Universo Primitivo
2022-2023 (1º Semestre)

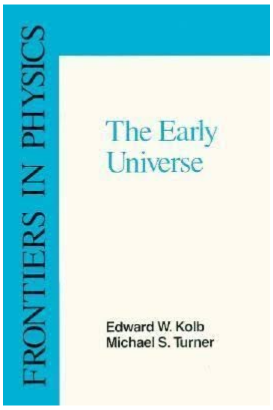
Mestrado em Física - Astronomia

Chapter 6

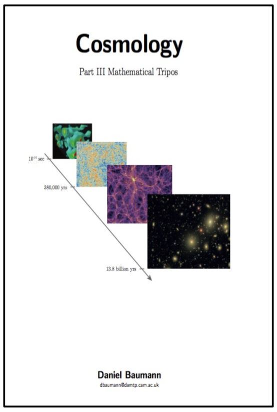
6 Big Bang Nucleosynthesis

- Initial Conditions;
- Nuclear statistical equilibrium;
- Neutron abundance;
- Helium abundance ;
- Comparison with observations
- BBN as a probe of cosmology and fundamental physics

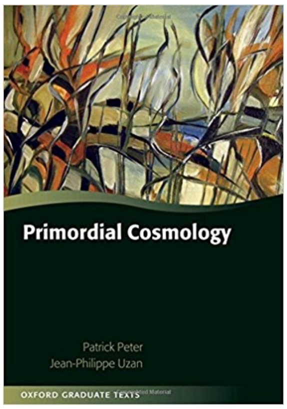
References



Ch. 4



Ch. 3.3



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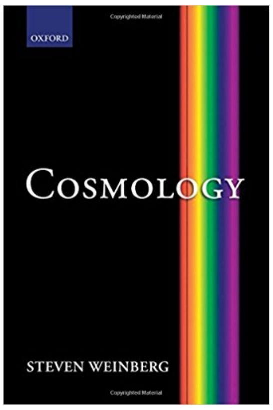
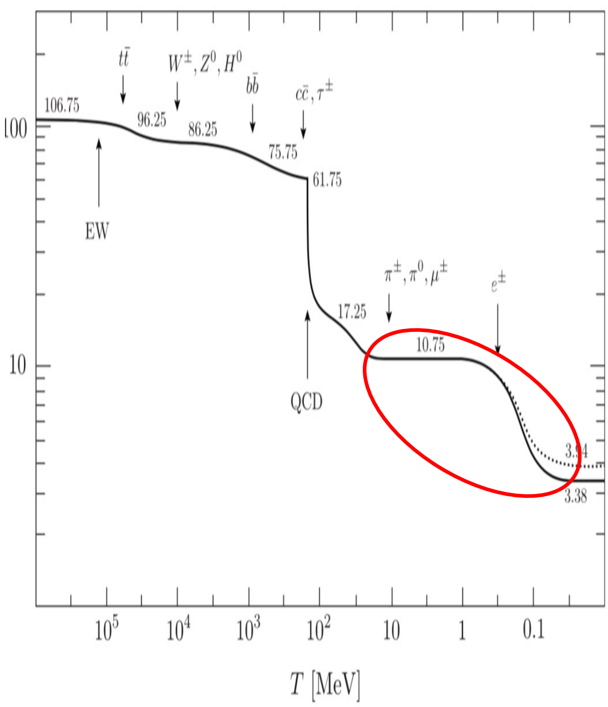


Ch. 3.2

Big-Bang Nucleosynthesis

Initial conditions



After QCD phase transition and $T \gtrsim 1$ MeV protons and neutrons…

- remain in equilibrium with the fluid due to weak interactions involving neutrinos.
- First atomic nuclei may form in equilibrium via 2-body nuclear reactions between protons + neutrons
- The proton to neutron ratio is $n/p = n^{eq}/p^{eq}$

By $T \sim 1 - 0.7$ MeV,

- weak interactions can no longer keep protons and neutrons in equilibrium
- Free neutrons decay into protons by $T/MeV \sim 0.8$, while atomic nuclei remain in equilibrium
- Neutrinos decouple, and n/p start to deviate from the equilibrium value, $n/p \neq n^{eq}/p^{eq}$

By $T \sim 0.7 - 0.5$ MeV,

- The production of deuterium, $n + p \to D + \gamma$, ceases when the number of neutrons decrease.
- Light atomic nuclei are then consistently formed by 2-body reaction involving deuterium nuclei (3 body reactions are very unlikely).

