ELETROMAGNETISMO E ANÁLISE DE CIRCUITOS

2023/24

RESOLUÇÃO DA 1ª CHAMADA

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a) A DISTÂNCIA ENTRE QUALQUER DAS CARGAS E O CENTRO DO QUADRADO É:

$$d = \sqrt{(0.05)^2 + (0.05)^2} = \sqrt{2} \times 0{,}05 \simeq 0.071\,\text{m}$$

O CAMPO ELÉTRICO TERÁ 4 CONTRIBUIÇÕES:

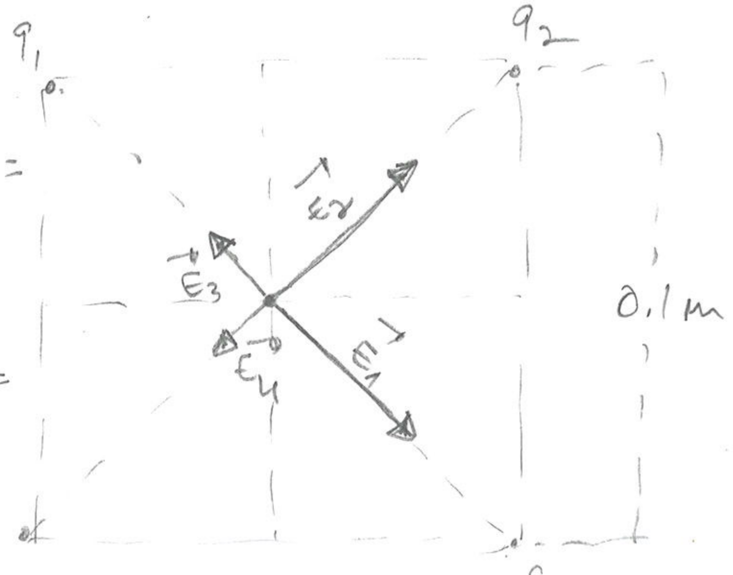
$$|\vec{E}_1| = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{d^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{10\times10^{-6}}{(0.071)^2}\right)\text{NC}^{-1} \simeq 1.78\times10^{7}\,\text{NC}^{-1}$$

$$|\vec{E}_2| = |\vec{E}_1| = 1.78\times10^{7}\,\text{NC}^{-1}$$

$$|\vec{E}_3| = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_3}{d^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{5\times10^{-6}}{(0.07)^2}\right)\text{NC}^{-1} \simeq 8.91\times10^{6}\,\text{NC}^{-1}$$

$$|\vec{E}_4| = |\vec{E}_3| = 8.91\times10^{6}\,\text{NC}^{-1}$$

SERÁ ENTÃO:



$$\vec{E}_1 = |\vec{E}_1|\cos(45°)\vec{e}_x - |\vec{E}_1|\sin(45°)\vec{e}_y =$$
$$= 1.26\times10^{7}\,\vec{e}_x - 1.26\times10^{7}\,\vec{e}_y$$

$$\vec{E}_2 = |\vec{E}_2|\cos(45°)\vec{e}_x + |\vec{E}_2|\sin(45°)\vec{e}_y =$$
$$= 1.26\times10^{7}\,\vec{e}_x + 1.26\times10^{7}\,\vec{e}_y$$

$$\vec{E}_3 = -|\vec{E}_3|\cos(45°)\vec{e}_x + |\vec{E}_3|\sin(45°)\vec{e}_y =$$
$$= -6.30\times10^{6}\,\vec{e}_x + 6.30\times10^{6}\,\vec{e}_y$$

$$\vec{E}_4 = -|\vec{E}_4|\cos(45°)\vec{e}_x - |\vec{E}_4|\sin(45°)\vec{e}_y = -6.30\times10^{6}\,\vec{e}_x - 6.30\times10^{6}\,\vec{e}_y$$

VIRÁ ENTÃO:

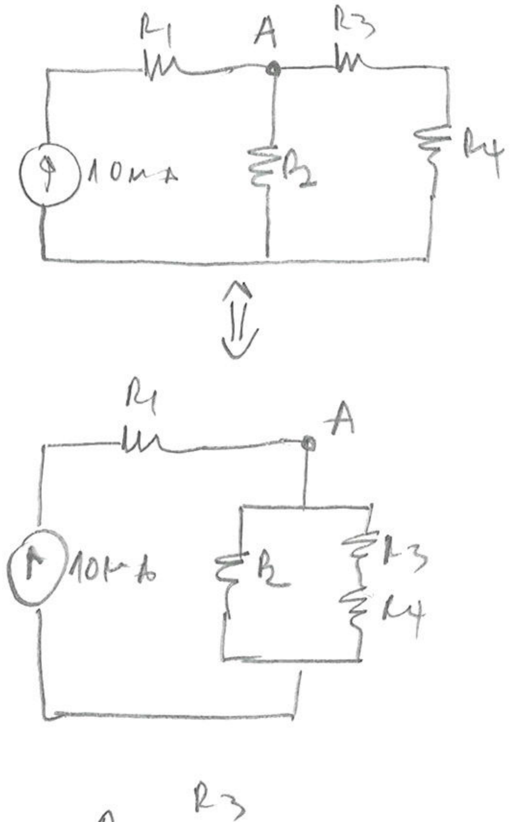
$$\vec{E} = \left(1.26\times10^{7} + 1.26\times10^{7} - 6.3\times10^{6} - 6.30\times10^{6}\right)\vec{e_x} +$$

