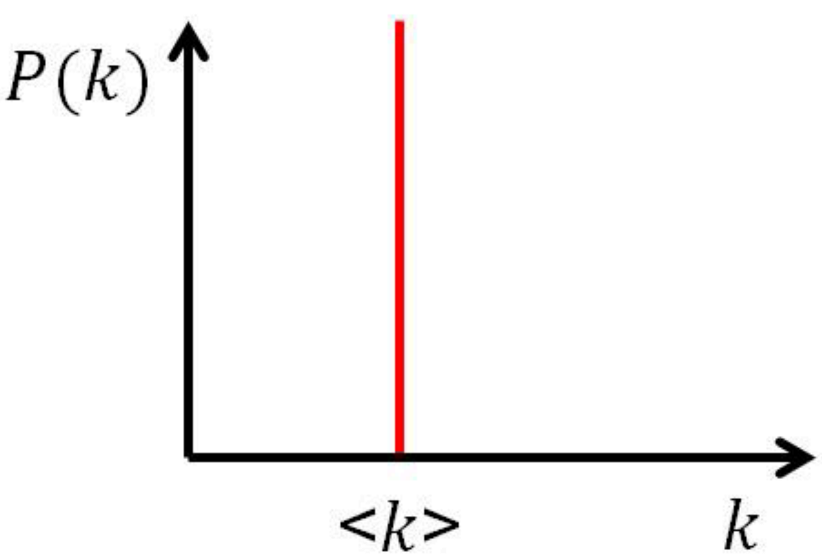
Network models

degree distribution

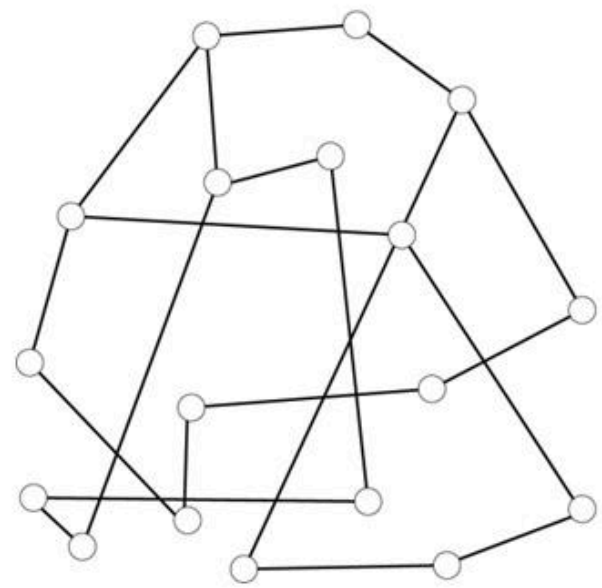
Fully connected



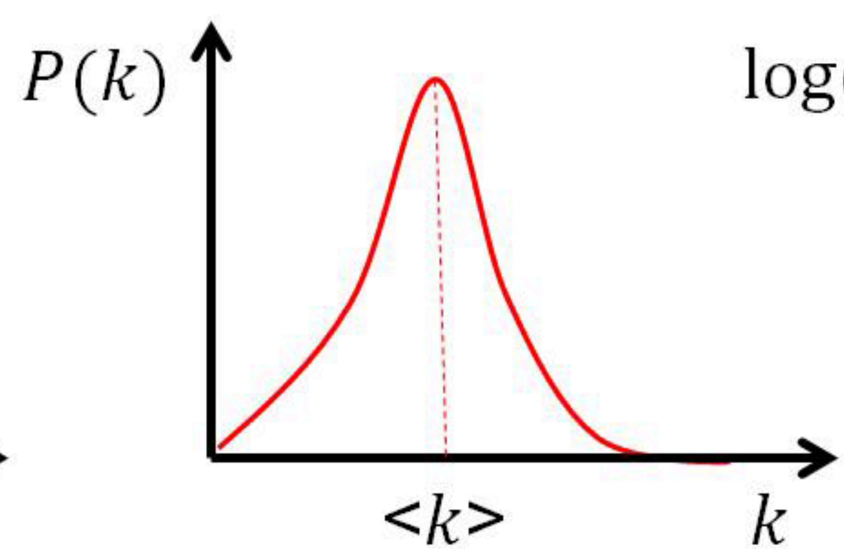
$$k = N - 1$$



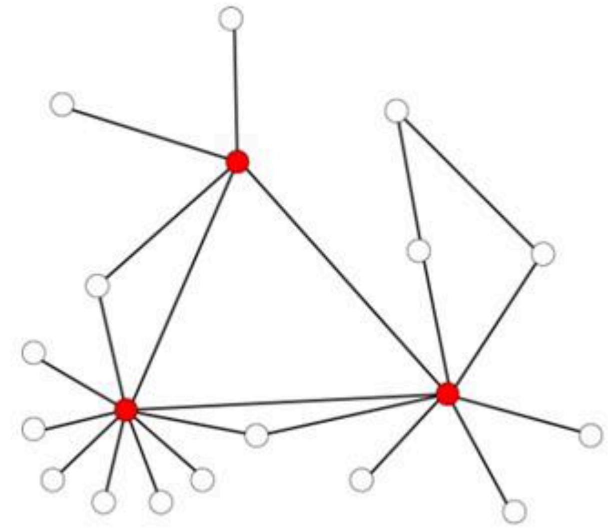
Random



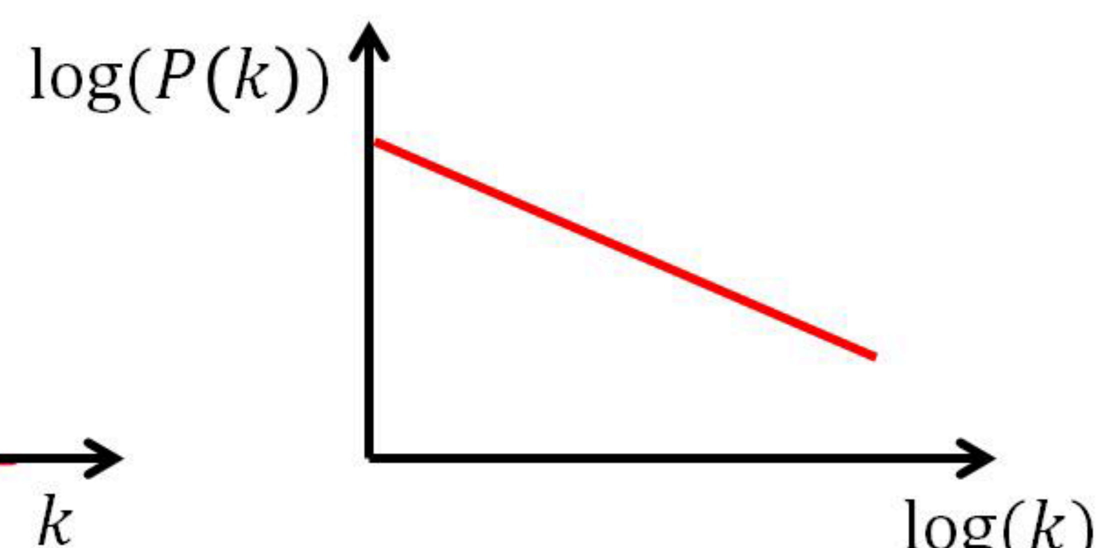
$$P(k) = \exp(-\lambda)\frac{\lambda^k}{k!}$$



BA network

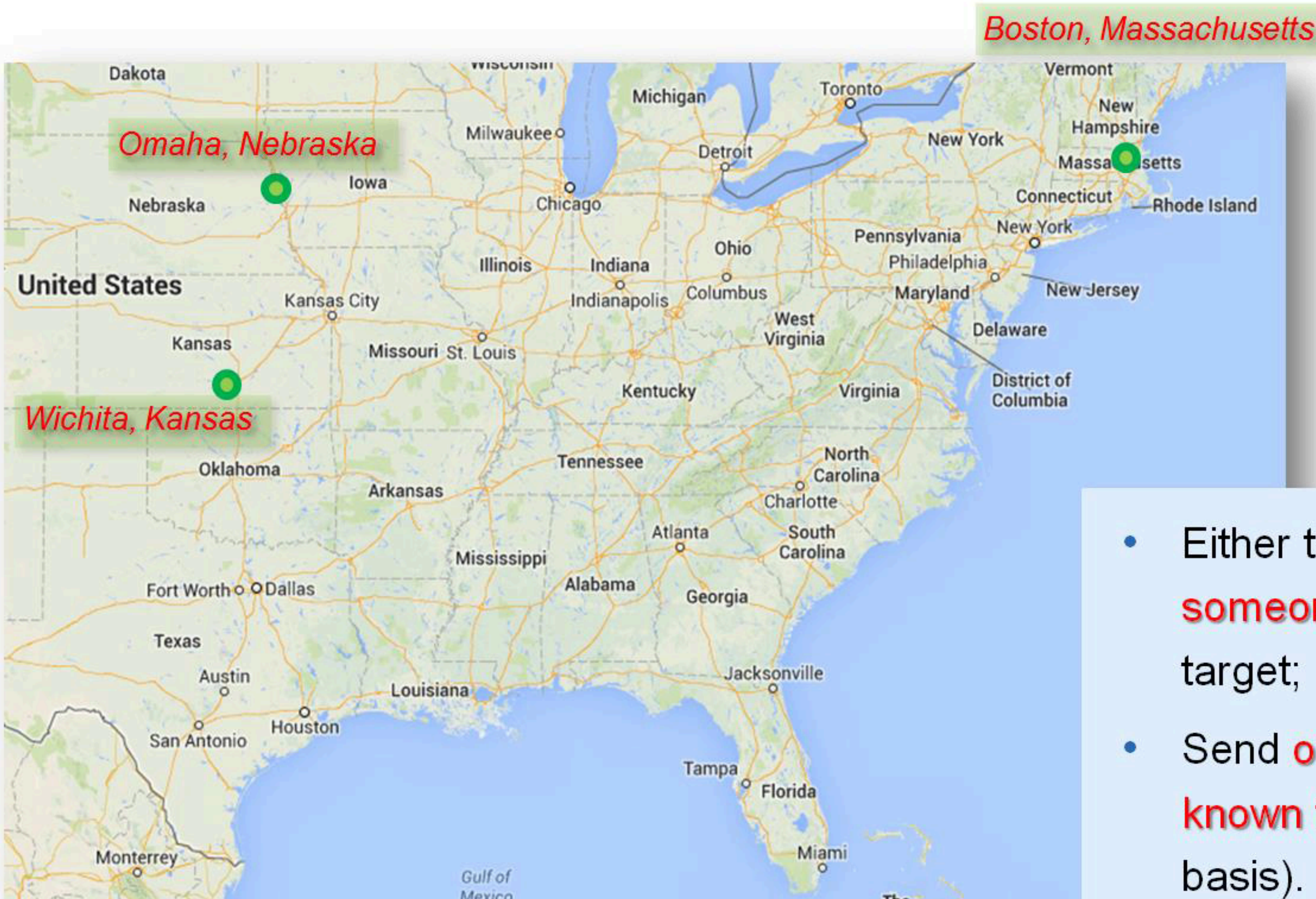


$$P(k) \sim k^{-\gamma}$$



Network models

small-world experiment (1967)



Stanley Milgram

(1933-1984)

- Either to the target or someone likely to know the target;
- Send only to personally known friends (first-name basis).

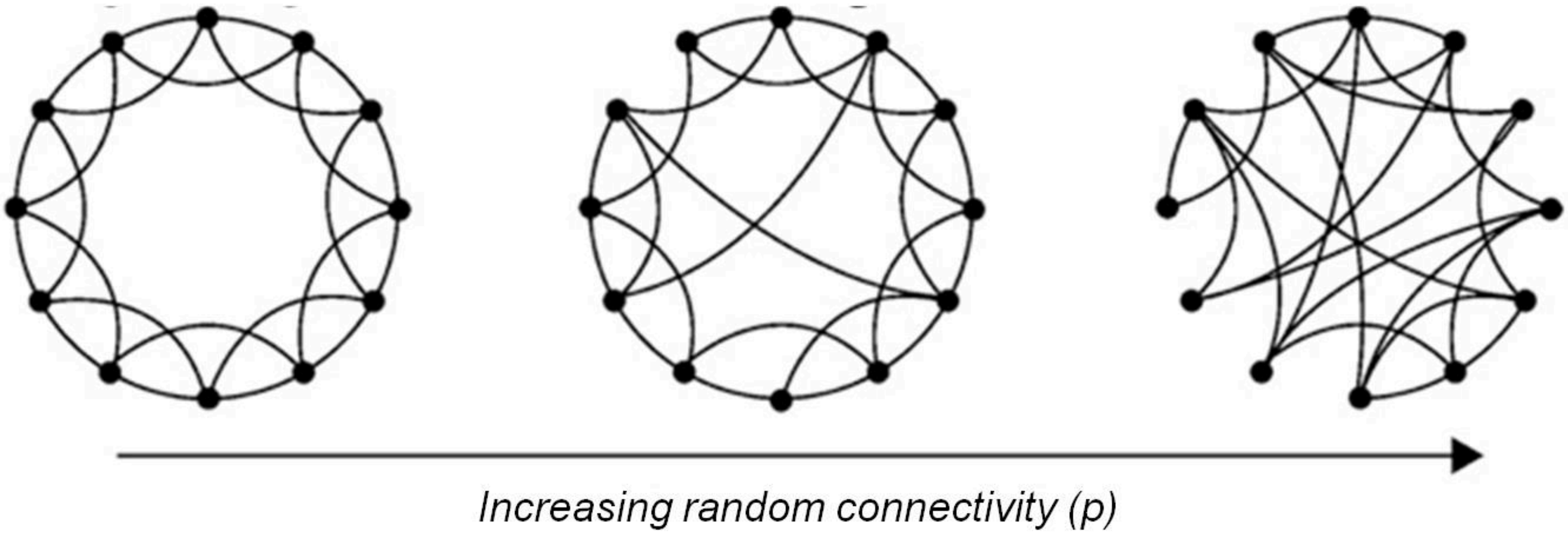
Network models

small world

$$\boxed{\langle \bar{\ell} \rangle \sim \ln N}$$

Duncan Watts

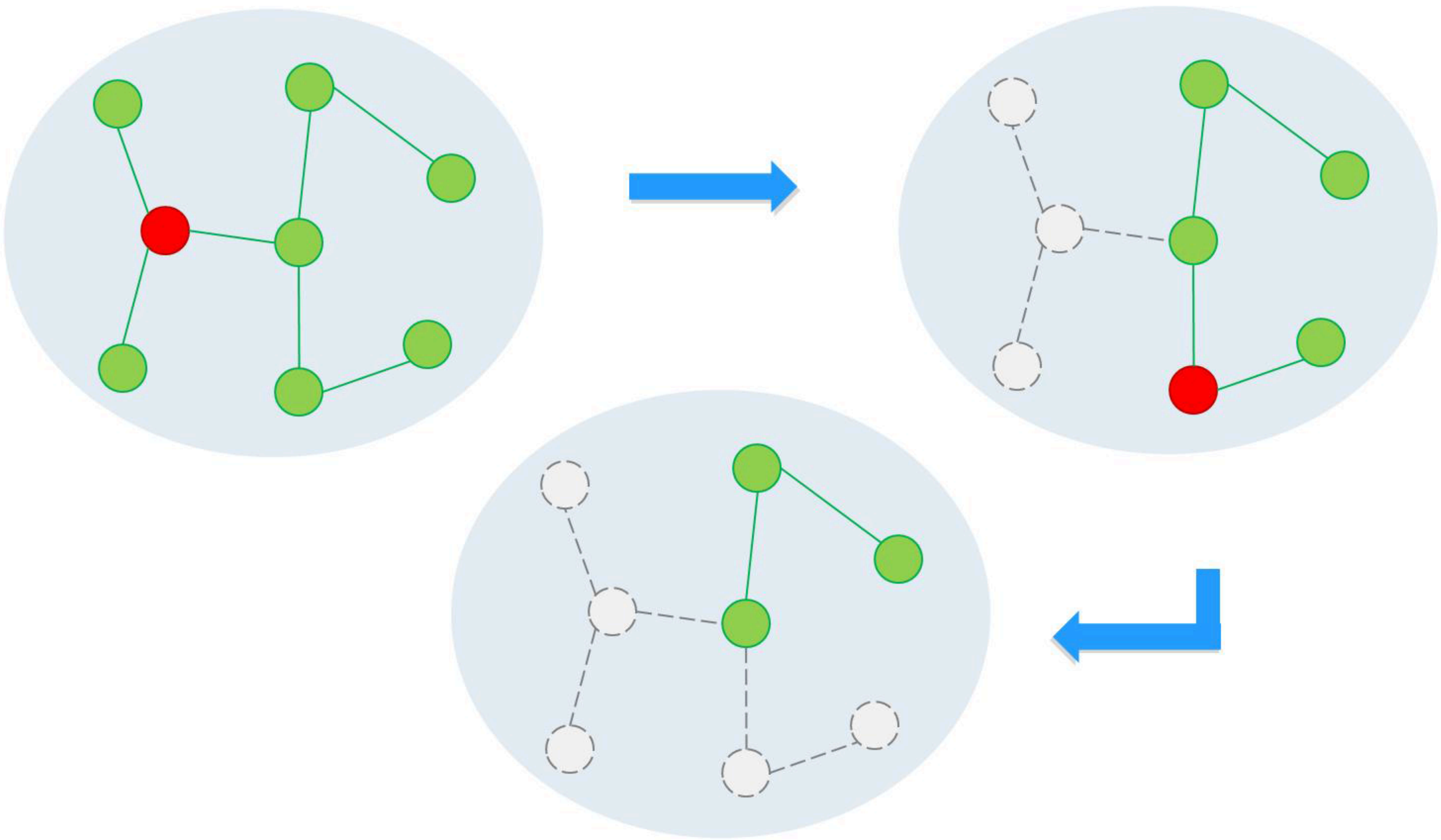
Steven Strogatz



Watts and Strogatz, *Nature* **393**, 6684 (1998)