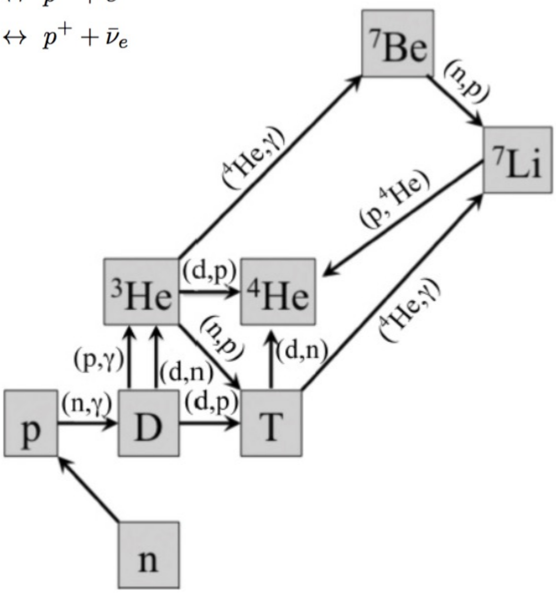
Big-Bang Nucleosynthesis

Initial conditions

$$\mathrm{p} + \mathrm{n} \to \gamma + \mathrm{D} \qquad \mathrm{D} + \mathrm{p} \to \gamma + {}^3\mathrm{He} \qquad \mathrm{D} + \mathrm{D} \to \mathrm{n} + {}^3\mathrm{He}$$
$$\mathrm{D} + \mathrm{D} \to \mathrm{p} + \mathrm{T} \qquad \mathrm{T} + \mathrm{D} \to \mathrm{n} + {}^4\mathrm{He} \qquad \mathrm{T} + {}^4\mathrm{He} \to \gamma + {}^7\mathrm{Li}$$
$${}^3\mathrm{He} + \mathrm{n} \to \mathrm{p} + \mathrm{T} \qquad {}^3\mathrm{He} + \mathrm{D} \to \mathrm{p} + {}^4\mathrm{He} \qquad {}^3\mathrm{He} + {}^4\mathrm{He} \to \gamma + {}^7\mathrm{Be}$$
$${}^7\mathrm{Li} + \mathrm{p} \to {}^4\mathrm{He} + {}^4\mathrm{He} \qquad {}^7\mathrm{Be} + \mathrm{n} \to \mathrm{p} + {}^7\mathrm{Li}$$

Main nuclear reactions that can be establish during this phase

$$n + \nu_e \leftrightarrow p^+ + e^-$$
$$n + e^+ \leftrightarrow p^+ + \bar{\nu}_e$$



Big-Bang Nucleosynthesis (BBN) is able to predict the observed abundances of light elements!

Big-Bang Nucleosynthesis

Nuclear Statistical equilibrium

Let us assume a given atomic nucleus $A = n + p$ nucleons (A is the **nuclear mass number**, n is the number of neutrons, and p is the number of protons of the nucleus. The nuclear charge is given by the **atomic number** $Z = p$.

The number density of this **nuclear species** at equilibrium is given by the non-relativistic expression derived in Series 2 (exercise 5.1):

$$\boxed{n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_A - m_A}{T}\right)}$$

where the chemical potential needs to account for the number of protons and neutrons that make up the nucleus

$$\mu_A = Z\mu_p + (A - Z)\mu_n$$

We can also write similar equations to the free (non-relativistic) protons and neutrons, noticing that both these particles have 2 degrees of freedom (spin).

Big-Bang Nucleosynthesis

Nuclear Statistical equilibrium

i.e., **neutrons and protons have also non-relativistic equilibrium densities** given by the previous expression ($\mathrm{A} = 1$, $g_A = 2$) :

$$n_n = 2\left(\frac{m_n T}{2\pi}\right)^{3/2} e^{-(m_n-\mu_n)/T}$$
$$n_p = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} e^{-(m_p-\mu_p)/T}.$$

The **nuclear binding energy**, B_A, of a nucleus with atomic mass, A, is defined as the difference between the total mass of free nucleons and the mass of the nucleus:

$$B_A = Zm_p + (A-Z)m_n - m_A$$
$$m_A = Zm_p + (A-Z)m_n - B_A$$

Using these expressions in n_A (the previous slide) and approximating $m_A = Am_B$ inside the (), with $m_p \simeq m_n \simeq m_B$, one obtains (series' exercise):

$$\boxed{n_A = \frac{g_A}{2^A} A^{3/2} \left(\frac{m_B T}{2\pi}\right)^{3(1-A)/2} n_p^Z n_n^{A-Z} e^{B_A/T}}$$

Big-Bang Nucleosynthesis

Nuclear Statistical equilibrium

This shows that abundance of a nuclear species, critically depends on :

- the abundance of protons and neutrons at a given T ;
- the binding energy to temperature ratio, B_A/T

It is useful to write the nuclear abundances in terms of **mass fraction abundances**, X_A, defined as:

$$\boxed{X_A \equiv \frac{n_A A}{n_B}} \quad \text{where} \quad \boxed{n_B = n_p + n_n + \sum_A A n_A}$$

