METFOG **2022/23**

Final Exam - 2^a Fase **06/07/2023** 9h -12h

1. [**4.0 val**] Consider the following Lagrangian, where ϕ_1 and ϕ_2 are real spin zero field and ψ is a spin 1/2 field,

$$\mathcal{L} = i\bar{\psi}\partial_\mu\gamma^\mu\psi - m_l\bar{\psi}\psi + \frac{1}{2}\partial_\mu\phi_1\,\partial^\mu\phi_1 - \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{2}\partial_\mu\phi_2\,\partial^\mu\phi_2 - \frac{1}{2}m_2^2\phi_2^2 + \lambda_1\phi_1^4 + \lambda_2\phi_2^4 + \kappa_1\bar{\psi}\psi\phi_1 + \alpha\phi_2^2\phi_1.$$

a) [**1.0 val**] Show the Lagrangian is invariant under the Z_2 symmetry $\phi_2 \to -\phi_2$. Explain why ϕ_2 can be seen as the DM particle.

b) [**1.0 val**] Write the missing terms of the Lagrangian that are compatible with the symmetry up to dimension 4.

c) [**2.0 val**] Draw the Feynman diagrams for the production of DM in the process $\psi\bar{\psi} \to \phi_2\phi_2\phi_1$ at tree-level and write the amplitude (not the cross section). What would one see at a collider with such a final state knowing that ϕ_1 decay mainly to $\bar{\psi}\psi$?

2. [**6.0 val**] Consider the Standard Model extended by a new fermion χ, a pseudoscalar P and a scalar S with a Lagrangian given by

$$\mathcal{L} = \mathcal{L} = \mathcal{L}_{SM} + \bar{\chi}(\gamma_\mu\partial^\mu - m_\chi)\chi - iy_\chi P\bar{\chi}\gamma_5\chi + \kappa_1 v S^2 h + \kappa_2 P S^2 + \frac{1}{2}\partial_\mu S\,\partial^\mu S - \frac{1}{2}m_S^2 S^2 + \frac{1}{2}\partial_\mu P\,\partial^\mu P - \frac{1}{2}m_P^2 P^2$$

where f is a SM fermion and S is the DM particle.

a) [**3.0 val**] Draw the Feynman diagram and calculate the amplitude for the scattering $Sq \to Sq$ where q is a generic quark. Extract the corresponding Wilson coefficient. (The Feynman rule for $\bar{q}qh$ is $-im_q/v$ and $v = 246$ GeV). Using the leading order form factor, write the expression for the direct detection cross section for a generic nucleon (no numbers involved!).

b) [**3.0 val**] Calculate the width $\Gamma(h \to SS)$ as a function κ_1. Assuming that the total width of the Higgs is $\Gamma_T(h) = 4.6$ MeV and that the present measurement on the Higgs invisible branching ratio is 0.11, find a bound on κ_1 for a DM mass of 10 GeV.

3. **[6.0 val]** Consider the Standard Model extended by a complex doublet H_D and with a portal Lagrangian given by

$$\mathcal{L}_{portal} = \lambda_D H_D^\dagger H_D H^\dagger H$$

where $H^\dagger = (0 \quad (v+h)/\sqrt{2})$ is the Higgs doublet in the unitary gauge and the dark doublet is written as $H_D^\dagger = (H_D^+ \quad (h_D - iA_D)/\sqrt{2})$. Consider that A_D is the lightest dark scalar and therefore the DM particle.

a) **[0.5 val]** Write the Feynman rules for the triple coupling $A_D A_D h$ and for the quartic coupling $A_D A_D hh$.

b) **[3.0 val]** Draw the Feynman diagrams and write the amplitude for the process $A_D A_D \to hh$ (note that there are three diagrams and that the Feynman rule for the SM triple Higgs coupling is $-3im_h^2/v$). Using only the term in the amplitude with the quartic coupling (hhA_DA_D) calculate the thermal averaged cross section for the process. Use the following approximations: $\sqrt{s} = 2m_{A_D}$, neglect the SM Higgs width, and use $< \sigma v >_{A_D A_D \to hh} = v\sigma_{A_D A_D \to hh}$ with $v = 2\sqrt{\frac{s}{4m_{A_D}^2} - 1}$. Remember the cross section can be written as

$$\sigma(A_D A_D \to hh) = \frac{1}{16\pi s}\sqrt{\frac{s - 4m_h^2}{s - 4m_{A_D}^2}}|T|^2$$

c) **[1.5 val]** Using the formulae below and setting $\lambda_D = 1$ and $m_{A_D} = 150$ GeV, check that the relic density is below the experimental value. Is this point excluded?

$$\Omega_\chi h^2 = m_{A_D} s_0 Y_0 \frac{8\pi G}{3H^2} \approx 2.742 \times 10^8 \frac{m_{A_D}}{GeV} Y_0 \qquad (\Omega h^2)_{DM}^{obs} = 0.1186 \pm 0.0020$$

$$Y_0 = 10/\lambda \qquad \lambda = N < \sigma v >_{A_D A_D \to hh} \qquad N = 1.6 \times 10^{20}$$

d) **[1.0 val]** What is the meaning of relic density? Is there a relic density for other particles other than DM?

4. **[4.0 val]** Consider a temperature dependent potential of the form

$$V(\phi, T) = 0.01(T^2 - T_0^2)\phi^2 - 0.1T\phi^3 + \frac{\lambda}{4}\phi^4$$

where $T_0 = 238$ K and $\lambda = m_h^2/(2v^2)$ and T is the temperature.

a) **[2.5 val]** Show that there is a first order phase transition and calculate the critical temperature.

b) **[1.5 val]** Find the value of the Higgs mass so that the transition could be considered strong, that is, $\frac{\phi_m(T_c)}{T_c} > 1.3$ with $\phi_m = .02\, T_c/\lambda$.

$$1\,\text{barn} = 0.00257\,\text{MeV}^{-2} = 10^{-24}\,\text{cm}^2$$