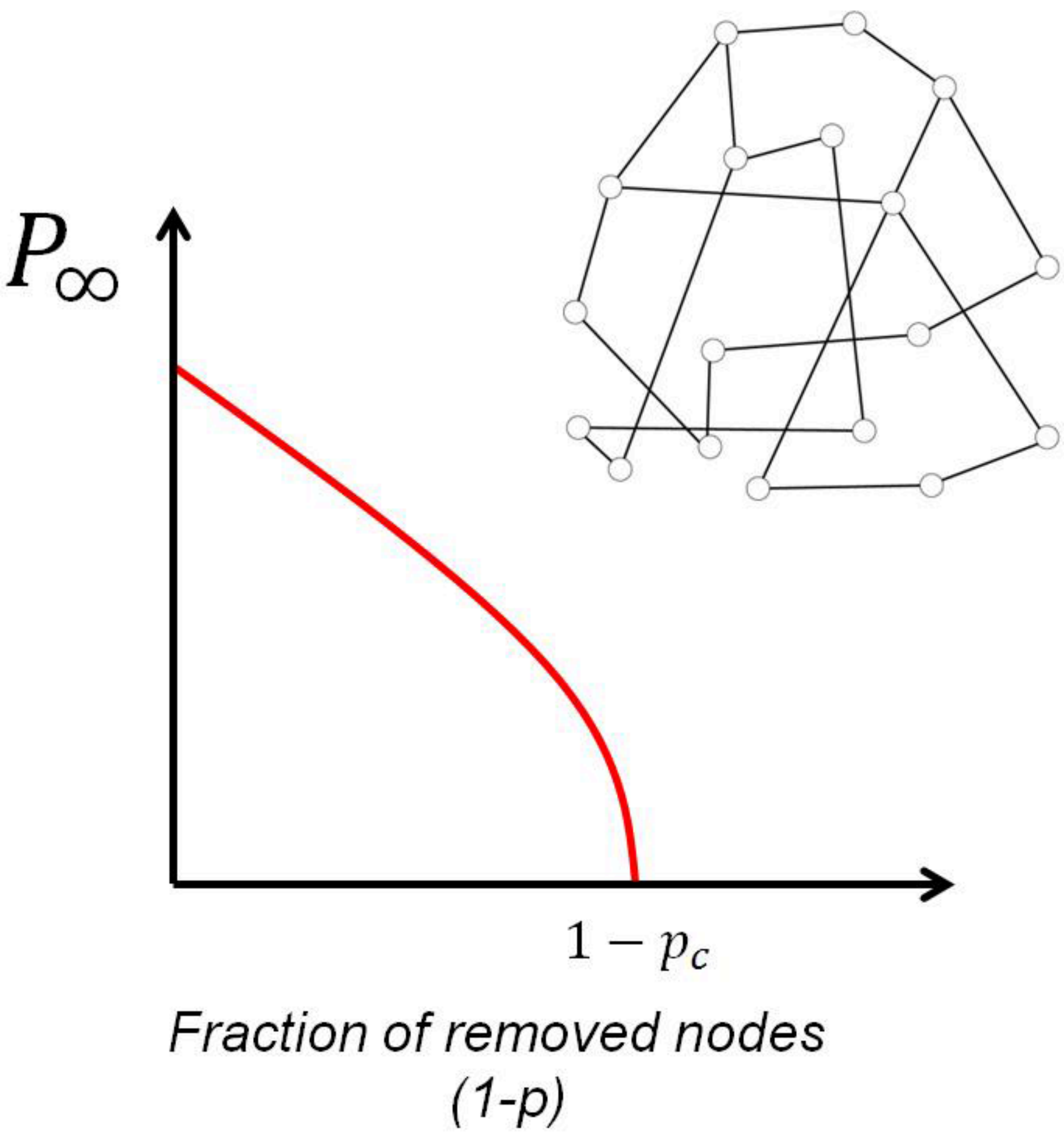
Network resilience

percolation: the quest of global connectivity



Network resilience

percolation: random graph



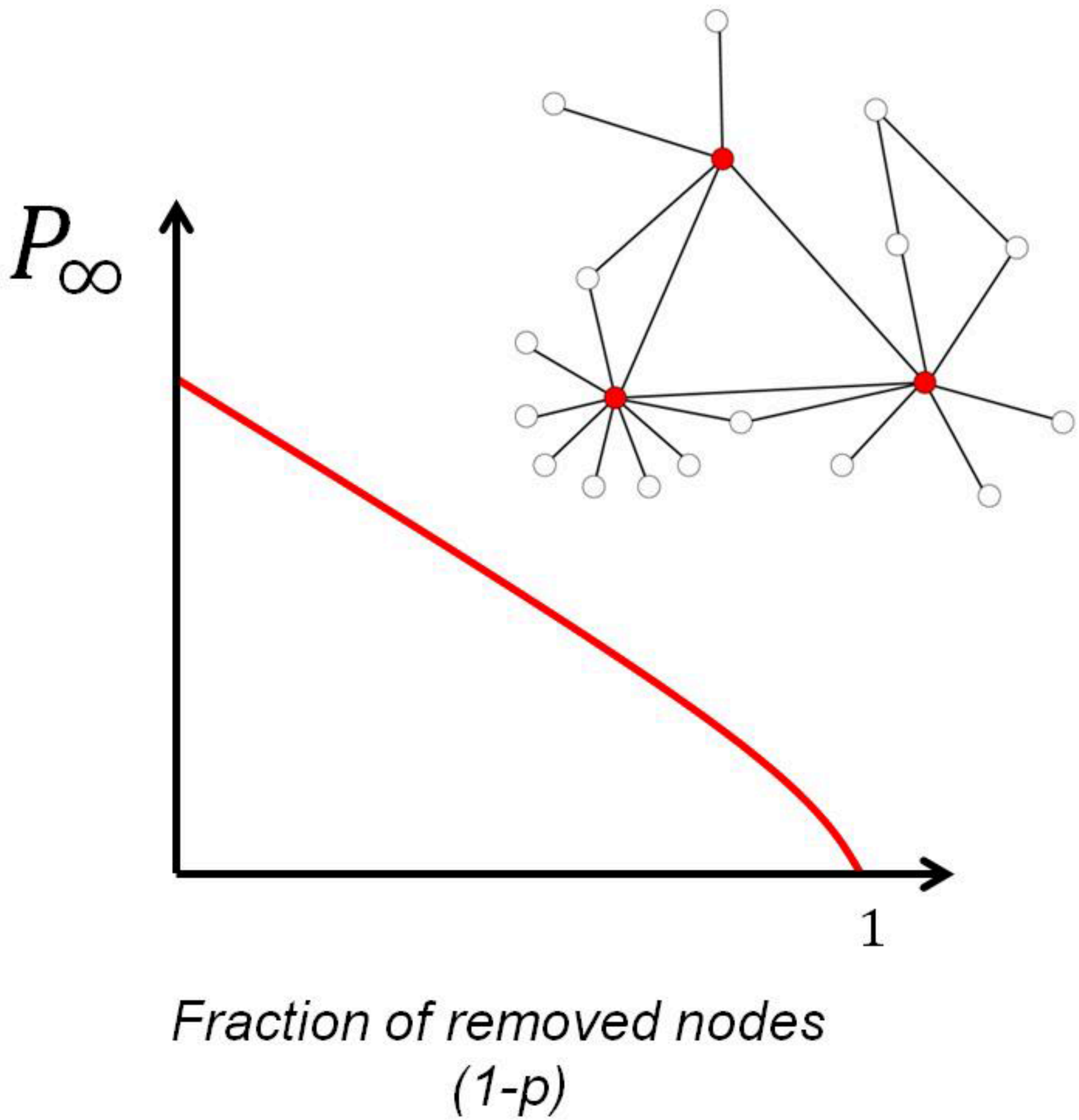
P_∞: Fraction of sites in the giant cluster

$$P_\infty \sim (p - p_c)^\beta$$

$$\boxed{p_c = \frac{1}{< k >}}$$

Network resilience

percolation: scale-free network (BA)



P_∞: Fraction of sites in the giant cluster

$$P_\infty \sim (p - p_c)^\beta$$

$$\boxed{p_c = 0}$$

Resilient to random attacks but vulnerable to targeted attacks

Properties and dynamics of network

several examples

World Airline Network

World Airline Network (WAN)

Airports	3237
Connections	18125
Flights	62202

$$< k^w >= 19.21$$

$$< l >= 4.05$$

$$l_{\max} = 12$$

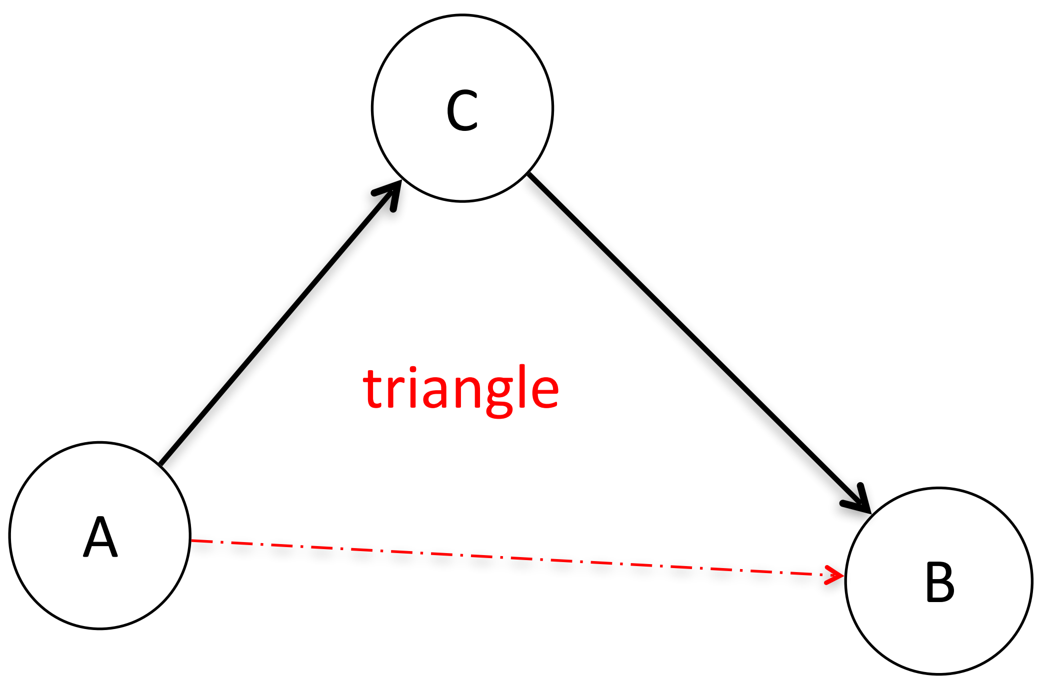
North America
South America
Europe
Africa
Asia
Australia

http://openflights.org/data.html

t-Core decomposition

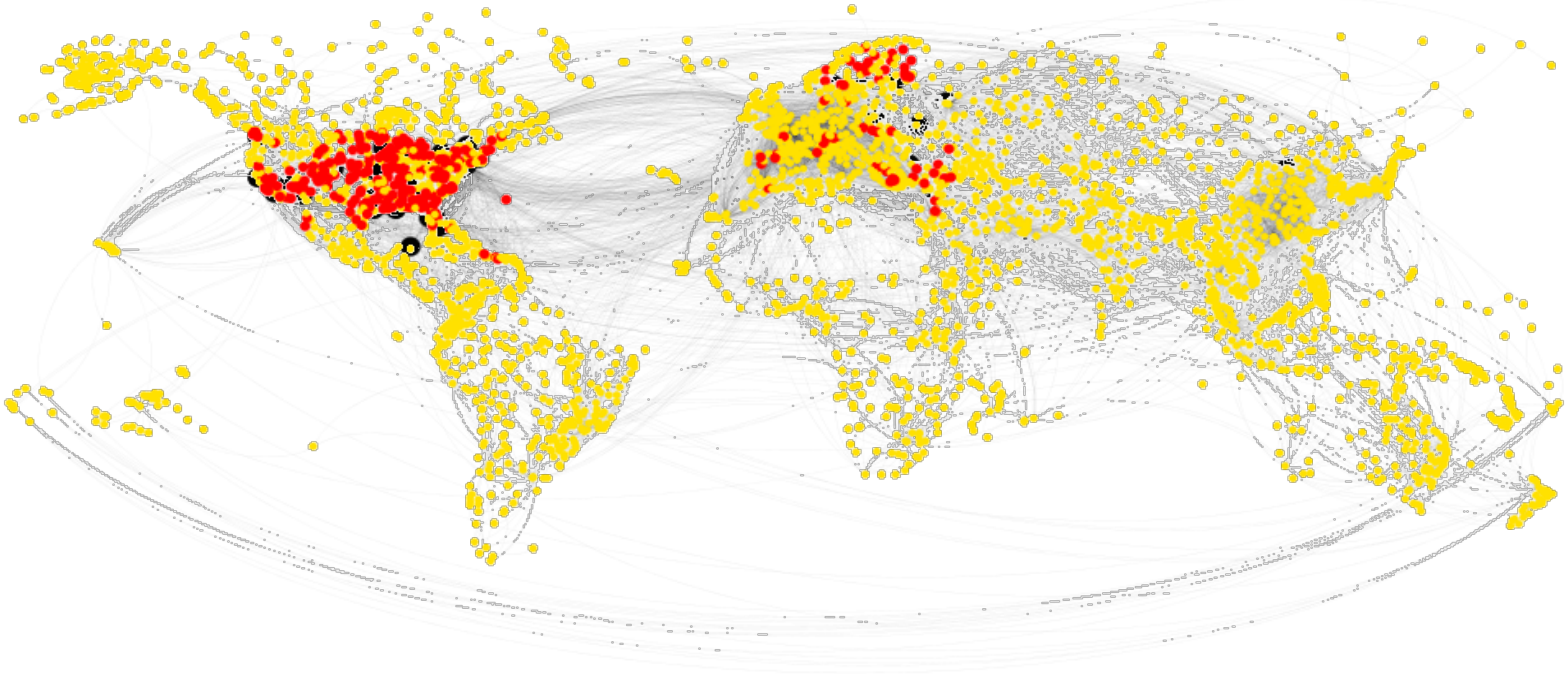
t = 0

t = 1



T. Verma, N. A. M. Araujo, and H. J. Herrmann. *Sci. Rep.* **4**, 5638 (2014)

t-Core decomposition

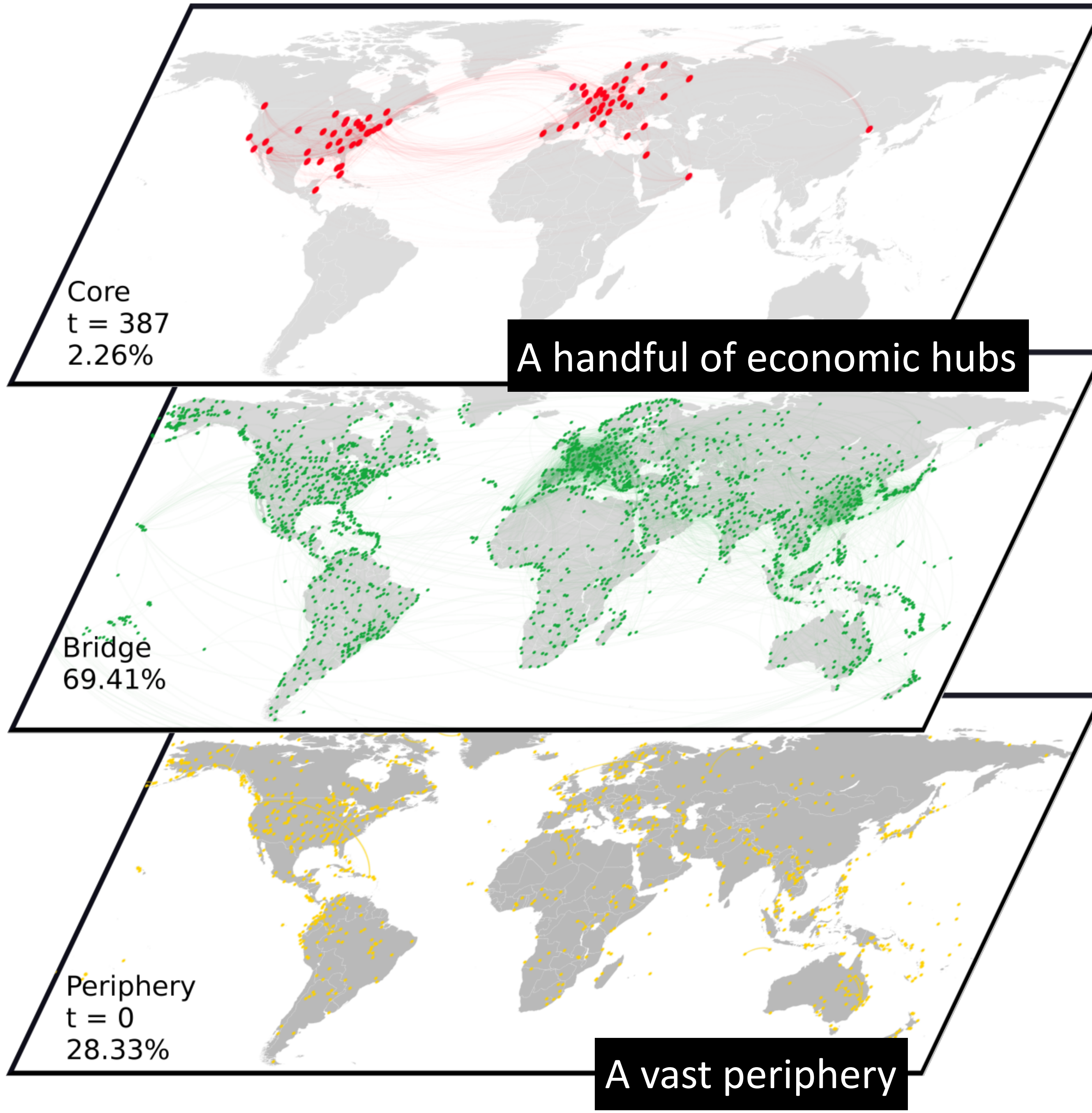


● 8.5% fall out of the cluster

Highly redundant connections

T. Verma, N. A. M. Araujo, and H. J. Herrmann. *Sci. Rep.* **4**, 5638 (2014)

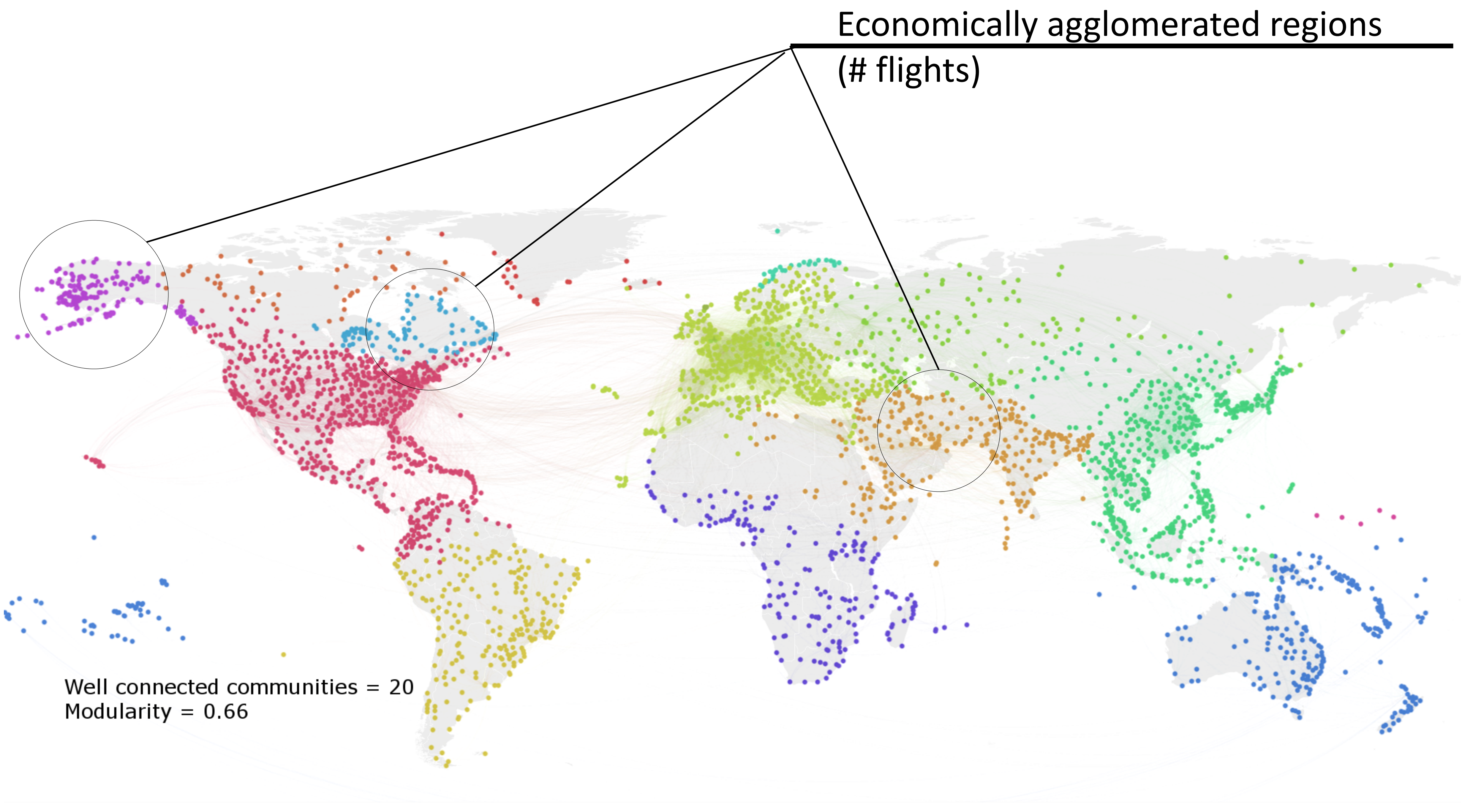
Core-periphery structure



T. Verma, N. A. M. Araujo, and H. J. Herrmann. *Sci. Rep.* **4**, 5638 (2014)

Communities

High modularity indicates presence of well formed communities



Core-periphery structures

World trade network

Fagiolo et al., J. Evol. Econ. 20, 479 (2010)