This definition allows on to write the following conservation equation of nuclear abundances

$$\sum_A X_A = 1$$

Using $n_A = X_A n_B/A$ in the expression of n_A in the previous slide, one has:

$$X_A = \frac{g_A}{2^A} A^{5/2} \left(\frac{m_B T}{2\pi}\right)^{3(1-A)/2} \frac{n_p^Z n_n^{A-Z}}{n_B} e^{B_A/T}.$$

Big-Bang Nucleosynthesis

Nuclear Statistical equilibrium

The density ratio in the previous expression can be written as:

$$\frac{n_p^Z n_n^{A-Z}}{n_B} = \frac{n_p^Z}{n_B^Z}\frac{n_n^{A-Z}}{n_B^{A-Z}} n_B^{A-1} = X_p^Z X_n^{A-Z} n_B^{A-1} = X_p^Z X_n^{A-Z} n_\gamma^{A-1} \eta^{A-1}$$

where we write $\boldsymbol{n_B = n_\gamma \, \eta}$ and define the **baryon to photon ratio** as:

$$\boxed{\eta \equiv n_B/n_\gamma} \qquad \text{where,} \qquad n_\gamma = \tfrac{2}{\pi^2}\zeta(3)T^3$$

η is a central quantity in BBN. It can be calculated at present ($T_0 = 2.7525$):

$$\eta = 2.74 \times 10^{-8} h^2 \Omega_B$$

Using these expressions in X_A (of the previous slide) one has (check all the steps!):

$$\begin{aligned} X_A &= \frac{g_A}{2^A} A^{5/2} \left(\frac{m_B T}{2\pi}\right)^{3(1-A)/2} X_p^Z X_n^{A-Z} \left(\frac{2}{\pi^2}\zeta(3)T^3\right)^{A-1} \eta^{A-1} e^{B_A/T} \\ &= g_A A^{5/2} 2^{-A+A-1-3(1-A)/2} \pi^{-2A+2-3(1-A)/2} \zeta(3)^{A-1} T^{(1-A)(-3+3/2)} m_B^{3(1-A)/2} \eta^{A-1} X_p^Z X_n^{A-Z} e^{B_A/T} \\ &= g_A \zeta(3)^{A-1} 2^{-5/2+3/2A} \pi^{-1/2-1/2A} A^{5/2} T^{-3(1-A)/2} m_B^{3(1-A)/2} \eta^{A-1} X_p^Z X_n^{A-Z} e^{B_A/T}, \end{aligned}$$

Big-Bang Nucleosynthesis

Nuclear Statistical equilibrium

Which can be written in a nicer way...

$$\boxed{X_A = F(A)\left(\frac{T}{m_B}\right)^{3(A-1)/2} \eta^{A-1} X_P^Z X_n^{A-Z} e^{B_A/T}}$$

where

$$F(A) = g_A A^{5/2} \zeta(3)^{A-1} \pi^{(1-A)/2} 2^{(3A-5)/2}$$

This expression allows one to explicitly compute the mass fraction abundances of any nuclear species **assuming nuclear statistical equilibrium**. In particular one has:

$$\begin{aligned} &\mathrm{D}: && X_2 = 16.3\left(\frac{T}{m_B}\right)^{3/2} \eta e^{B_2/T} X_n X_p, && B_2 = 2.22\ \mathrm{MeV} \\ &{}^3\mathrm{He}: && X_3 = 57.4\left(\frac{T}{m_B}\right)^{3} \eta^2 e^{B_3/T} X_n X_p^2, && B_3 = 7.72\ \mathrm{MeV}(^3\mathrm{He}) \\ &{}^3\mathrm{H}: && && B_3 = 6.92\ \mathrm{MeV}(^3\mathrm{H}) \\ &{}^4\mathrm{He}: && X_4 = 113\left(\frac{T}{m_B}\right)^{9/2} \eta^3 e^{B_4/T} X_n^2 X_p^2, && B_4 = 28.3\ \mathrm{MeV} \\ &{}^{12}\mathrm{C}: && X_{12} = 3.22 \times 10^5\left(\frac{T}{m_B}\right)^{33/2} \eta^{11} e^{B_{12}/T} X_n^6 X_p^6, && B_{12} = 92.2\ \mathrm{MeV} \end{aligned}$$

Big-Bang Nucleosynthesis

Nuclear Statistical equilibrium

These abundances are constrained by the conservation equation which, if one neglects other elements, reads:

$$X_n + X_p + X_2 + X_3 + X_4 + X_{12} = 1$$

The neutron and proton fractions are related. Their mass fractions in equilibrium can be easily obtained. We know that

$$\boxed{\left(\frac{n_n}{n_p}\right)_{\rm eq} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-(m_n-m_p)/T}}$$

Dividing the numerator and the denominator on left hand side of this equation by n_b one obtains:

$$\left(\frac{n_n}{n_p}\right)_{\rm eq} = \left(\frac{X_n}{X_p}\right)_{\rm eq} \simeq e^{-Q/T} \qquad (Q = m_n - m_p)$$

Note that expressions for X_n derived in the previous slides assume the approximation $m_B = m_p = m_n$. If this approximation is taken rigorously then $X_n/X_p = 1$. However the mass difference ($Q = m_n - m_p$) in the exponential should not be ignored whereas it is smaller impact on the ratio of masses of the right hand side of n_n/n_p .

Big-Bang Nucleosynthesis

Nuclear Statistical equilibrium

Also note that, since the mass fraction abundances add up to one, e.g.,

$$X_n + X_p + X_2 + X_3 + X_4 + X_{12} = 1$$

and **nuclear synthesis occurs via 2-body reactions** (such as those in slide 5):

- Heavier **nuclear species are only effectively produced after the lighter ones are produced**. This is the case of Helium-4 which is only produced via 2-body reaction involving Deuterium or Hydrogen-3
- If the abundance fraction of a given nuclear species increases, this happens at the expenses of the some other species (which has its fraction reduced)

So one can define an **estimate of the temperature at which a given nuclear species is effectively produced by setting $X_A \sim 1$**. this can only happen for $T \ll B_A$ so that the exponential term in X_n compensates η (the baryon to photon ratio) term, which is small.

$$X_A = F(A)\left(\frac{T}{m_B}\right)^{3(A-1)/2} \eta^{A-1} X_P^Z X_n^{A-Z} e^{B_A/T} \sim 1$$

Big-Bang Nucleosynthesis

Nuclear Statistical equilibrium

From this previous expression on can derive an approximate equation to compute the temperature of effective production of a given nuclear species, T_A. Setting $X_A \sim X_n \sim X_p \sim 1$, taking the logarithm of X_A and droping $\ln F(A)$, which is also $F_A \sim 1$, gives:

$$0 = \frac{3}{2}(A-1)\ln\left(\frac{T_A}{m_B}\right) + (A-1)\ln\eta + \frac{B_A}{T_A}$$

This can be used with iterative numerical methods to estimate T_A,

$$\begin{aligned} T_A &\approx -\frac{B_A}{\frac{3}{2}(A-1)\ln\left(\frac{T_A}{m_B}\right) + (A-1)\ln\eta} \\ &= \frac{B_A}{A-1}\frac{1}{\ln\eta^{-1} + \frac{3}{2}\ln\left(\frac{m_B}{T_A}\right)}. \end{aligned}$$

For example, using this expression for Deuterium, one obtains:

$$T_D = \frac{2.22}{1}\frac{1}{\ln(2\times 10^{-8}\Omega_B h^2)^{-1} + \frac{3}{2}\ln\left(\frac{1\text{ GeV}}{T_D}\right)}$$

Similar equations can be derived for other nuclear species.

Big-Bang Nucleosynthesis

Nuclear Statistical equilibrium

Solving these type of equations, one obtains the following effective temperatures of production (in equilibrium) of the Deuterium, Tritium, and Helium-4:

$$T_D \approx 0.07\text{MeV}\ ; \quad T_{^3\text{H}} \approx 0.11\text{ MeV}\ ; \quad T_{^4\text{He}} \approx 0.28\text{ MeV}.$$

These temperatures can be converted to time using the Friedmann equation expressed in terms of temperature of the effective degrees of freedom in energy

$$\frac{\dot{a}}{a} = H = \sqrt{\frac{\hbar c}{3M_{pl}^2}\frac{\pi^2}{30}g_* T^4} = \frac{\pi}{3}\left(\frac{g_*}{10}\right)^{1/2}\frac{T^2}{M_{pl}}$$