Autonomous Internet network

Rossa et al., Sci. Rep. 3, 1467 (2013)

Financial interbank lending markets

Elliott et al., SSRN, 2175056 (2013)

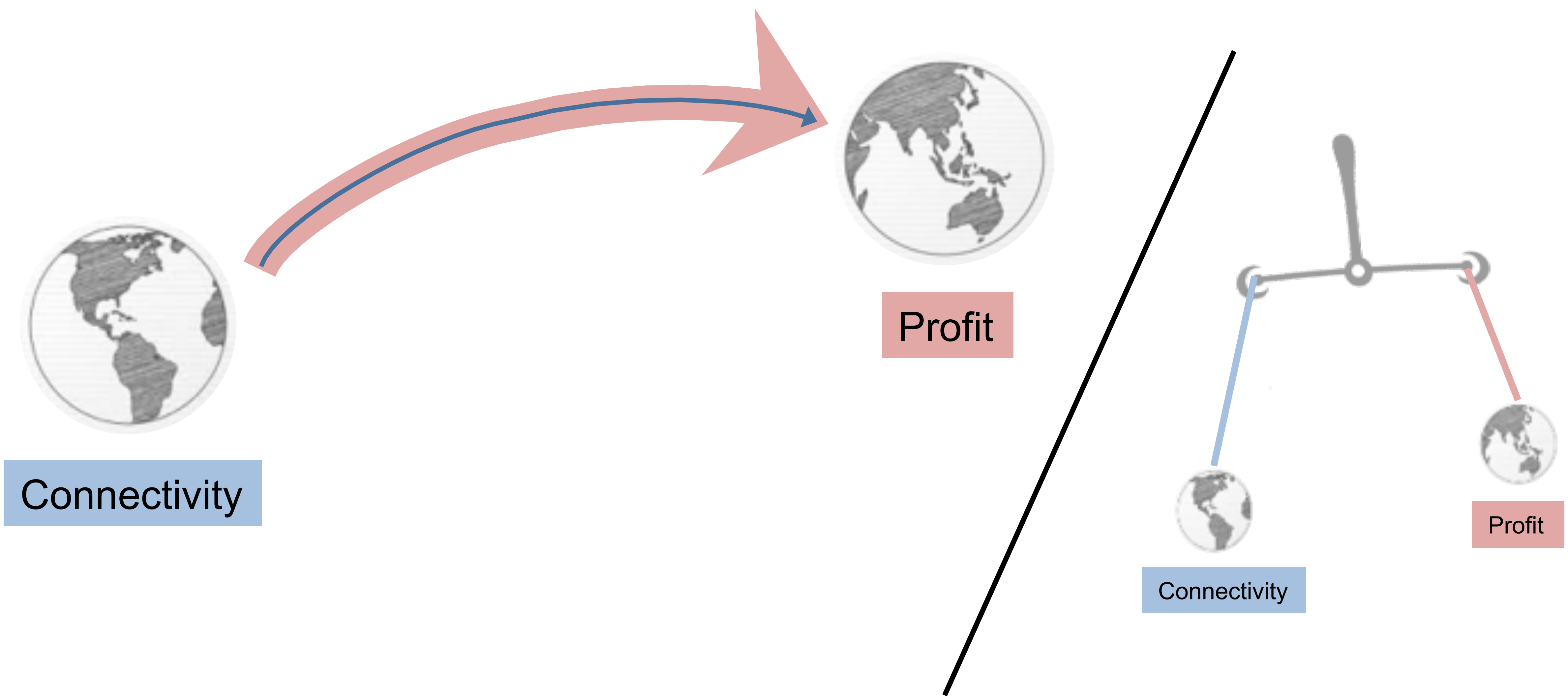
Friendship, voting, and collaboration networks

Rombach et al., SIAM J. Appl. Math. 74, 167 (2014)

Other social networks

Avin et al., arXiv: 1111.3374

Emergence of core-peripheries by pruning

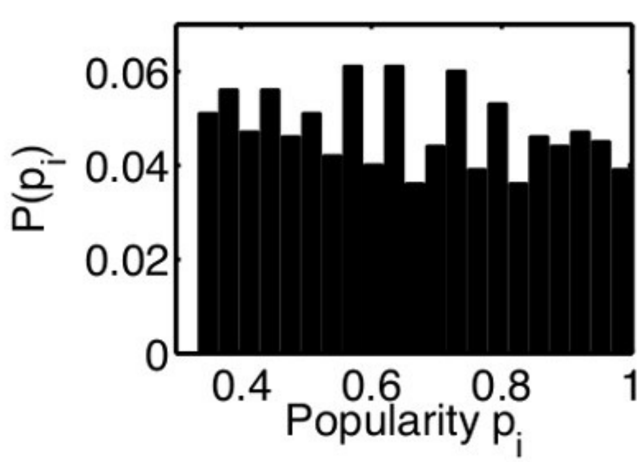


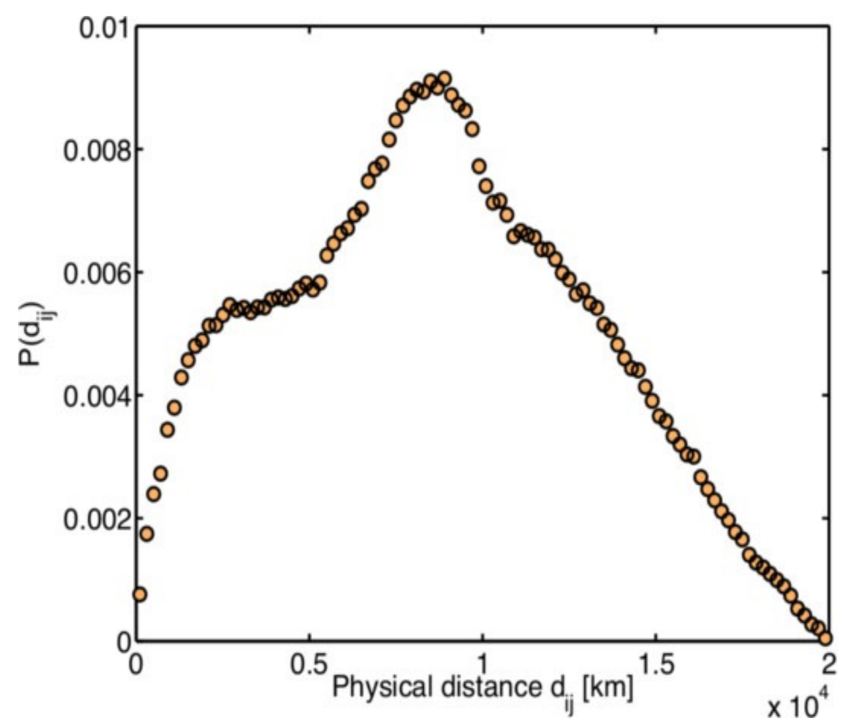
T. Verma, F. Russmann, N. A. M. Araújo, J. Nagler, and H. J. Herrmann, *Nature Comm.* ***7***, 10441 (2016)

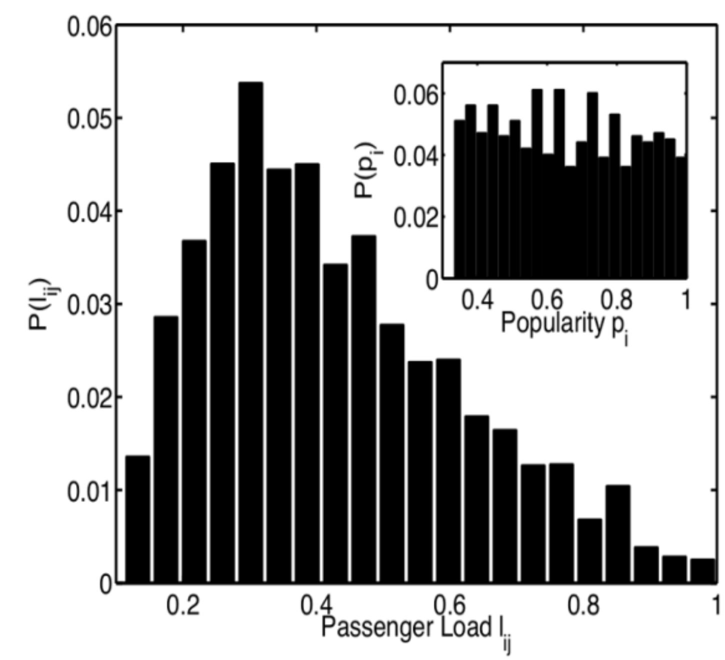
Emergence of core-peripheries *by pruning*

Utility : $u_{ij}=l_{ij}-c_{ij}$

Node popularity : p_i	Distance distribution	Load : $l_{ij} = p_i p_j$
• Randomly assign values from the distribution	• Randomly assign values from the distribution	• Indication of profit • Threshold parameter : v



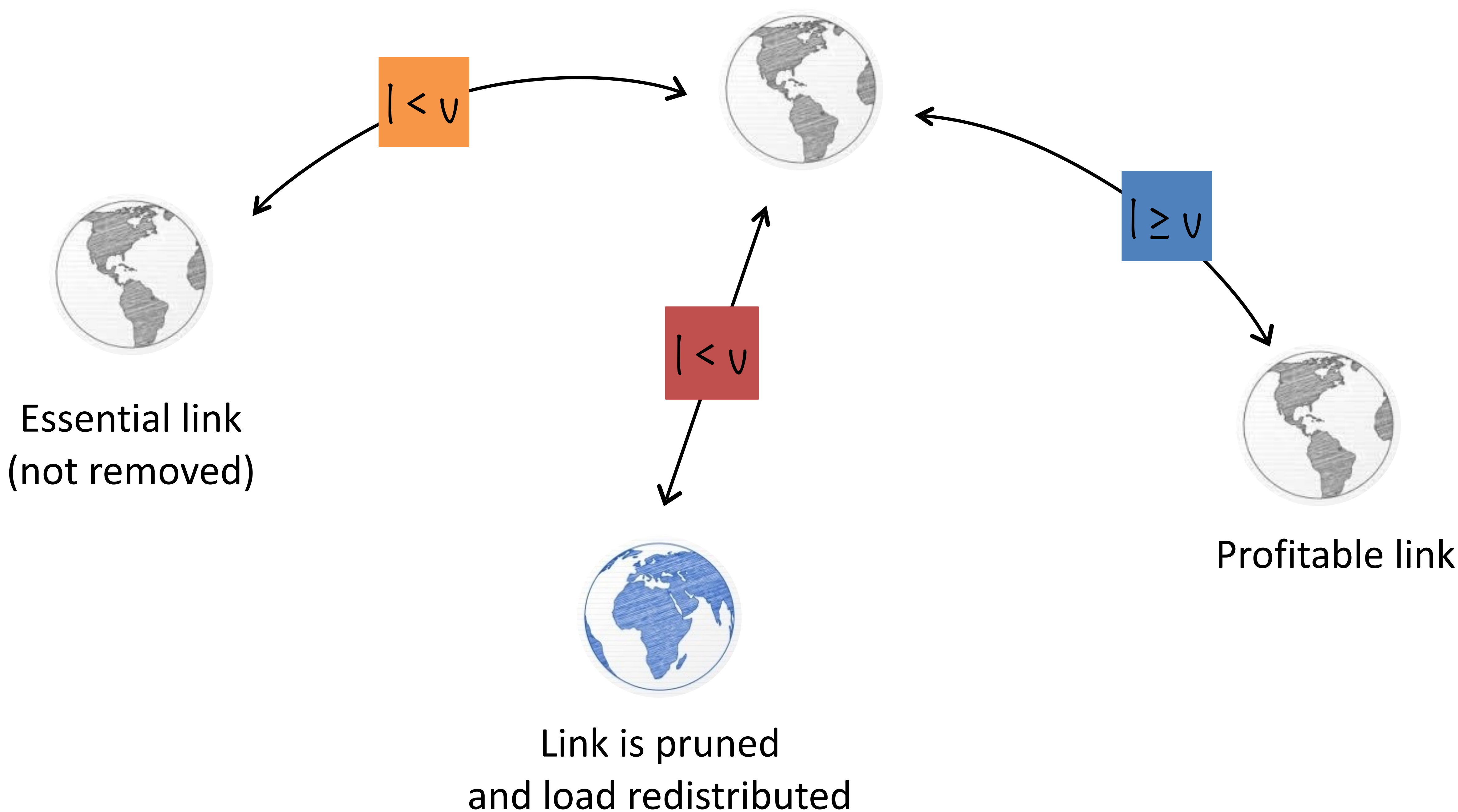




N = 1000 nodes

T. Verma, F. Russmann, N. A. M. Araújo, J. Nagler, and H. J. Herrmann, *Nature Comm.* ***7***, 10441 (2016)

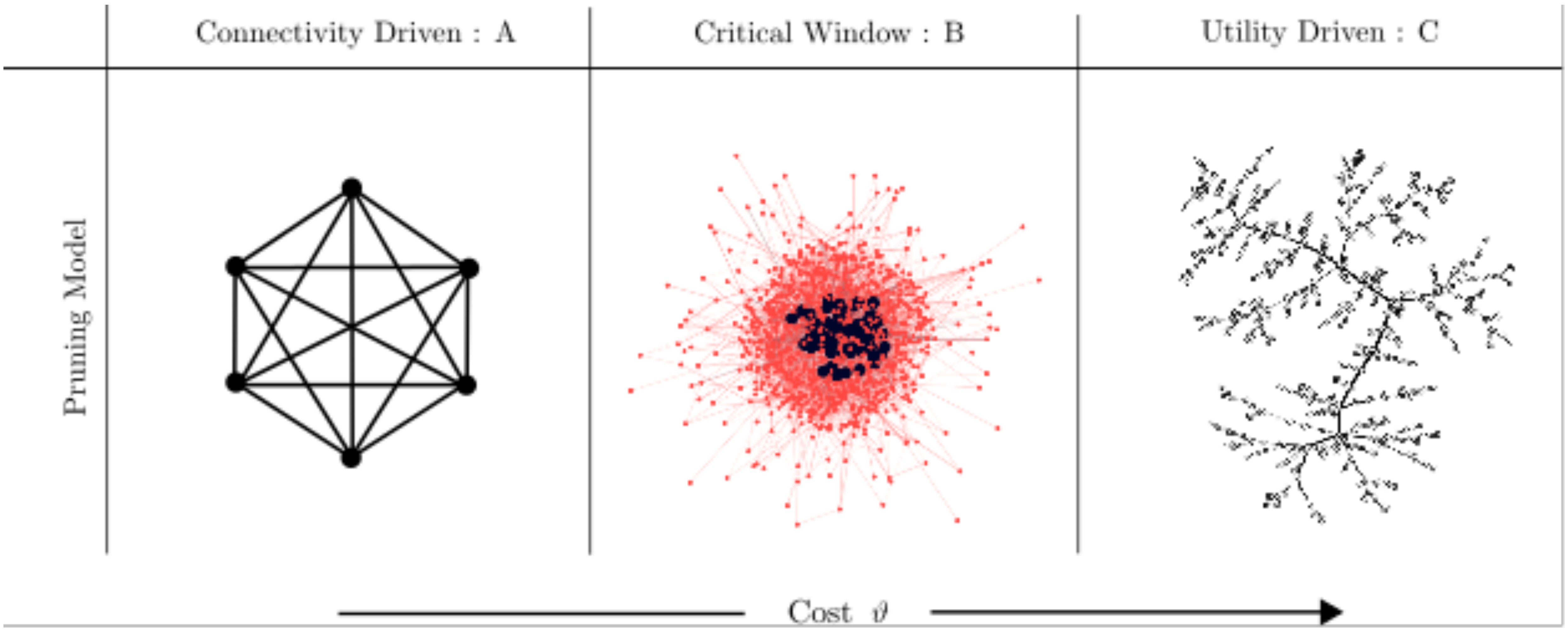
Emergence of core-peripheries
by pruning



ν: *cost*

T. Verma, F. Russmann, N. A. M. Araújo, J. Nagler, and H. J. Herrmann, *Nature Comm.* ***7***, 10441 (2016)

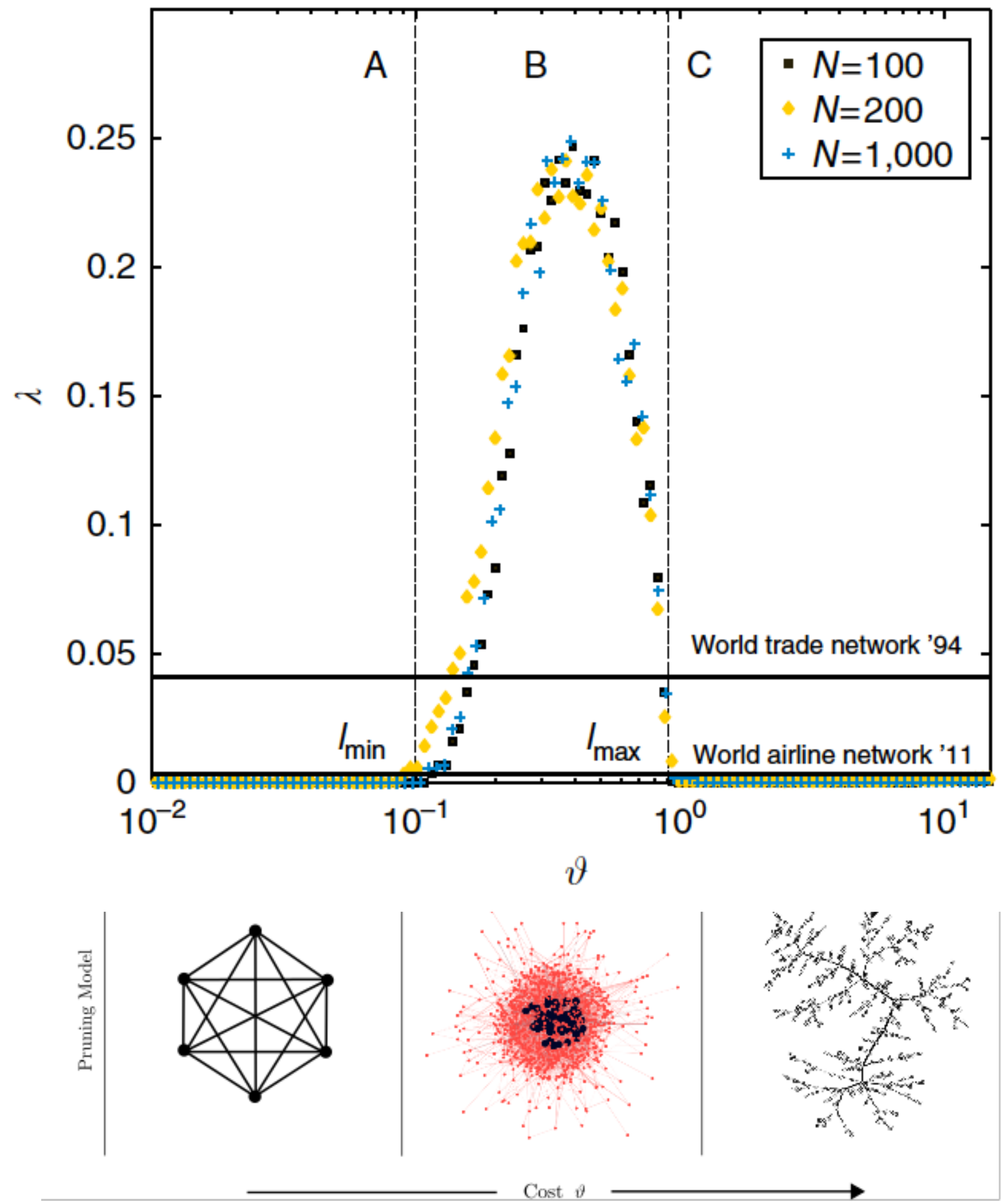
Emergence of core-peripheries by pruning