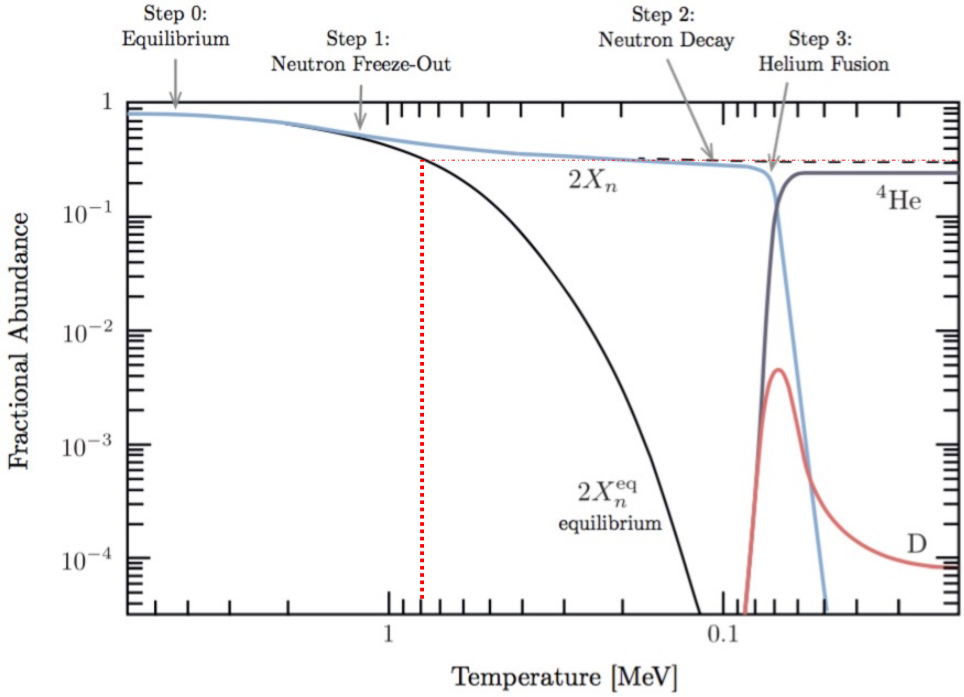
where we assume radition domination.
Taking $g_* = 3.38$ one can derive the following expression for the beginning of the nucleosynthesis,

$$\boxed{t_{\text{nuc}} = 132\text{ s}\left(\frac{0.1\text{ MeV}}{T_{\text{nuc}}}\right)^2.}$$

Big-Bang Nucleosynthesis

Neutron's abundance

The production of nuclear elements within the mechanism of Big-Bang nucleosynthesis is directly related with the abundance of free neutrons, and the evolution of n_B or the baryon to photon ratio. One can tell the story of neutrons in a few steps:



Neutrons decouple from the fluid and abandon equilibrium. They also decay into Protons.

Big-Bang Nucleosynthesis

Neutron's abundance

Step 0 (Equilibrium): Above $T \sim 1$ MeV protons and neutrons are in equilibrium via the nuclear reactions

$$n + \nu_e \leftrightarrow p^+ + e^-$$
$$n + e^+ \leftrightarrow p^+ + \bar{\nu}_e$$

The relative abundance of neutrinos to protons is then given by the equilibrium prediction (neglecting the chemical potential of the leptons, $\mu_n \sim \mu_p$):

$$\left(\frac{n_n}{n_p}\right)_{\rm eq} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-(m_n - m_p)/T}$$

where $m_n - m_p = Q = 1.293$ MeV is the mass difference between neutrons and protons. So the fraction of neutrons at equilibrium can be approximated by:

$$X_n^{\rm eq} \simeq \frac{n_n^{\rm eq}}{n_p^{\rm eq} + n_n^{\rm eq}} = \frac{n_n^{\rm eq}/n_p^{\rm eq}}{1 + n_n^{\rm eq}/n_p^{\rm eq}} \simeq \frac{e^{-Q/T}}{1 + e^{-Q/T}}$$

where $n_B \simeq n_n + n_p$ is used in the first equality and $m_n / m_p \simeq 1$ is used in the last equality. At $T = 0.8$ MeV this gives,

$$X_n^{\rm eq}(0.8\,{\rm MeV}) = 0.17 \sim 1/6$$

Big-Bang Nucleosynthesis

Neutron’s abundance

Step 1 (Decoupling): As neutrinos decouple and positron-electron annihilation occurs, neutrons are forced to also decouple from the fluid. From the previous sides one expects that the **freeze out abundance of neutrons** should be close to:

$$\boxed{X_n^\infty \sim X_n^{\rm eq}(0.8\,{\rm MeV}) \sim \frac{1}{6}}$$

To confirm this expectation one needs to integrate the Boltzmann equation for the interactions that keep neutrons and protons in contact with the plasma. As seen in Chapter 4, the **Boltzmann equation** for the 2-body interaction $1+2 \rightleftarrows 3+4$ is:

$$\frac{1}{a^3}\frac{d(n_1 a^3)}{dt} = -\langle\sigma v\rangle \left[n_1 n_2 - \left(\frac{n_1 n_2}{n_3 n_4}\right)_{\rm eq} n_3 n_4 \right]$$

For interactions of the form $n + l_1 \leftrightarrows p^+ + l_2$, where l_i is a lepton **tightly bound to the plasma** one obtains:

$$\boxed{\frac{1}{a^3}\frac{d(n_n a^3)}{dt} = -\Gamma_n \left[n_n - \left(\frac{n_n}{n_p}\right)_{\rm eq} n_p \right]}$$

Since leptons are tightly bound to the fluid one has: $n_{l_i} = n_{l_i}^{eq}$, and $\Gamma_n = < n_{l_1} \sigma v >$.