T. Verma, F. Russmann, N. A. M. Araújo, J. Nagler, and H. J. Herrmann, *Nature Comm.* ***7***, 10441 (2016)

Emergence of core-peripheries by pruning

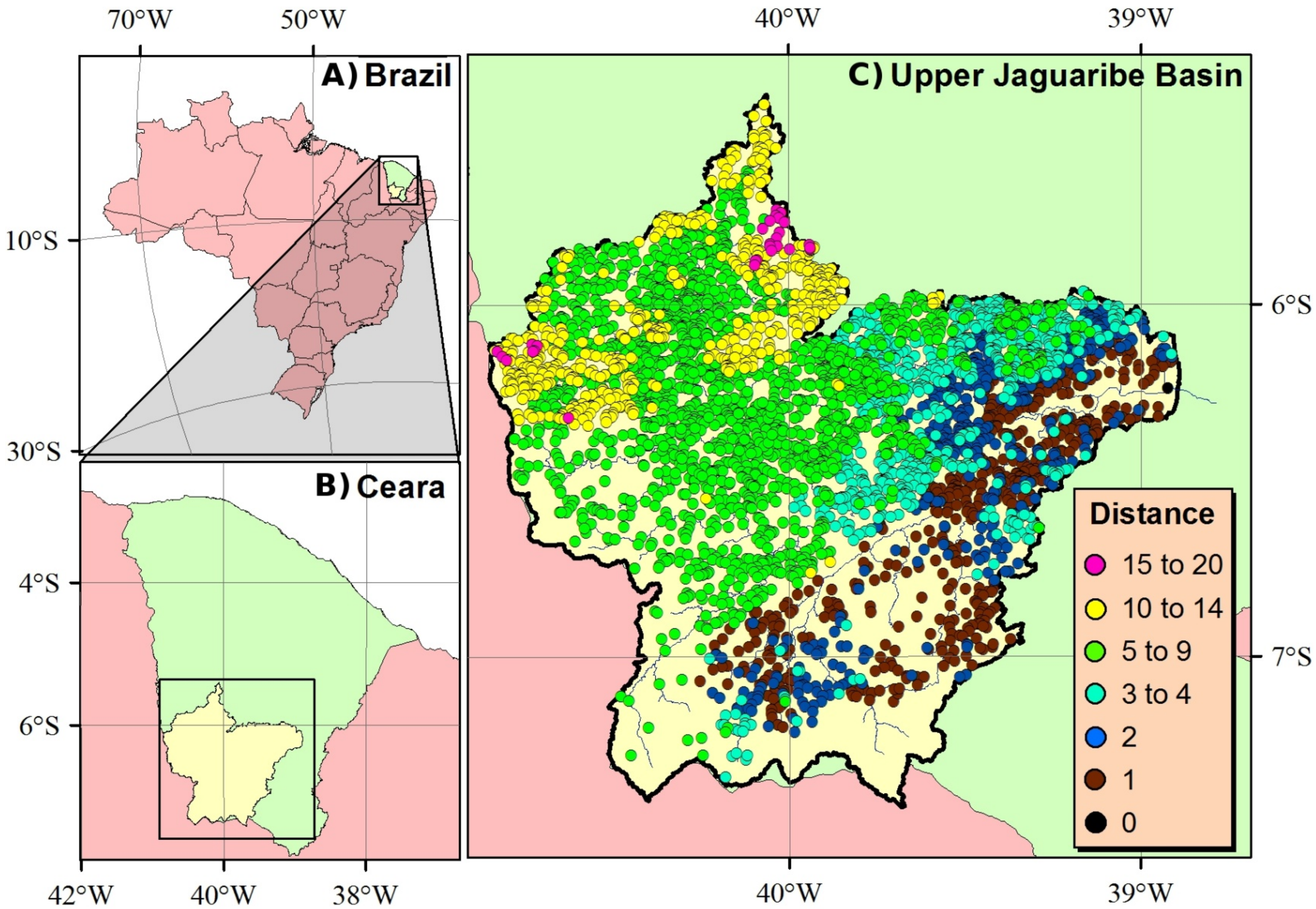
$$\lambda = (\tau_{max} - \tau_{min}) \frac{S_{\tau_{min}}}{S_{\tau_{max}}}$$



T. Verma, F. Russmann, N. A. M. Araújo, J. Nagler, and H. J. Herrmann, *Nature Comm.* ***7***, 10441 (2016)

Water reservoirs

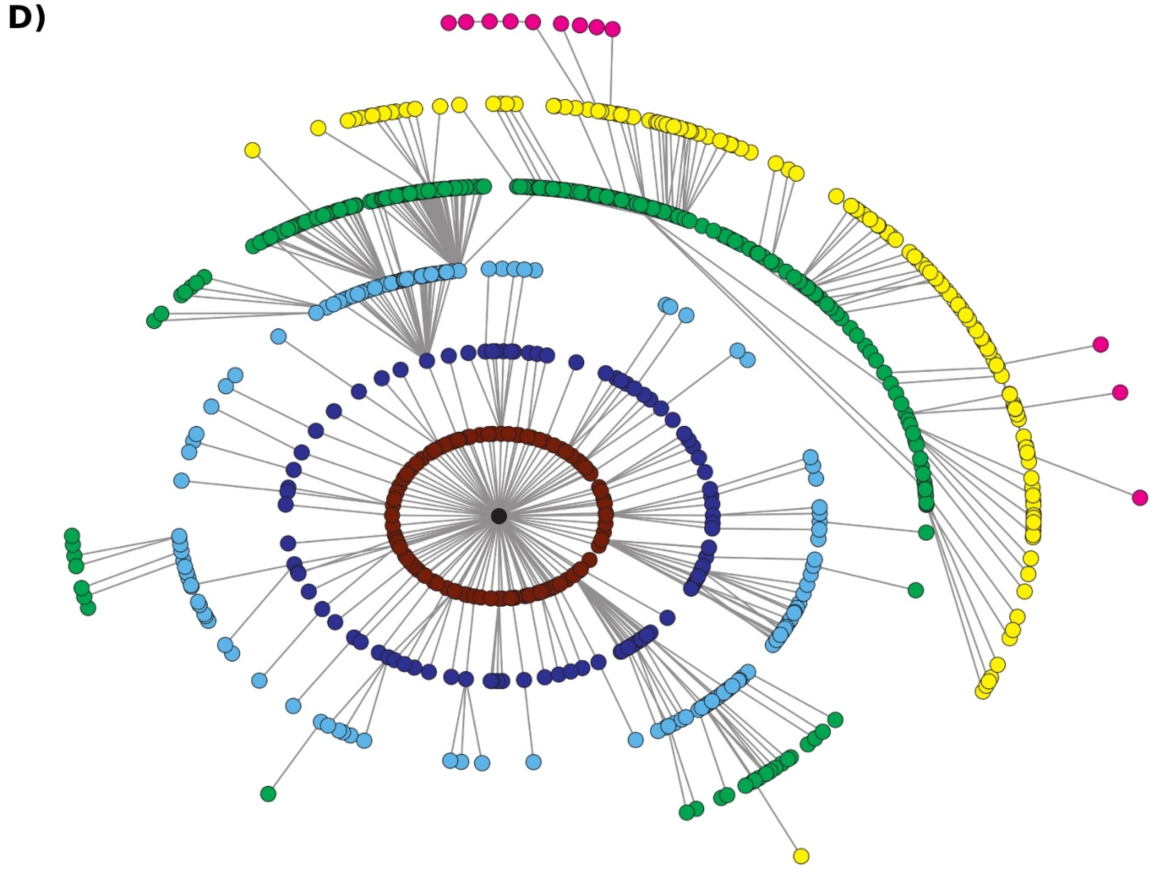
The Upper Jaguaribe Basin



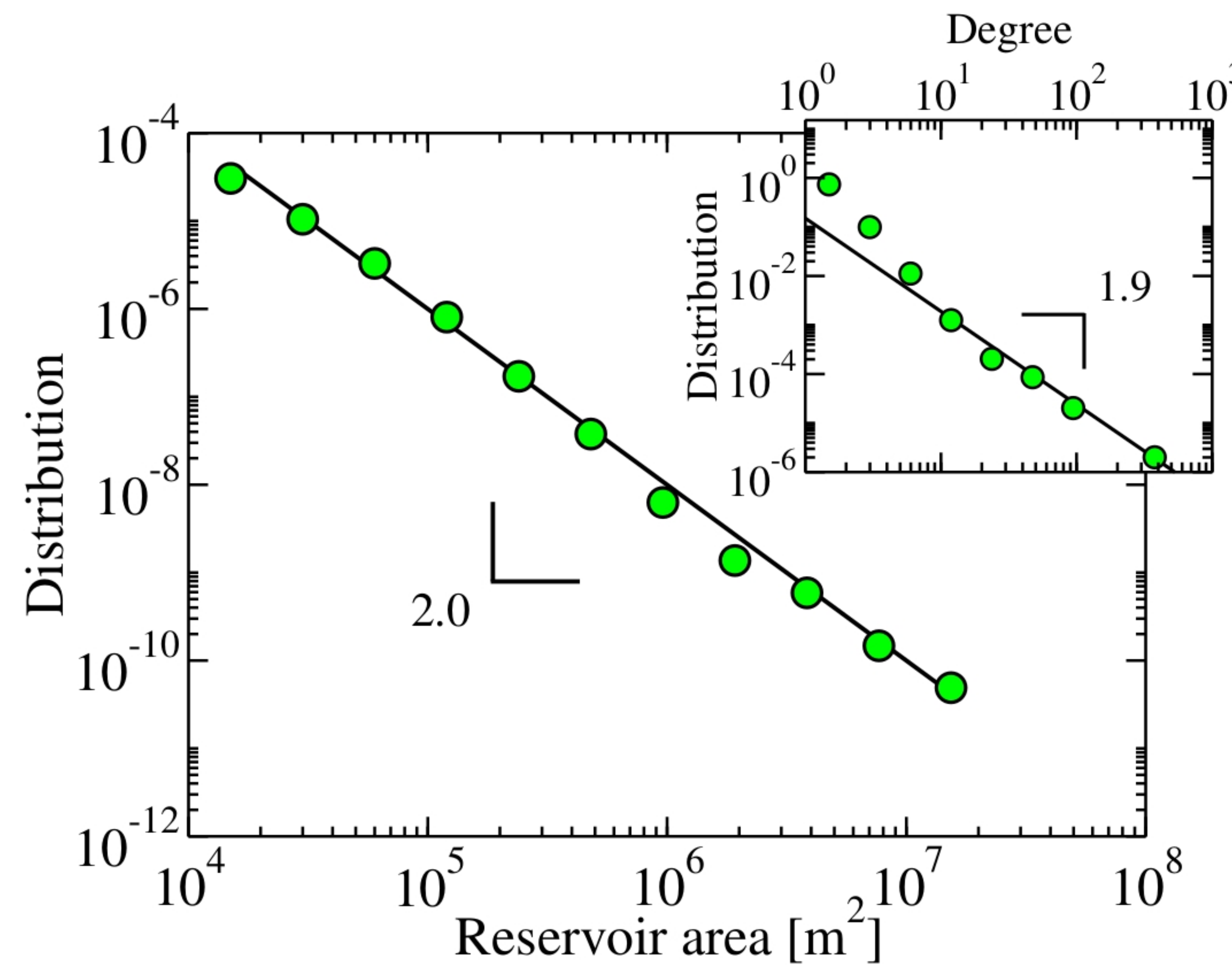
- Catchment area of about 25 000 km^2
- Almost 4 000 reservoirs
- Capacity from 2 500 m^3 to 2 billion m^3
- Affects half a million people
- Average annual rainfall: 860 mm
- Evaporation: 2 000 mm

G. L. Mamede, N. A. M. Araújo, C. M. Schneider, J. C. de Araújo, H. J. Herrmann. *PNAS* **109**, 7191 (2012)

The topology of the network



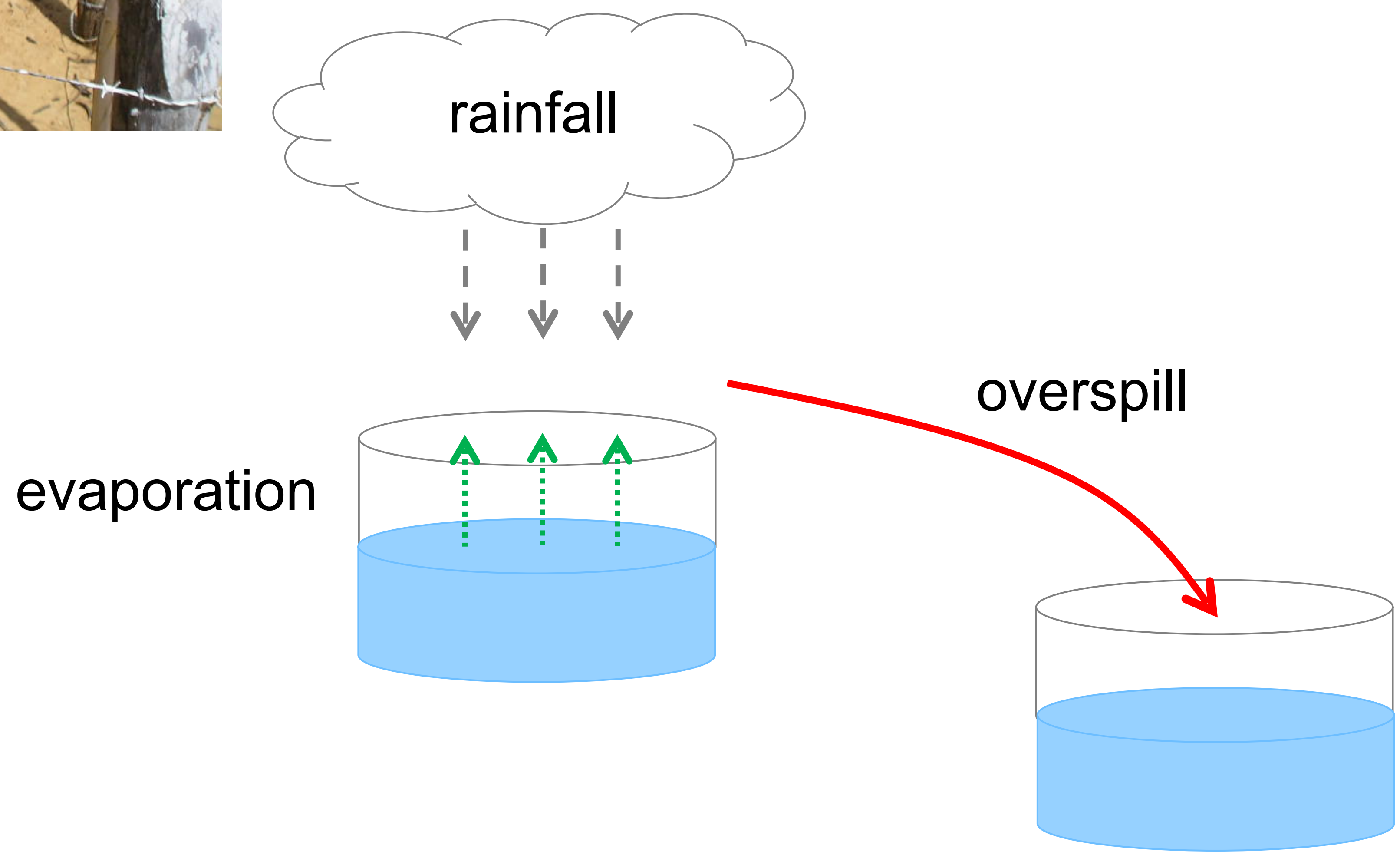
Size and degree distributions



G. L. Mamede, N. A. M. Araújo, C. M. Schneider, J. C. de Araújo, H. J. Herrmann. *PNAS* **109**, 7191 (2012)

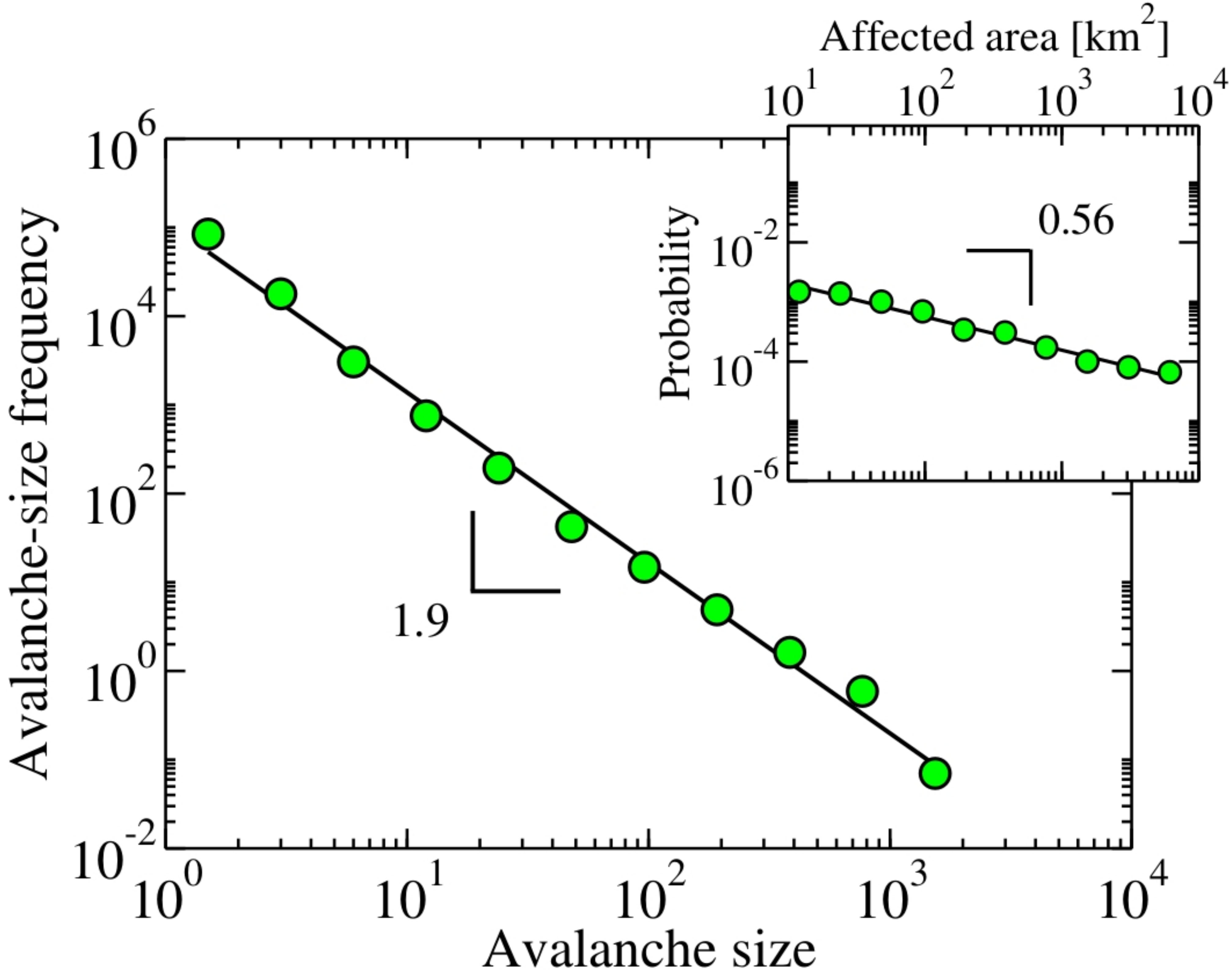
Climatic data

The hydrological model



G. L. Mamede, N. A. M. Araújo, C. M. Schneider, J. C. de Araújo, H. J. Herrmann. *PNAS* **109**, 7191 (2012)

Avalanche-size distribution



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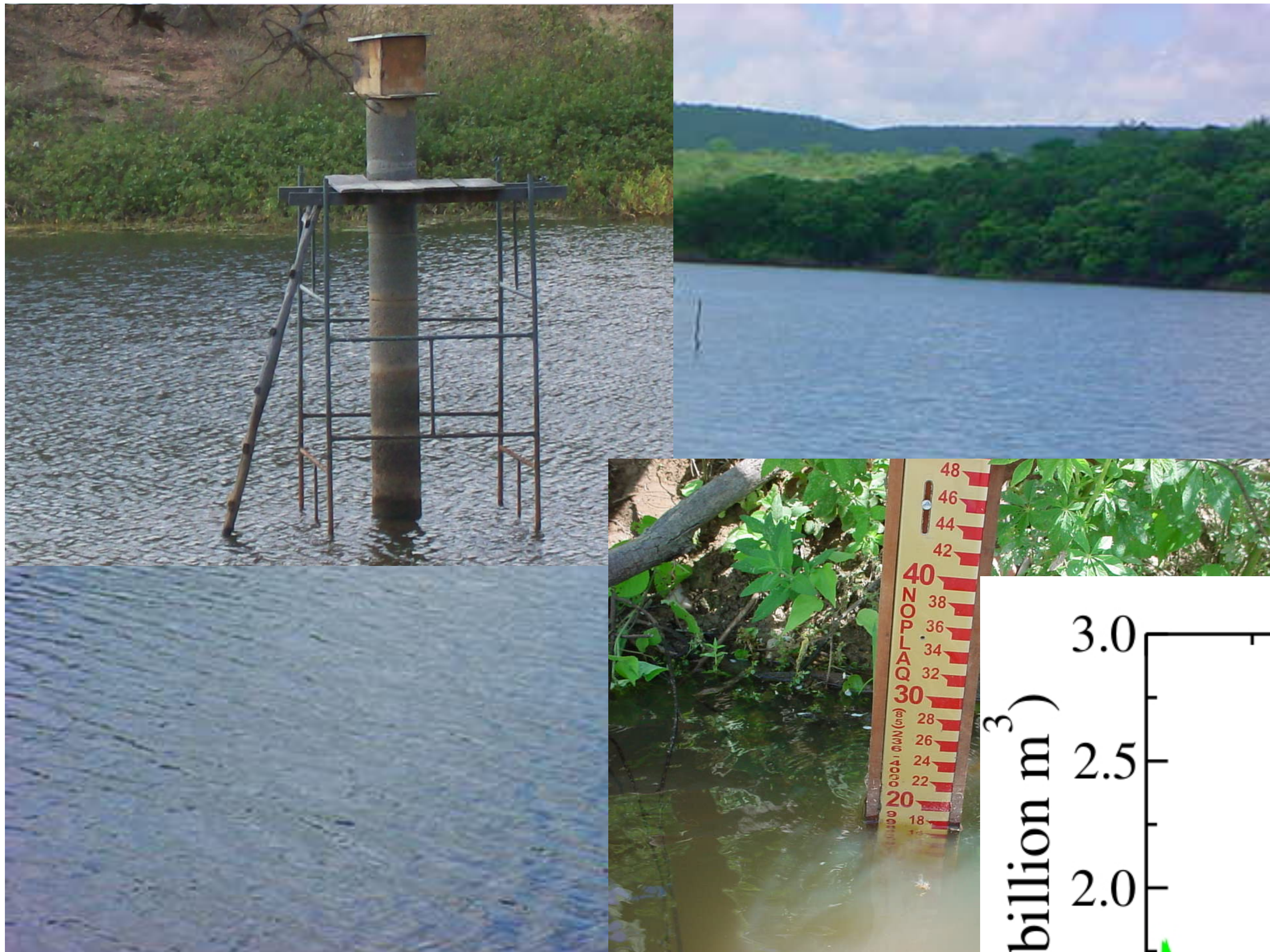
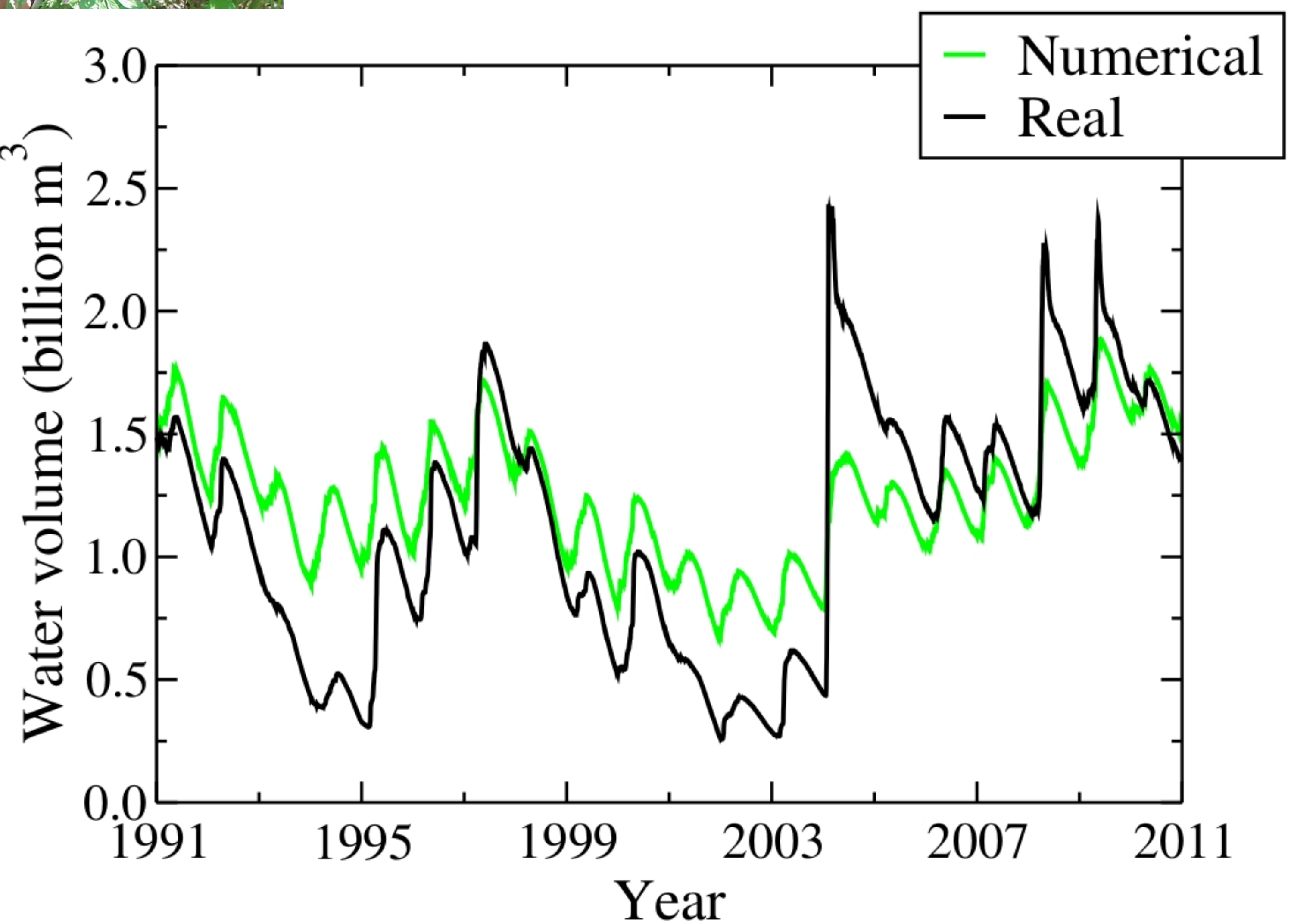
Reservoir water level



Reservoir water level



G. L. Mamede, N. A. M. Araújo, C. M. Schneider, J. C. de Araújo, H. J. Herrmann. *PNAS* **109**, 7191 (2012)

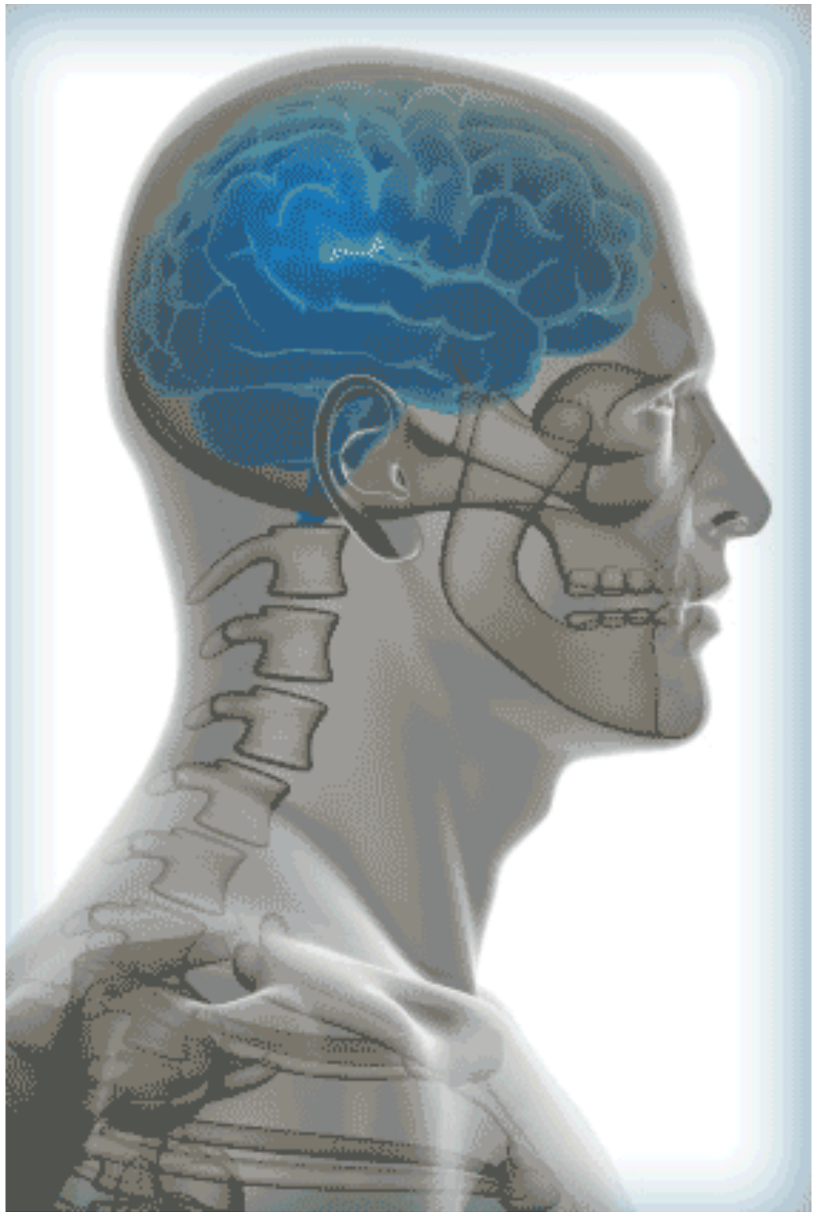
Synchronization on coupled networks

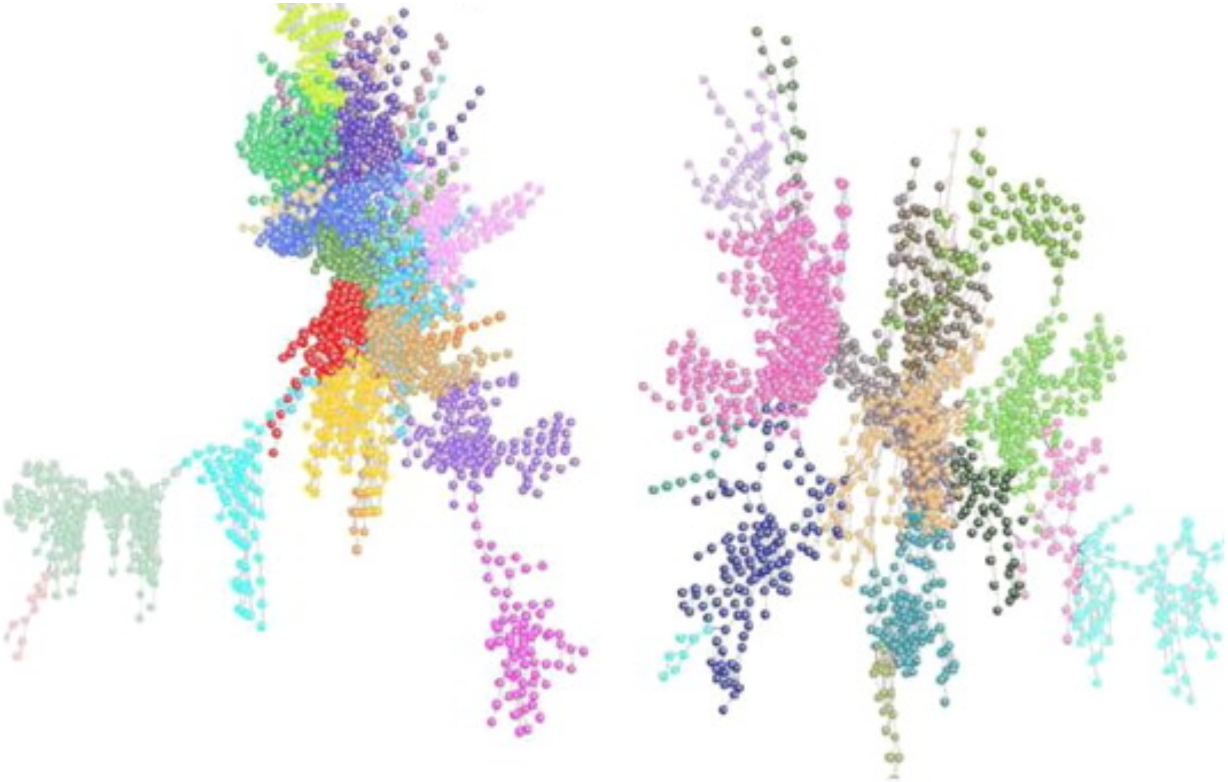
Virtual choir

Eric Whitacre

Chris Hadfield (International Station) and Barenaked Ladies

brain





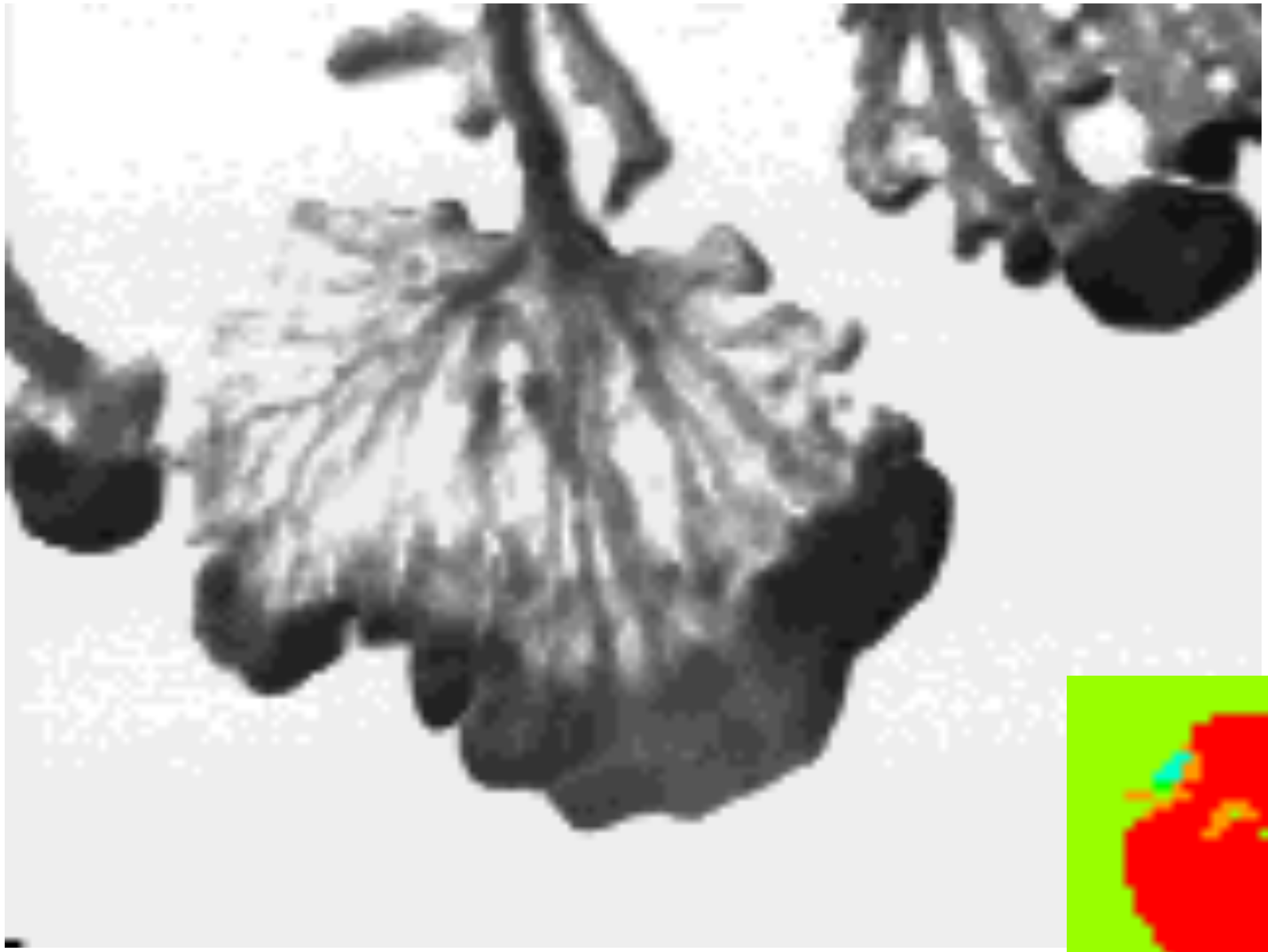
Brain Modules Identified

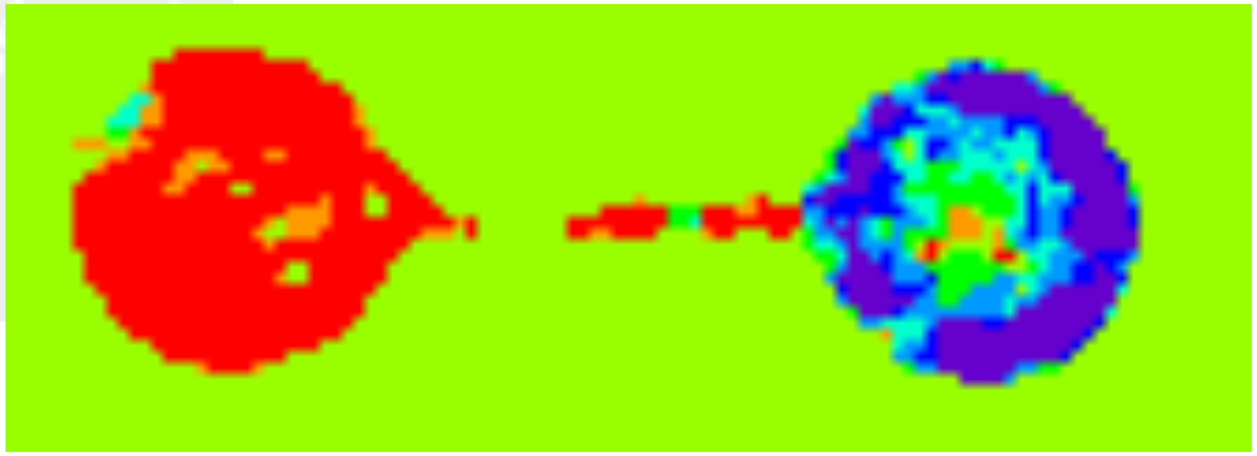
L. K. Gallos, H. A. Makse, and M. Sigman, *Proc. Nat. Acad. Sci. USA* **109**, 2825 (2012).