Big-Bang Nucleosynthesis

Neutron’s abundance

Step 1 (Decoupling): The solution of the Boltzmann equation is numerical. To compute the free neutron’s fraction, X_n, one needs to use its definition (in slide 8) and compute the densities of all baryon species in the fluid at a given time.

However one can simplify the calculation of X_n using the following approximations:

- before neutron decoupling $n_b \simeq n_n + n_p$
- the total number of baryons is conserved, i.e., $n_b a^3 = constant$.

Using these assumptions the Boltzmann equation can be written as:

$$\frac{dX_n}{dt} = -\Gamma_n \left[X_n - (1 - X_n) e^{-Q/T} \right]$$

To perform this integration, it is useful to make a change of variable $x = Q/T$, giving

$$\frac{dX_n}{dx} = \frac{\Gamma_n}{H_1} x \left[e^{-x} - X_n (1 + e^{-x}) \right]$$

where H_1 is the x −independent part of the Hubble rate written as a function of x.

$$H = \sqrt{\frac{\rho}{3M_{\rm pl}^2}} = \underbrace{\frac{\pi}{3}\sqrt{\frac{g_\star}{10}}\frac{Q^2}{M_{\rm pl}}}_{\equiv H_1 \approx 1.13\, s^{-1}} \frac{1}{x^2}\,, \qquad \text{with} \quad g_\star = 10.75\,.$$

Big-Bang Nucleosynthesis

Neutron's abundance

Step 1 (Decoupling):

The exact form of Γ_n depends on the lepton particles being considered. Its calculation can be done in Quantum Field Theory. It can be, generically, approximated by:

$$\Gamma_n(x) = \frac{255}{\tau_n} \cdot \frac{12 + 6x + x^2}{x^5}$$

where $\tau_n = 886.7$ s is the neutron half-time decaying period.

With these expressions the **numerical integration of the Boltzmann** equation (blue curve) would give:

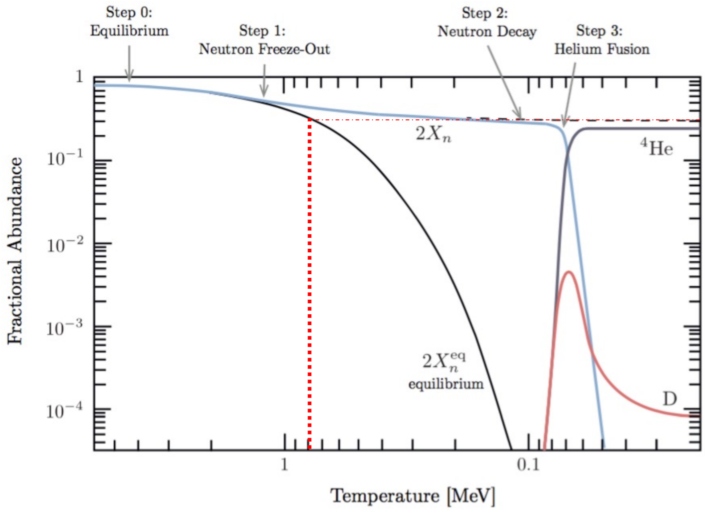
$$X_n^\infty \equiv X_n(x = \infty) = 0.15$$

only if neutrons don't decay! (Step 2)
This is similar to the result in slide 17.
So just **before Neutron decay** one has:

$$n_B \simeq n_p + n_n \iff 1 \simeq X_p + X_n$$

and

$$\boxed{X_p \simeq 1 - X_n = 0.85 \;\; ; \;\; X_n/X_p \simeq 0.17}$$



Big-Bang Nucleosynthesis

Neutron's abundance

Step 2 (Neutron decay): The decoupled neutrons also decay into protons via the process:

$$\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e$$

which has a half-time decaying period of $\tau_n = 886.7 \pm 0.8$ sec.. **This can only start effectively enough when the universe is as old as this decaying period**).

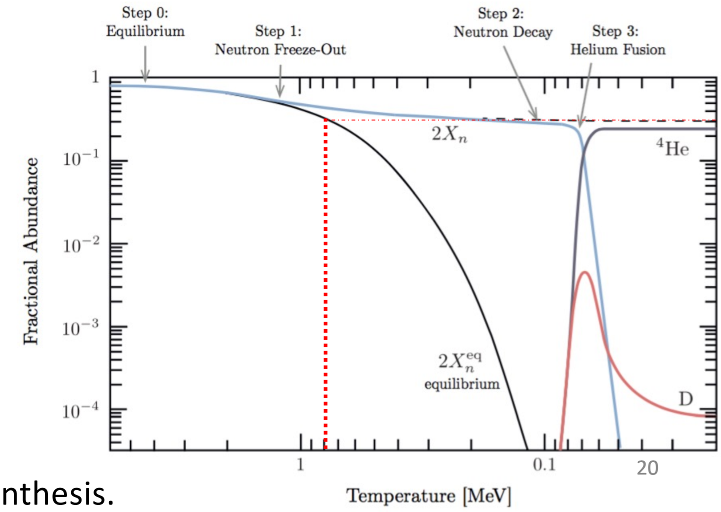
To include neutron decay into the calculation we simply multiply the freeze-out abundance by an exponential term characteristic of nuclear decaying processes:

$$\boxed{X_n(t) = X_n^\infty e^{-t/\tau_n} = \frac{1}{6} e^{-t/\tau_n}}$$

Where t is related to temperature via a temperature time relation, as:

$$t = 132 \text{ s} \left(\frac{0.1 \text{ MeV}}{T}\right)^2.$$

This decaying mechanism has strong implications for the nuclear species synthesis.



Big-Bang Nucleosynthesis

Neutron's abundance

Step 2 (Neutron decay)

Let us now compute mass fraction abundances by $\boldsymbol{t = t(T \sim 0.07\ MeV) = 269.4}$ **s**

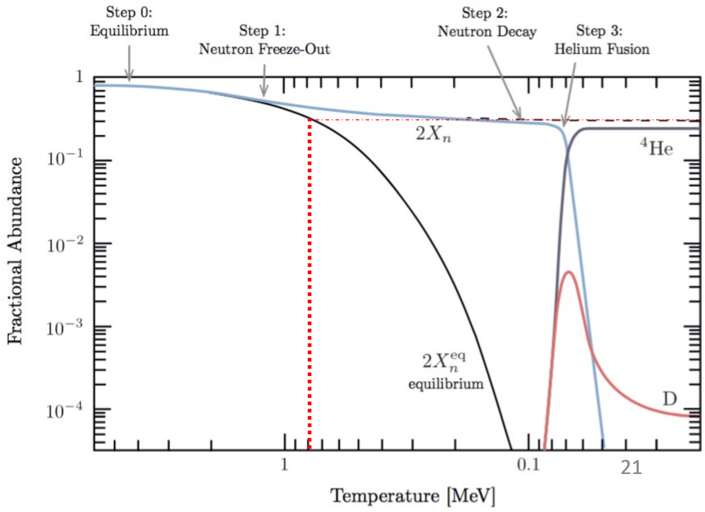
$$X_n(0.07\ MeV) = \frac{1}{6}\ \exp\left(-\frac{260.4\ s}{886.7\ s}\right) = 0.123$$

$$X_p(0.07\ MeV) = 1 - X_n = 0.877$$

and therefore:

$$\frac{X_n(0.07\ MeV)}{X_p(0.07\ MeV)} = 0.140 \sim \frac{1}{7}$$

this is just before Hellium-4 starts to be produced in an efficient way.



Big-Bang Nucleosynthesis

Helium abundance

Step 3 (Helium fusion): Helium is produced via the reactions:

$$\mathrm{D} + p^+ \ \leftrightarrow\ {}^3\mathrm{He} + \gamma\ ,$$

$$\mathrm{D} + {}^3\mathrm{He}\ \leftrightarrow\ {}^4\mathrm{He} + p^+$$

that **require the existence of Deuterium**, which is produced via: $n + p^+ \ \leftrightarrow\ \mathrm{D} + \gamma$

So, helium cannot be produced before a sufficient amount of deuterium is formed.

The **helium fraction abundance by the end of BBN** can be estimated as follows:

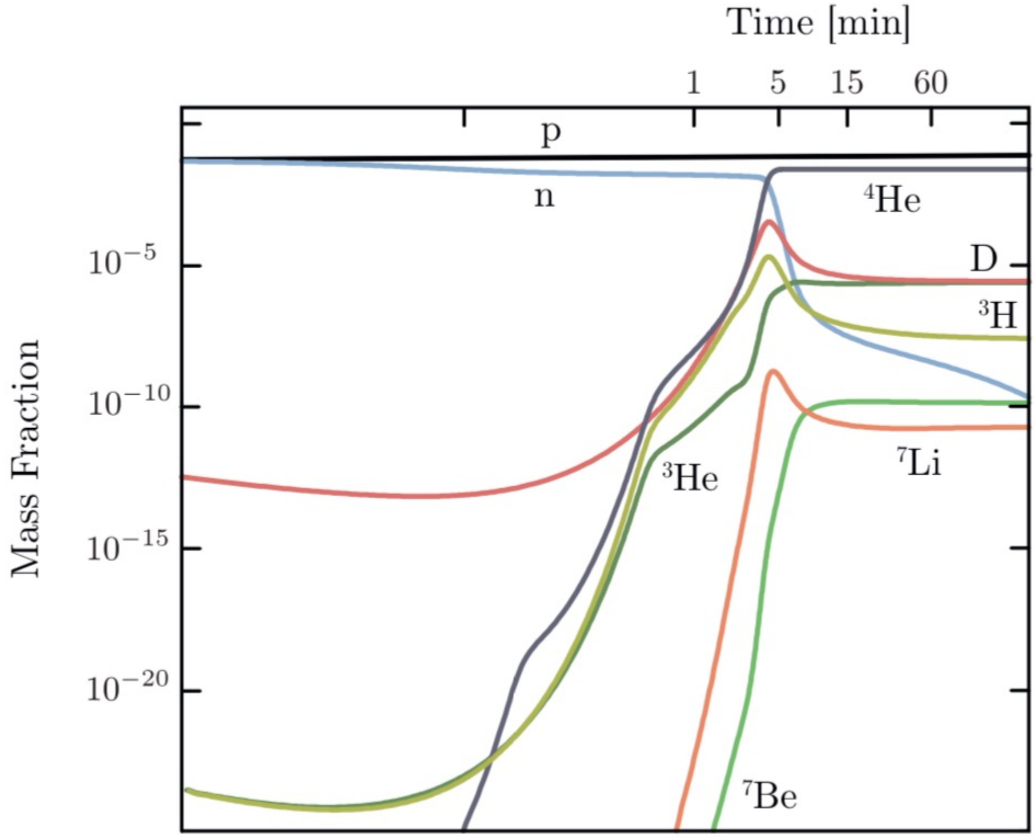
- Until before neutron decay ($T \sim 0.07$) all baryons are in the form of free protons and neutrons: $\boxed{n_B^i \simeq n_p^i + n_n^i}$
- By the end of BBN **hydrogen** (p) and **helium-4** nuclei are the 1st and 2nd most abundant elements (other nuclei are residual). So, baryon conservation allows to write: $\boxed{n_p^f + 4n_{^4\mathrm{He}}^f = n_p^i + n_n^i}$
- By the end of BBN **about half of the initial neutrons are inside helium nuclei** (because each nucleus of helium contains 2 neutrons): $\boxed{n_{^4\mathrm{He}}^f = n_n^i/2}$

Under these approximations, the **Helium mass fraction abundance** becomes:

$$X_{^4\mathrm{He}} = \frac{4n_{^4\mathrm{He}}^f}{n_p^f + 4n_{^4\mathrm{He}}^f} = \frac{4n_n^i/2}{n_p^i + n_n^i} = \frac{2n_n^i}{n_p^i + n_n^i} = \frac{2n_n^i/n_p^i}{1 + n_n^i/n_p^i} = \frac{2X_n^i/X_p^i}{1 + X_n^i/X_p^i} \simeq \frac{2/7}{1+1/7} \simeq \frac{1}{4}$$

Big-Bang Nucleosynthesis

Numerical evolution of mass fraction abundances of light elements:



Big-Bang Nucleosynthesis

Comparison with observations:

Helium 4: constraints from ionized gas (metal poor) clouds

Deuterium: constraints from metal poor quasar absorption systems

Helium 3: is hard to constraint. Limits estimated from solar system and HII (metal abundant) regions in our galaxy

Lithium 7: constraints from low metallicity population II stars in our galaxy.

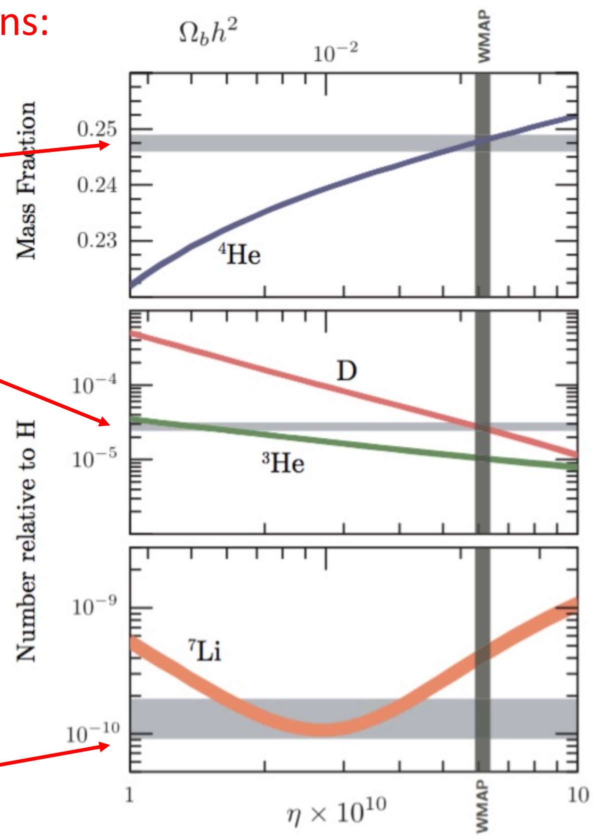


Figure 3.10: *Theoretical predictions (colored bands) and observational constraints (grey bands).*