plasmodial slime mol

http://faculty.clintoncc.suny.edu/faculty/michael.gregory

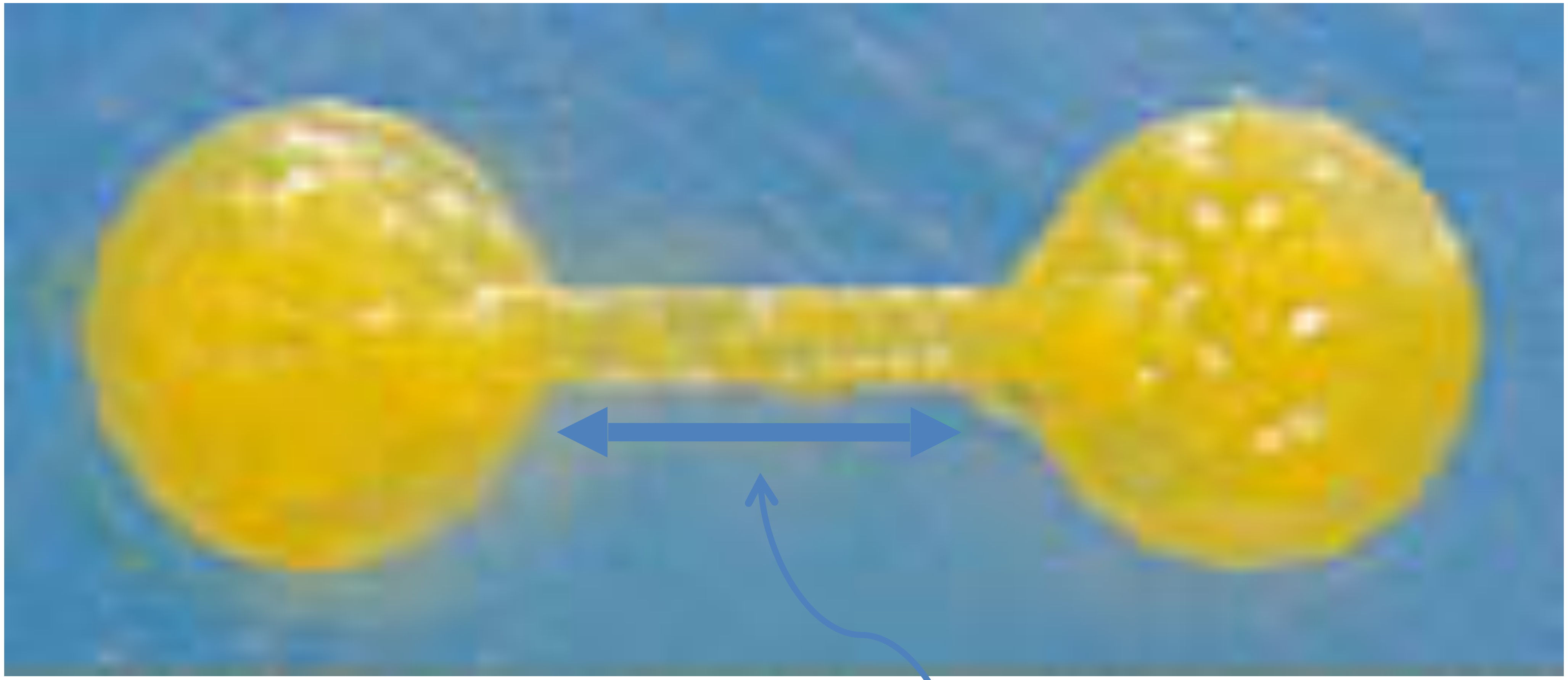
plasmodial slime mol





A. Takamatsu, T. Fujii, and I. Endo, *Phys. Rev. Lett.* **85**, 2026 (2000).

plasmodial slime mol

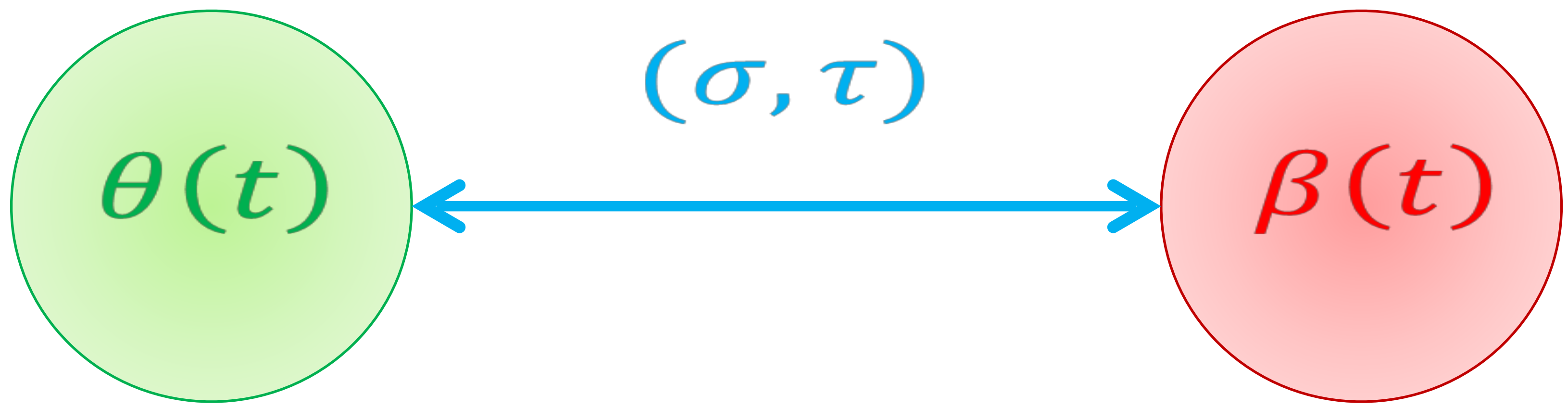


Coupling strength and delay between sections

A. Takamatsu, T. Fujii, and I. Endo, *Phys. Rev. Lett.* **85**, 2026 (2000).

Time delay

two coupled oscillators: Schuster and Wanger



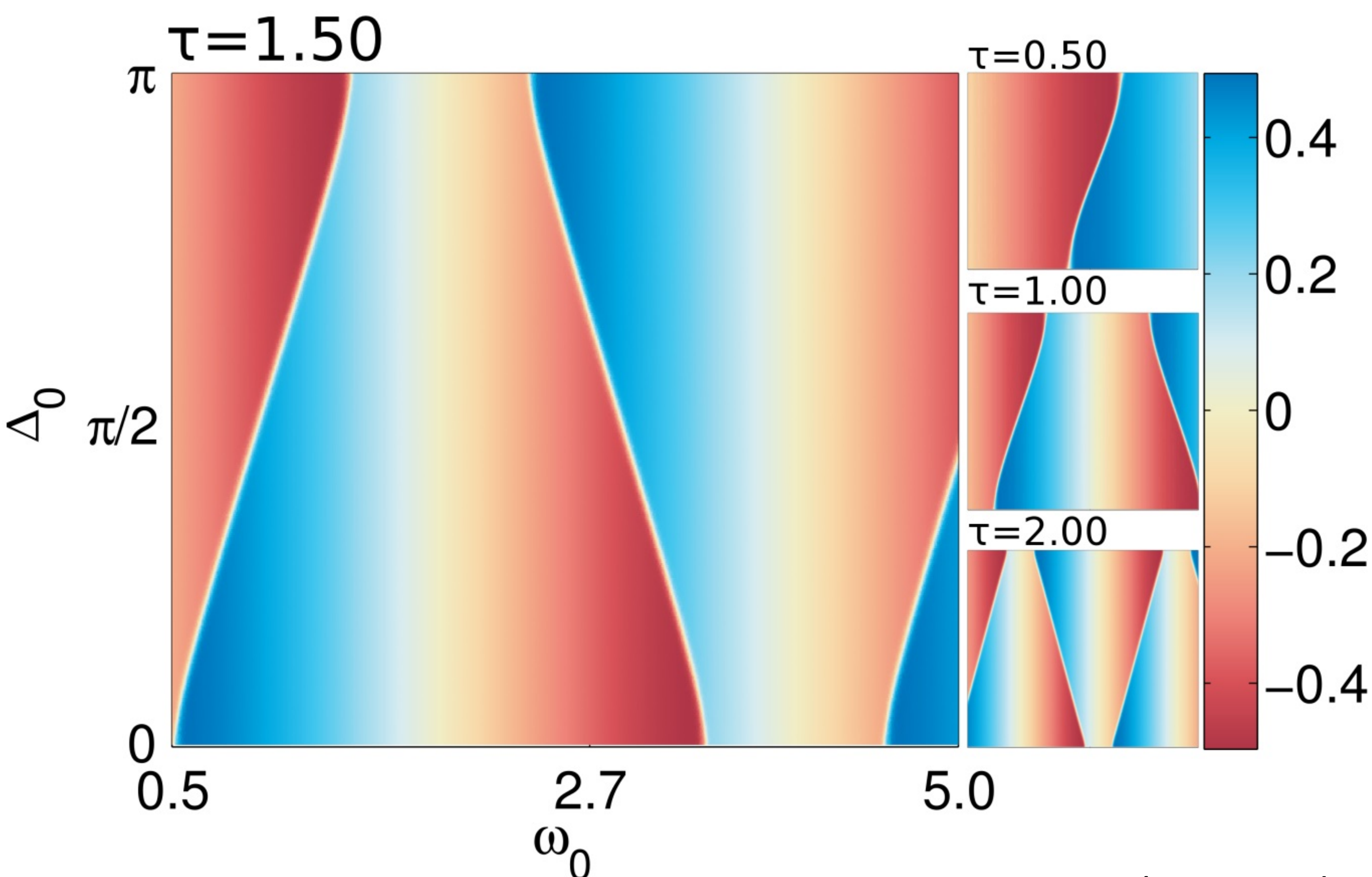
$$\dot{\theta}(t) = \omega_\theta + \sigma \sin[\beta(t-\tau) - \theta(t)]$$

$$\dot{\beta}(t) = \omega_\beta + \sigma \sin[\theta(t-\tau) - \beta(t)]$$

H. G. Schuster and P. Wagner, *Prog. Theor. Phys.* **81**, 939 (1989).

Time delay
two coupled oscillators: Schuster and Wanger

$$\omega = \omega_0 - \sigma \sin(\omega\tau)$$



In phase when:

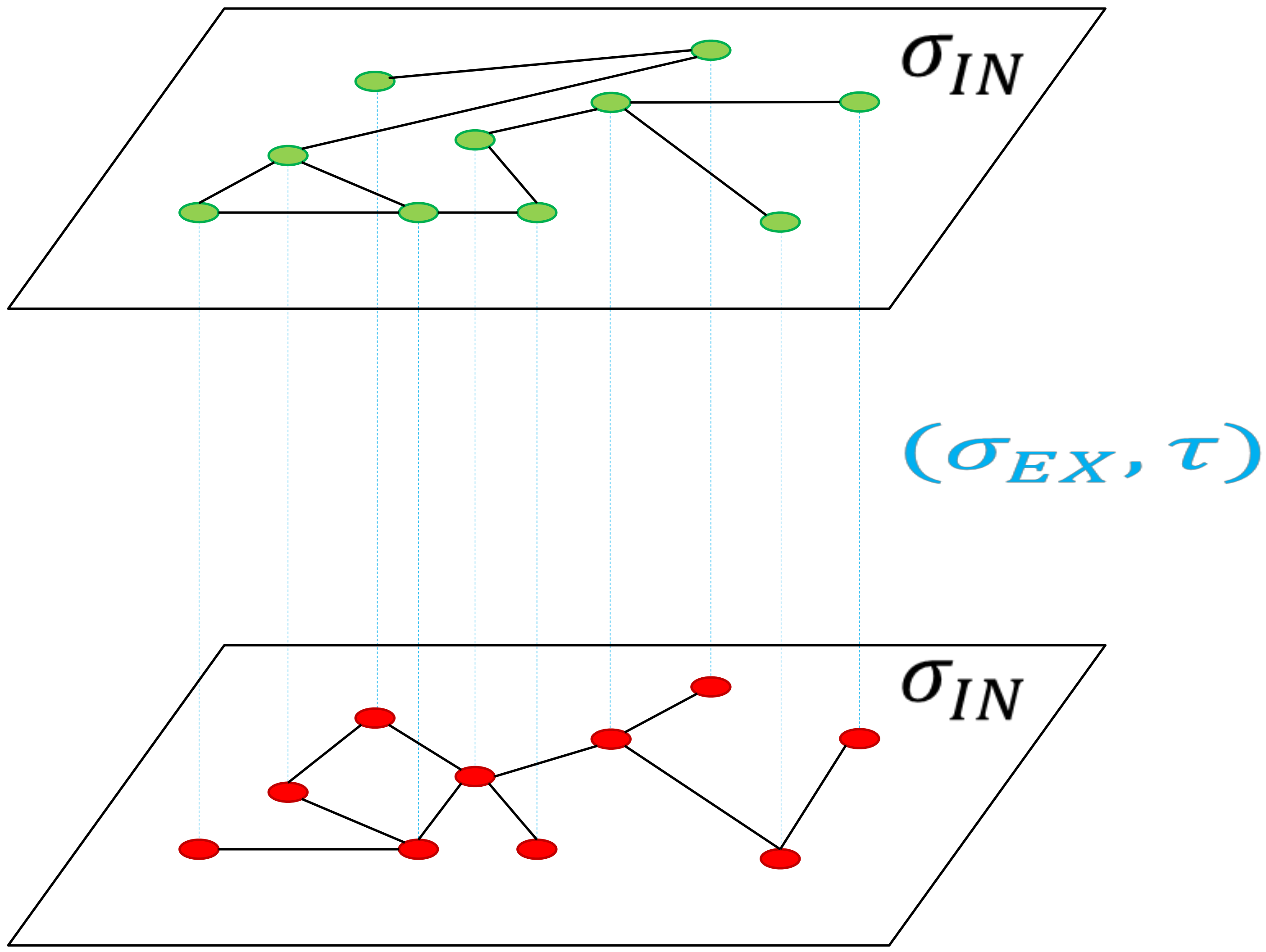
$$\cos(\omega\tau) > 0$$

H. G. Schuster and P. Wagner, *Prog. Theor. Phys.* **81**, 939 (1989).

Time delay

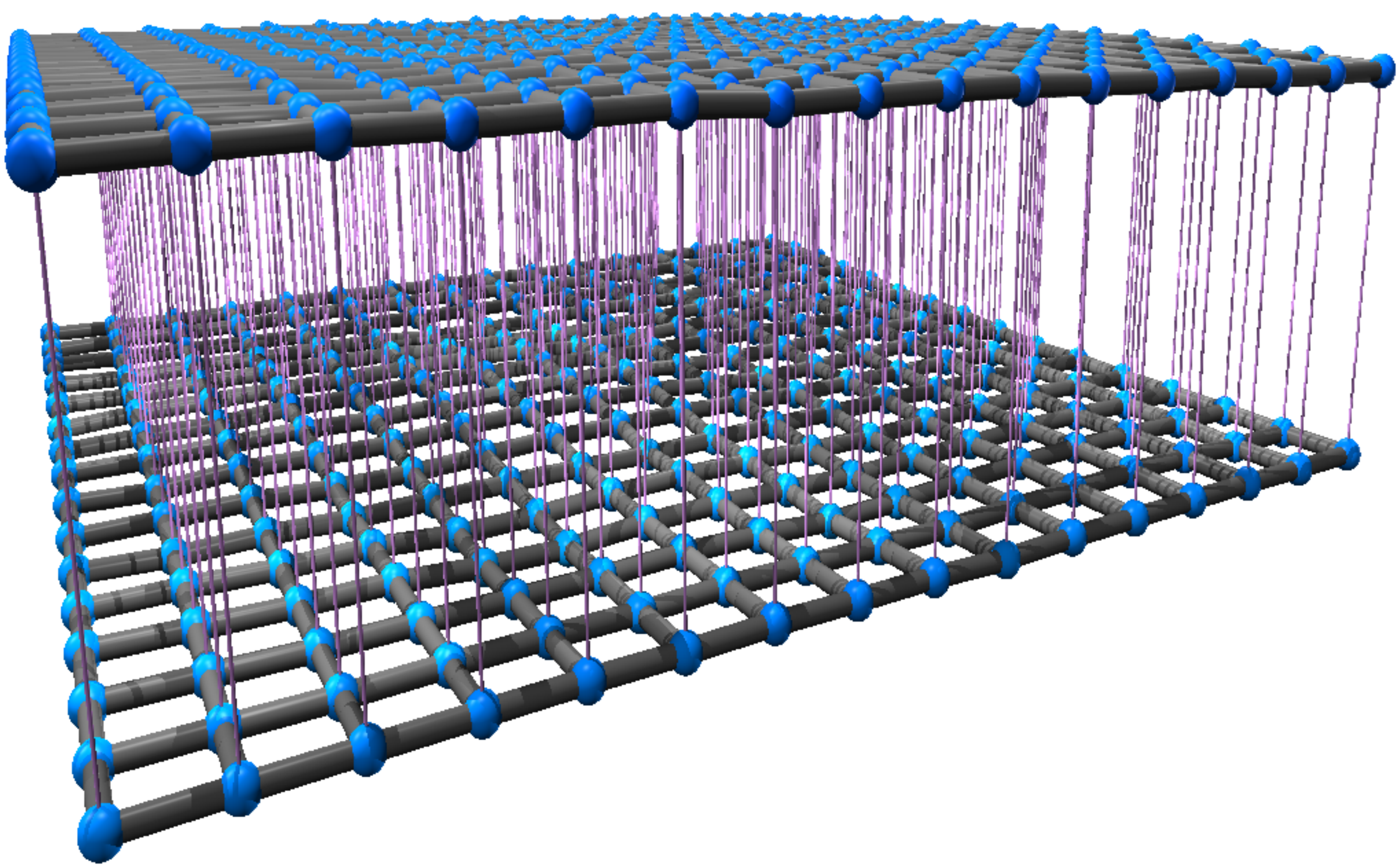
multiplex network with time delay

$$\dot{\theta}_i = \omega_i + \sigma_{EX} \sin\left(\beta_j(t-\tau) - \theta_i\right) + \sigma_{IN} \sum_{l \epsilon N(i)} \sin(\theta_l - \theta_i)$$

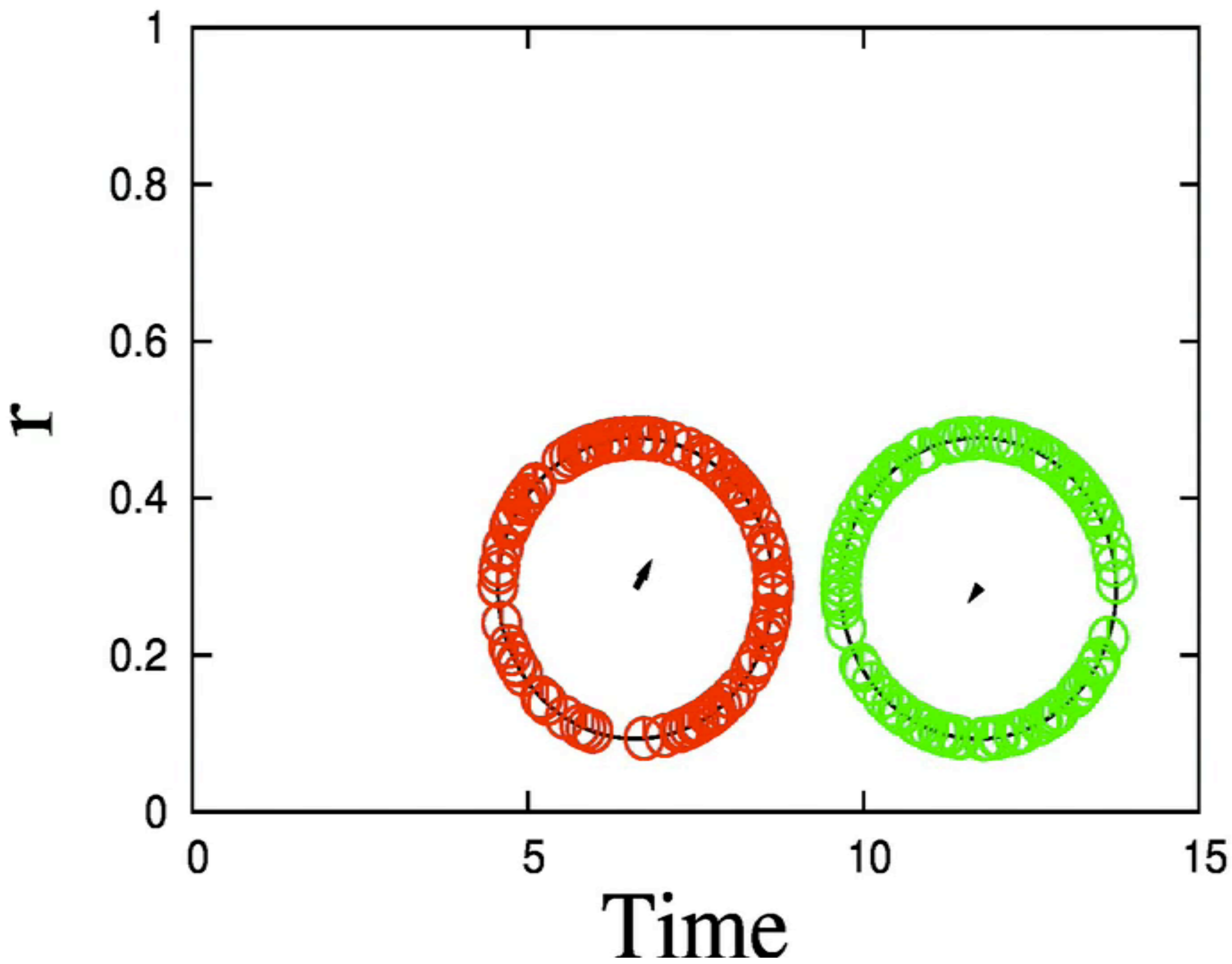


$$\dot{\beta}_j = \omega_j + \sigma_{EX} \sin\left(\theta_i(t-\tau) - \beta_j\right) + \sigma_{IN} \sum_{l \epsilon N(j)} \sin\left(\beta_l - \beta_j\right)$$

Time delay
two coupled square lattices

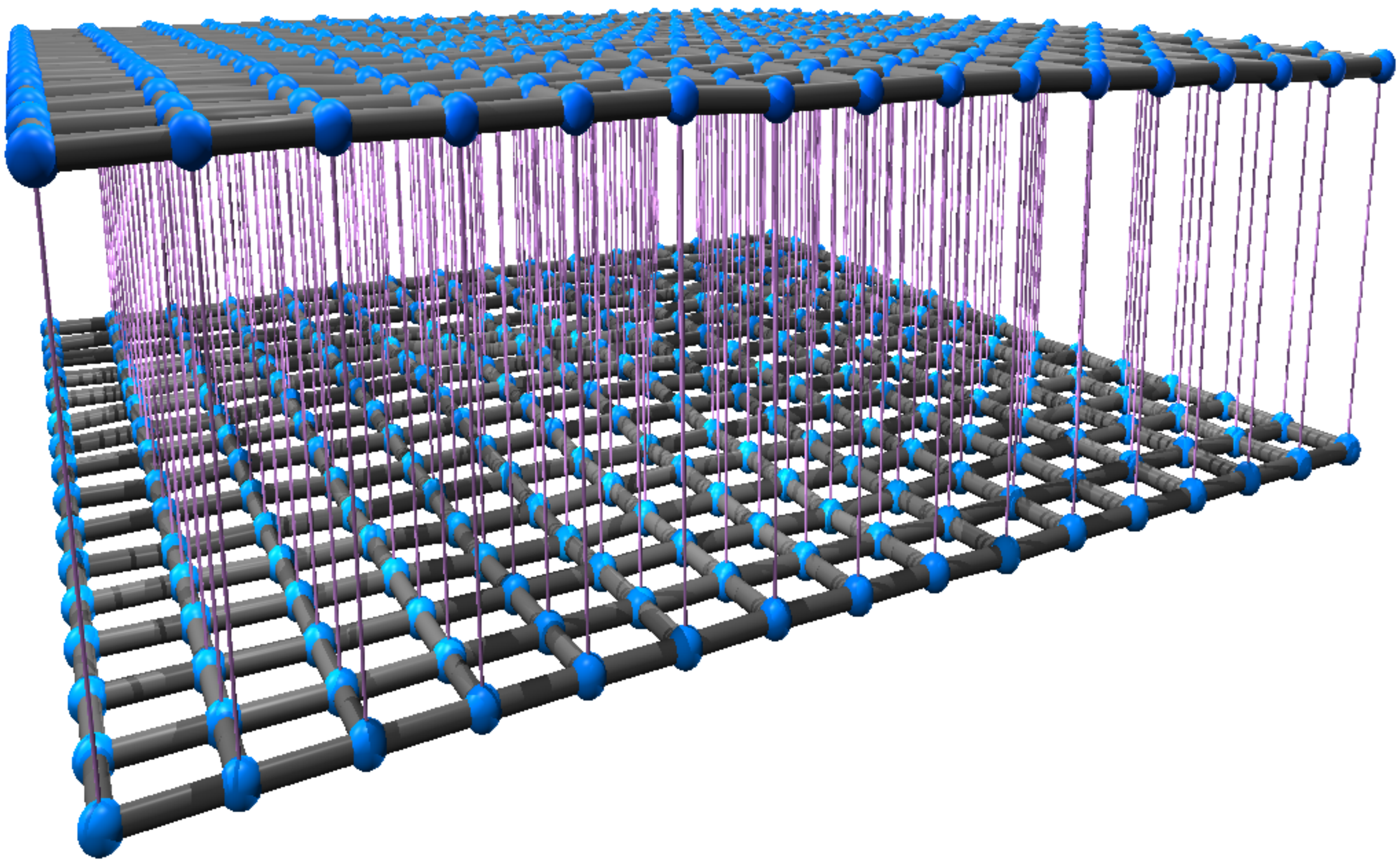


Time delay
no external coupling



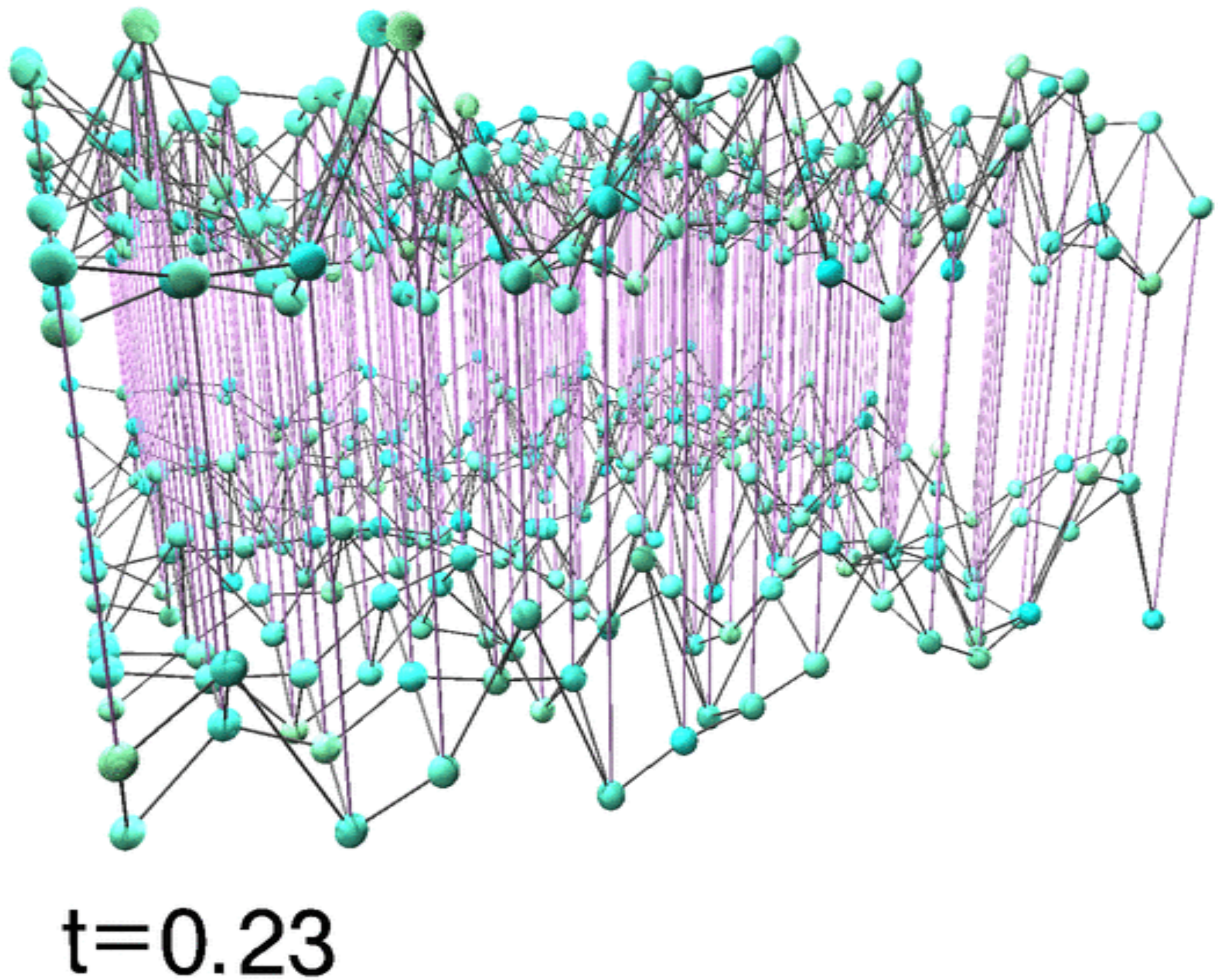
Time delay

two coupled square lattices



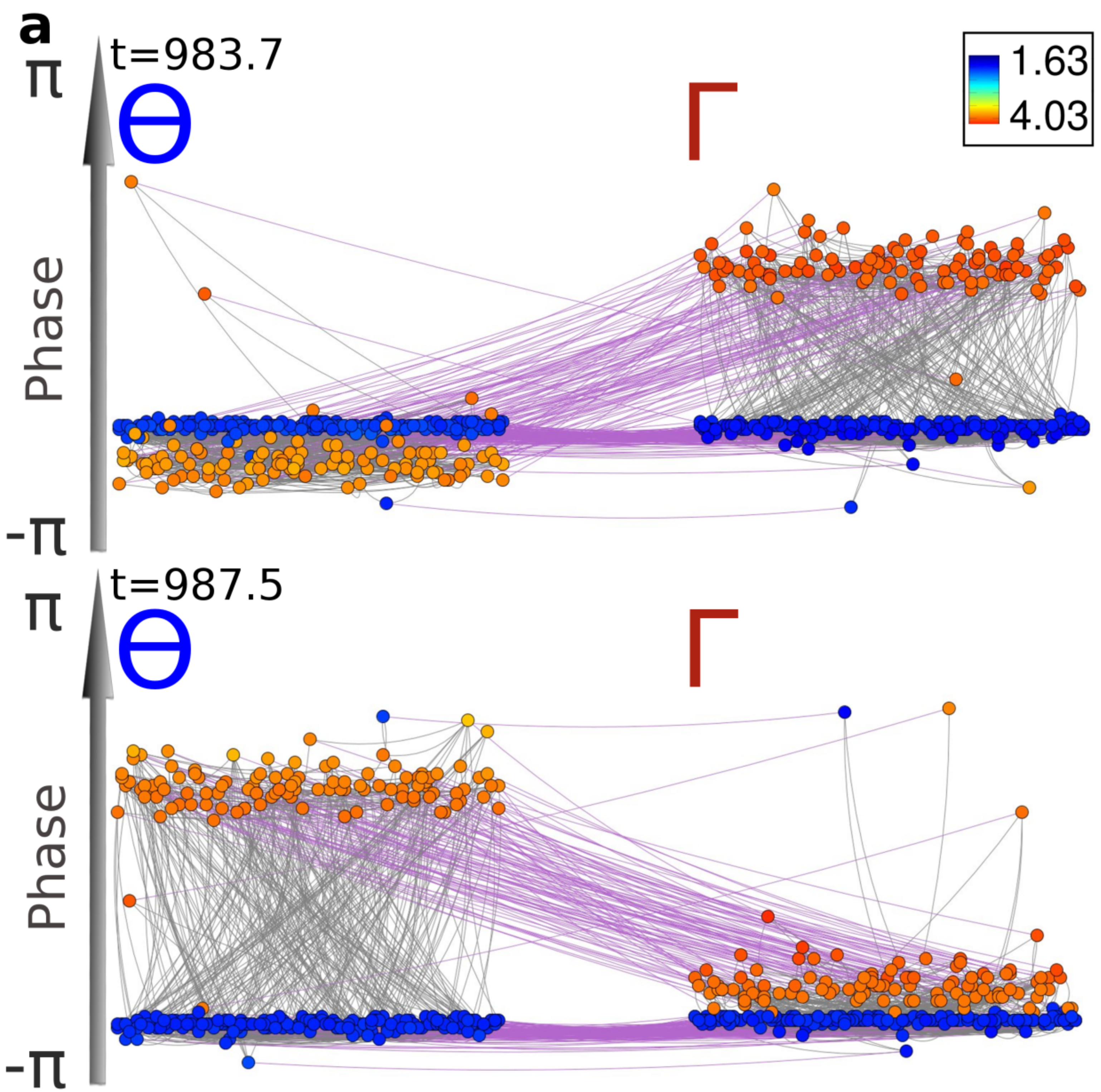
Time delay

weak internal coupling



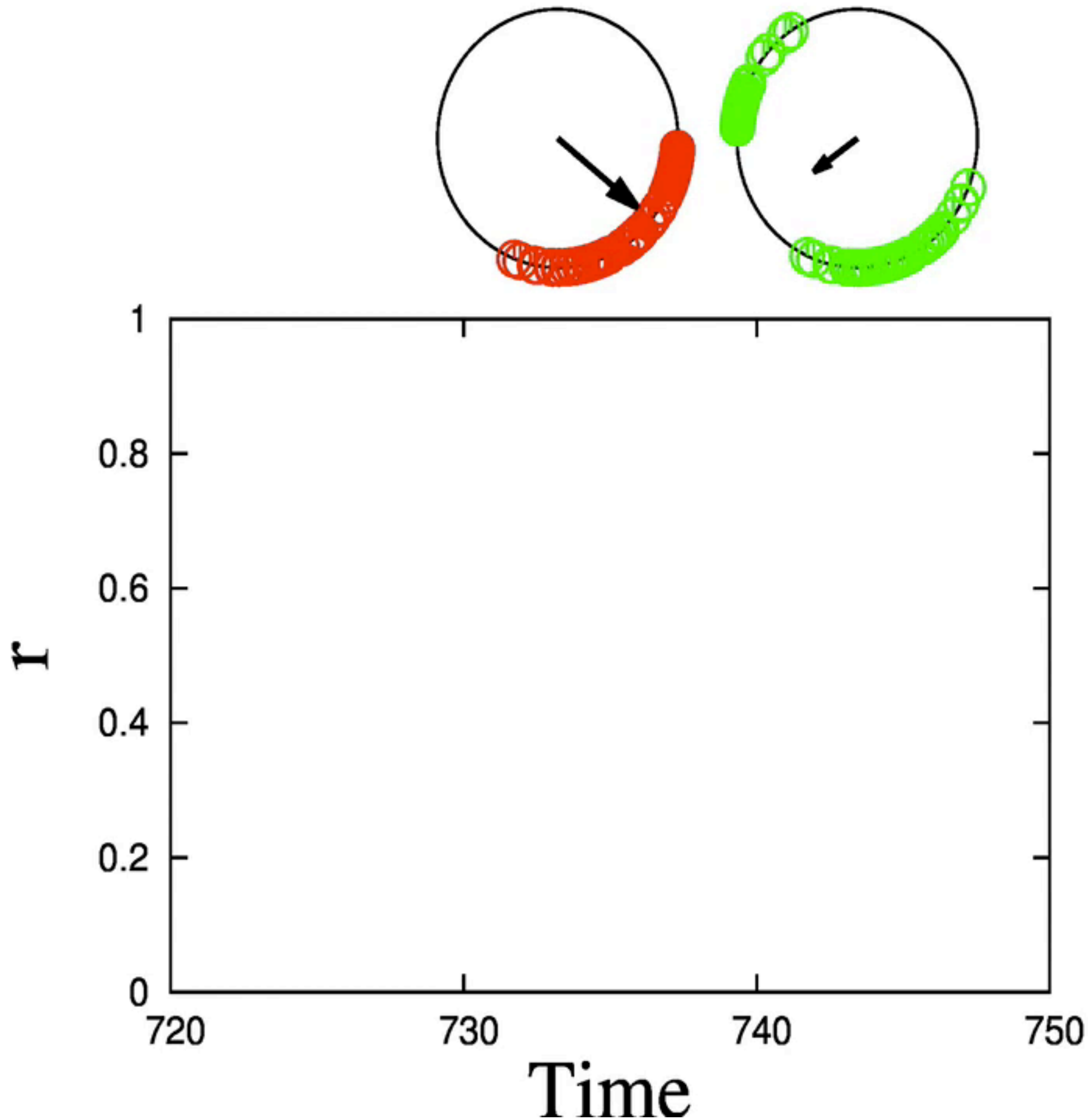
Time delay

weak internal coupling: breathing synchronization



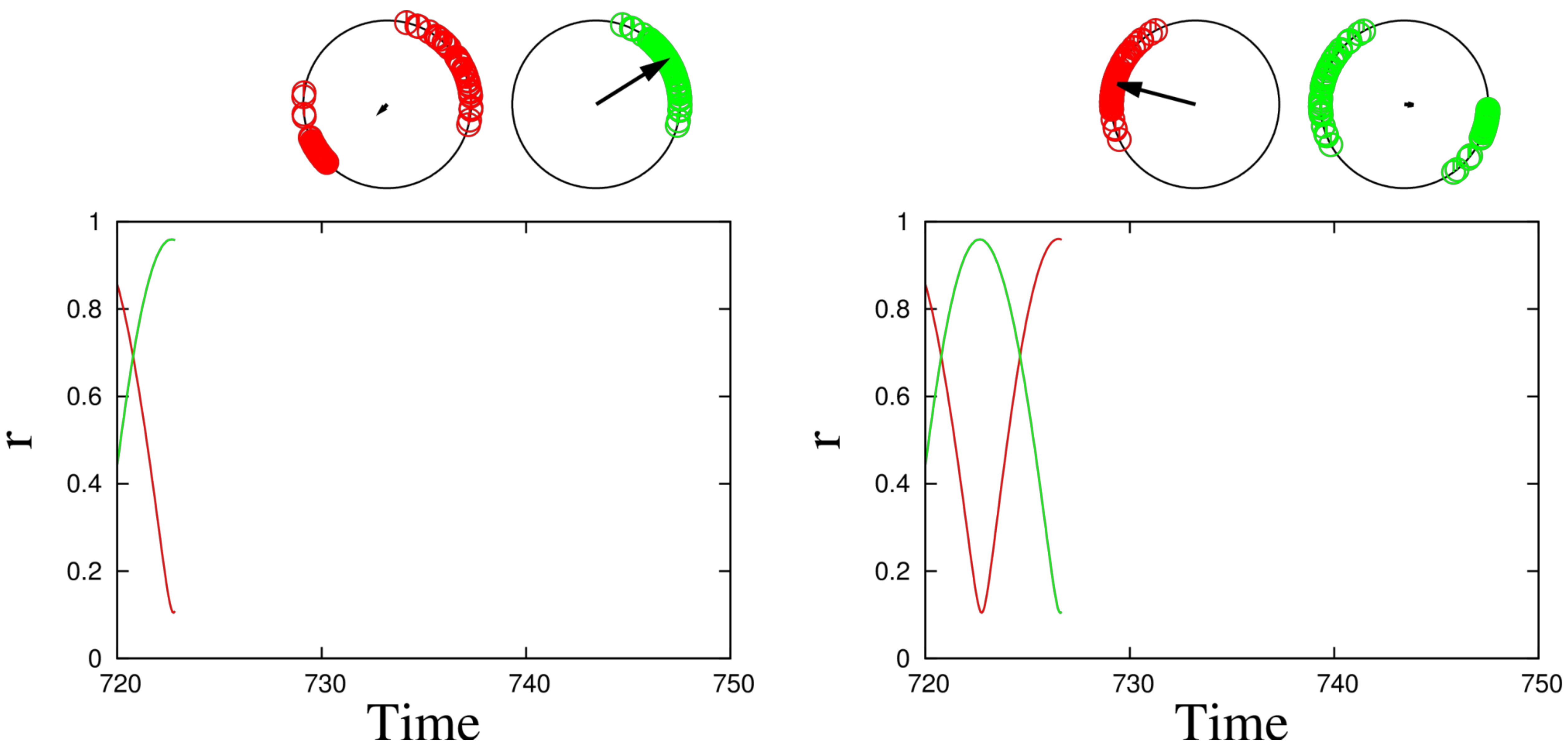
Time delay

weak internal coupling: breathing synchronization

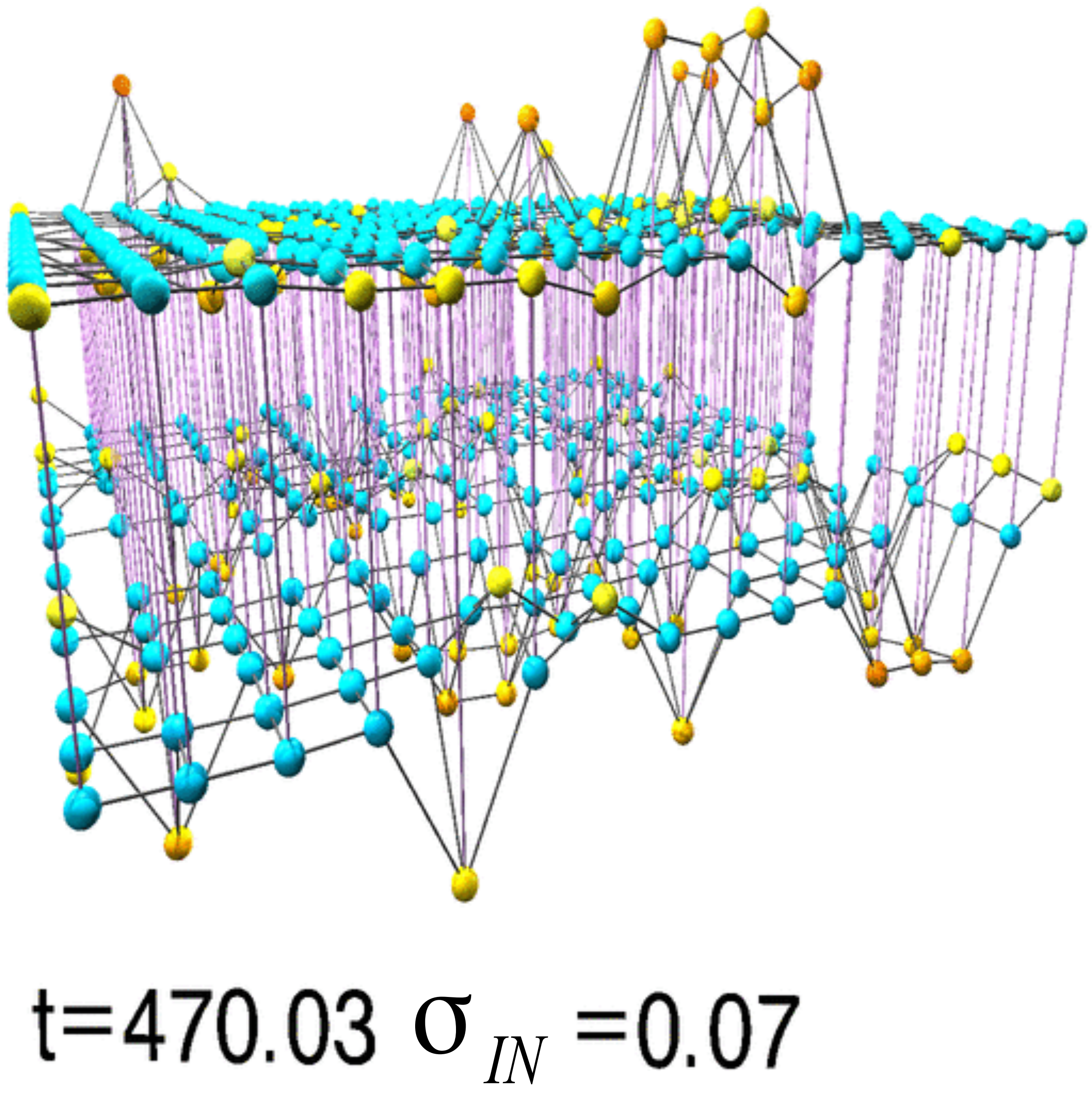


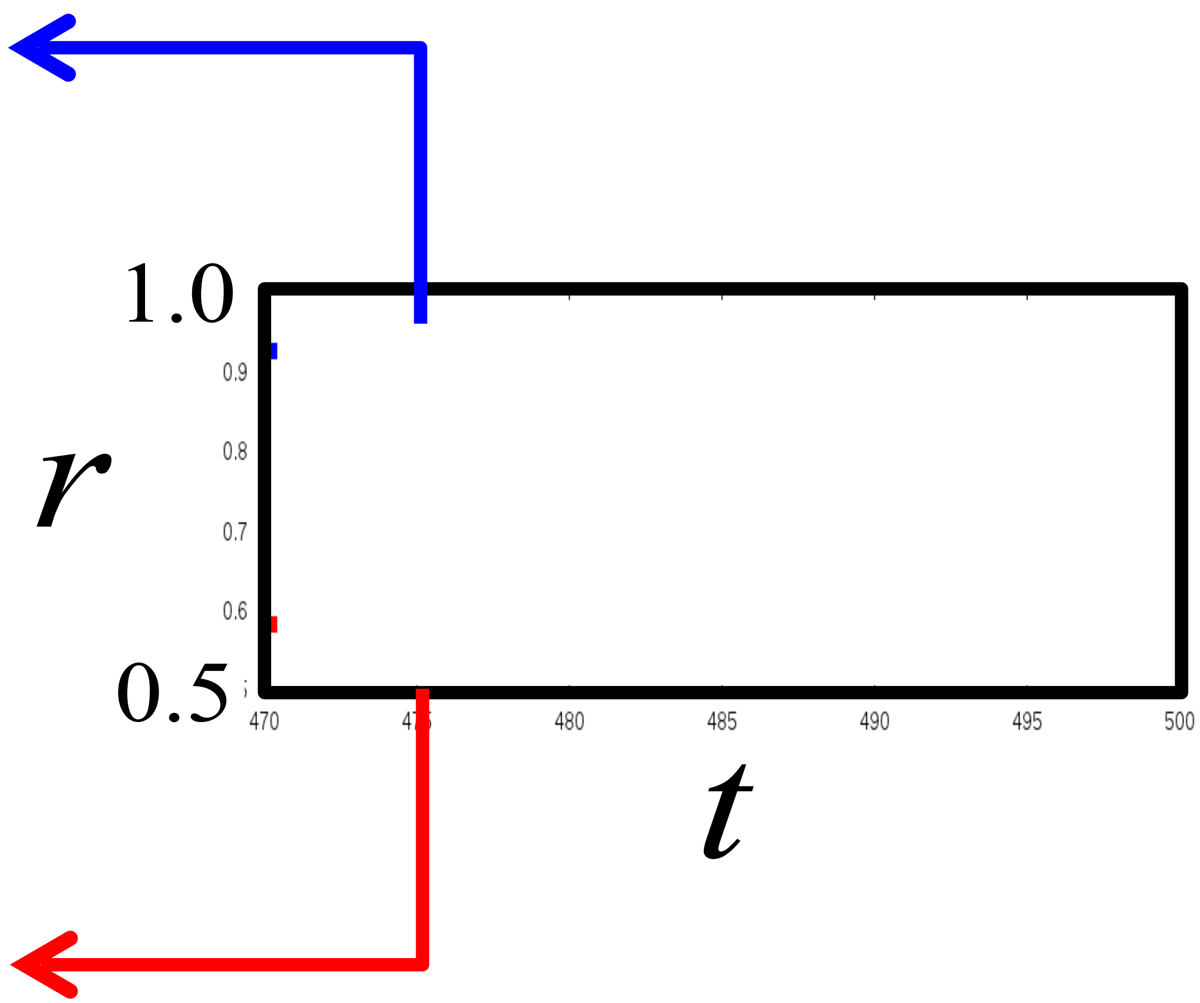
Time delay

weak internal coupling: breathing synchronization

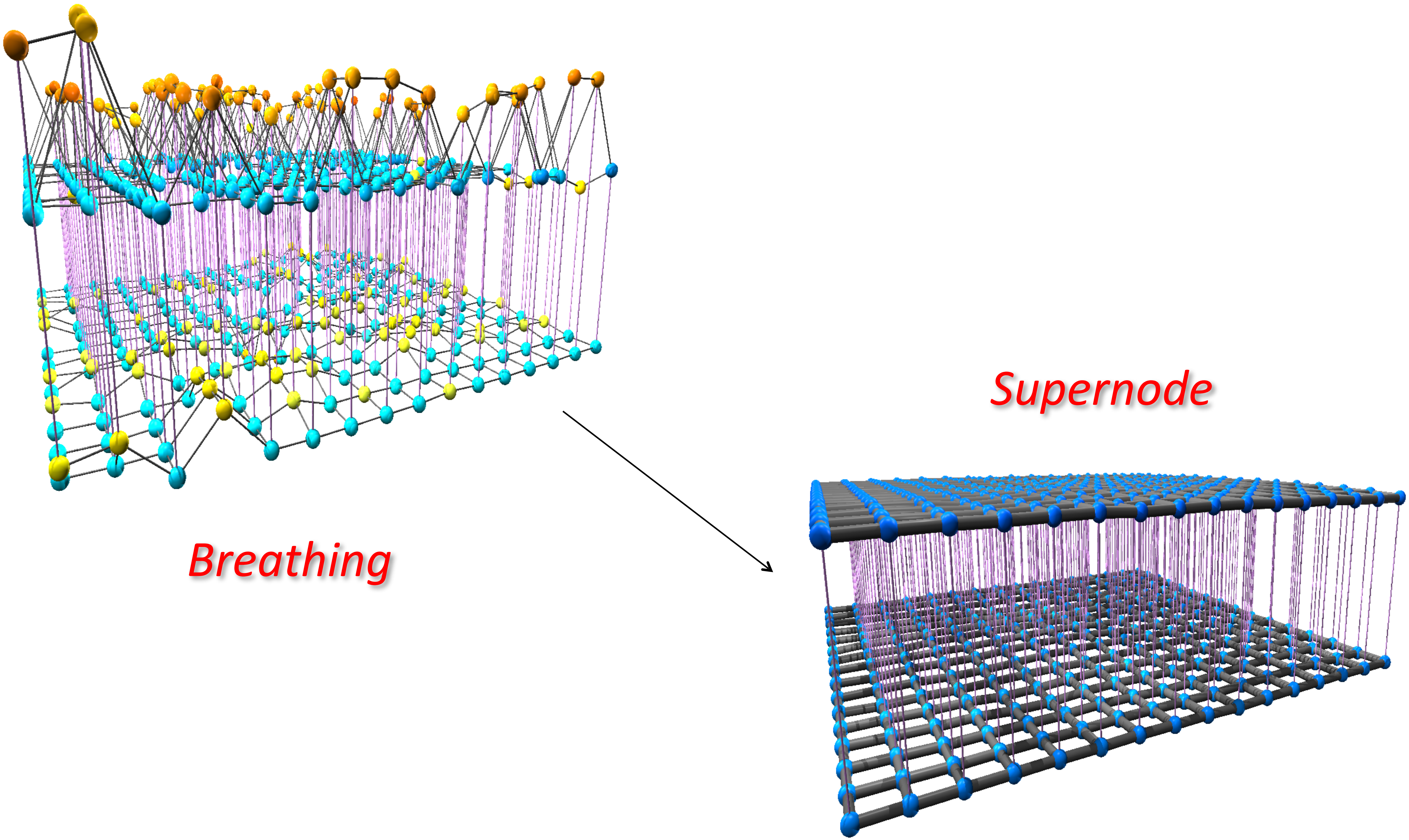


Time delay
increasing internal coupling

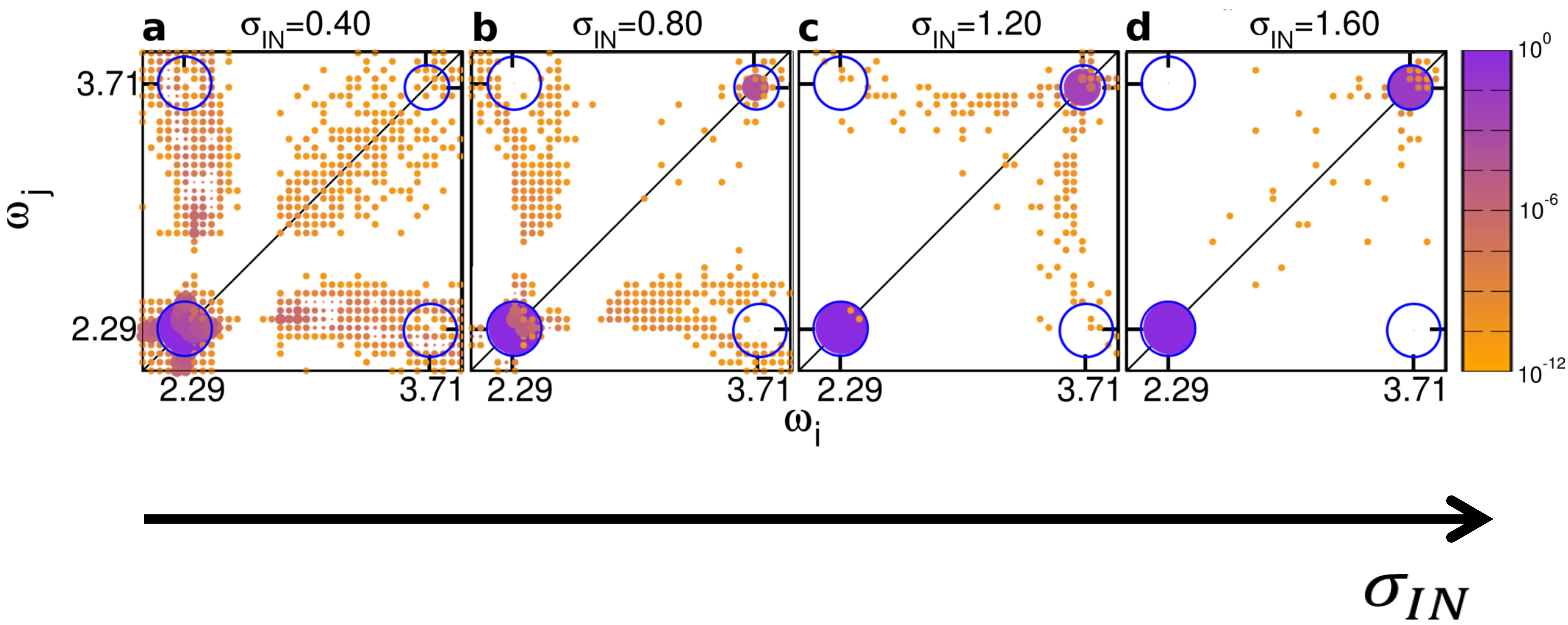




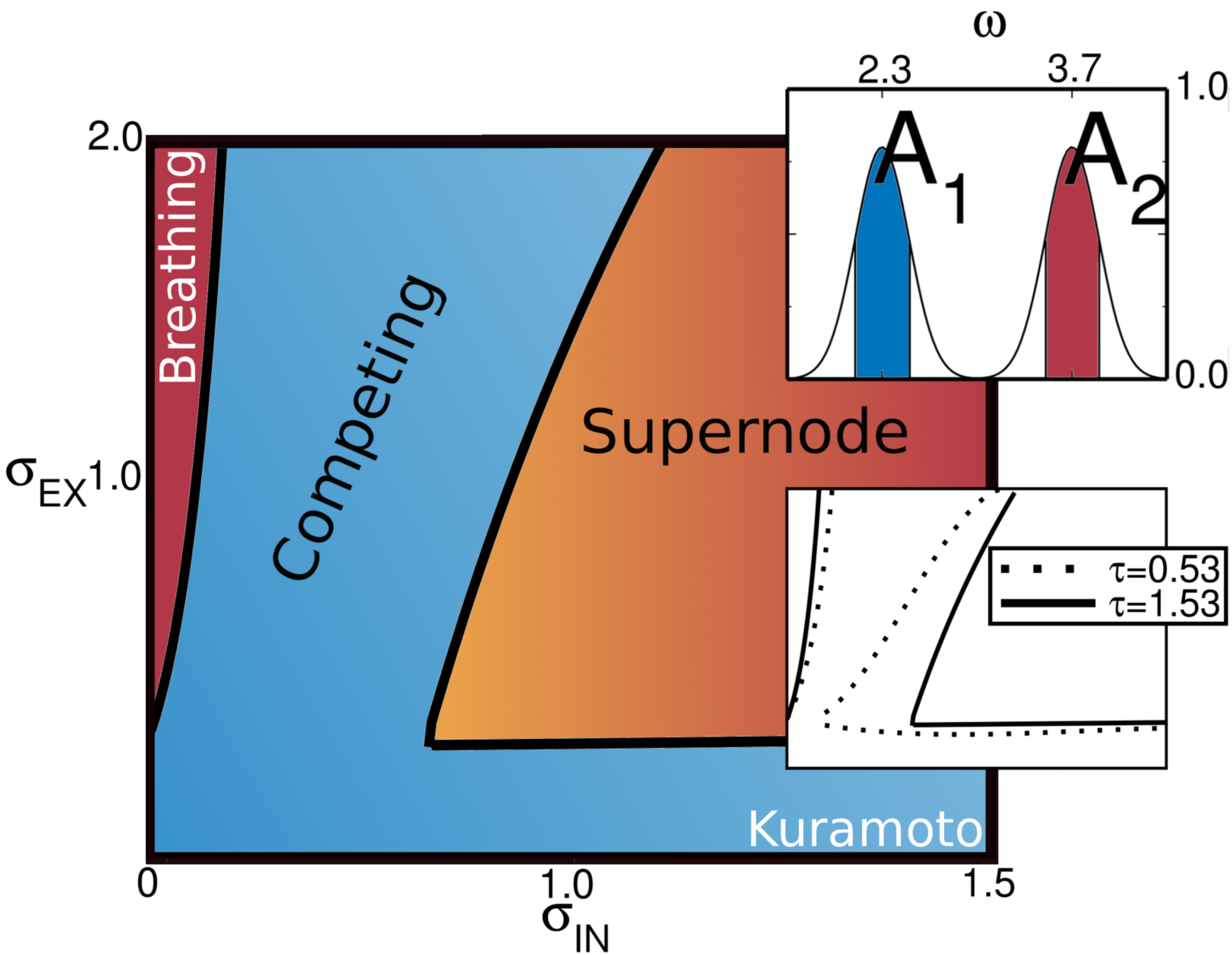
Time delay
increasing internal coupling



Time delay
increasing internal coupling



Time delay diagram



Time delay
bifurcation in frequency