$$+\left(\cancel{-1.26\times10^{7}} + \cancel{1.26\times10^{7}} + \cancel{6.30\times10^{6}} \cancel{-6.30\times10^{6}}\right)\vec{e_y}$$

$$\vec{E} = 1.26\times10^{7}\,\vec{e_x}\ (NC^{-1})$$

b) O CAMPO ELÉTRICO SERÁ NULO SE SE VERIFICAREM AS CONDIÇÕES $q_1 = q_3 \wedge q_2 = q_4$

2 a) UMA VEZ QUE HÁ DUAS FONTES NO CIRCUITO TEREMOS QUE UTILIZAR O PRINCÍPIO DA SOBREPOSIÇÃO



$$(V_A)_1 = 10\,mA \times R_{eq}$$

$$R_{eq} = R_2 \,||\, (R_3 + R_4) =$$

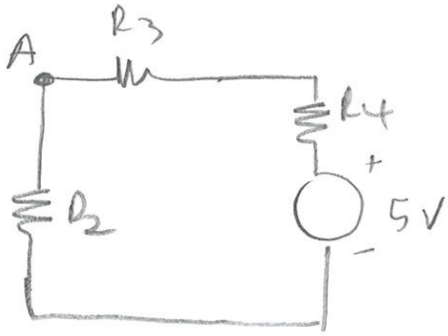
$$= 220 \,||\, (3300 + 330) =$$

$$= \frac{220\times3630}{220+3630}\,\Omega \simeq$$

$$\simeq 207\,\Omega$$

$$(V_A)_1 = 10\times10^{-3}\times 207\ V \simeq$$

$$\simeq 2.07\,V$$

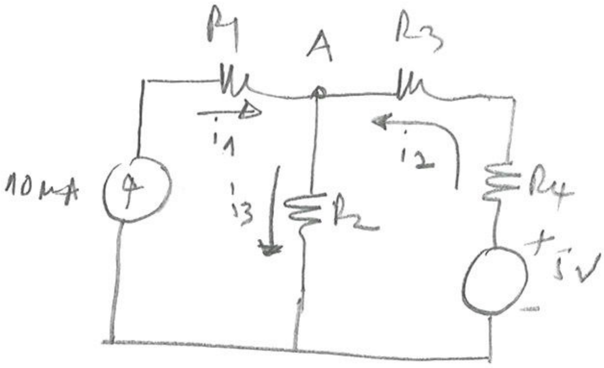


$$(V_A)_2 = \frac{R_2}{R_2+R_3+R_4}\times 5\,V =$$

$$= \frac{220}{(220+3300+330)}\times 5\,V \simeq 0.29\,V$$

$$V_A = (V_A)_1 + (V_A)_2 \simeq 2.36\,V$$

b) A corrente na resistência R_1 é necessariamente $i_1 = 10\,mA$, dirigida da esquerda para a direita.



Sabendo que $V_A = 2{,}4V$, a corrente i_2 que atravessa R_3 e R_4 será dada pela relação:

$$i_2 = \frac{5V - V_A}{(R_3 + R_4)} = \frac{2.64}{(3300 + 330)}\ A \simeq 0.73\ mA$$

Obtemos então:

$$i_3 = i_1 + i_2 = 10.73\ mA$$

Esta mesma conclusão poderia ser obtida diretamente através da relação:

$$i_3 = \frac{V_A}{R_2} = \frac{2.36}{220}\ A \simeq 10.73\ mA$$

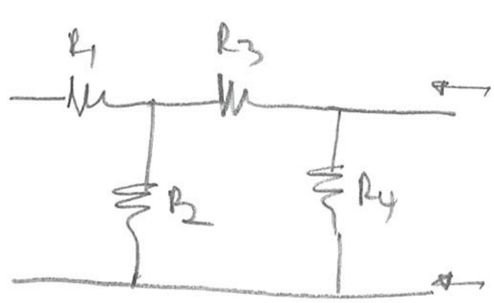
c) $$V_{FC} = V_A + R_1 \times i_1 = (2.36 + 470 \times 10 \times 10^{-3})\ V \simeq 7.1\ V$$

d) $$V_{Th} = V_S = 5V - R_4 \times i_2 = (5 - 330 \times 0.73 \times 10^{-3})\ V \simeq 4.8\ V$$

$$R_{Th} = R_4 \,||\, (R_2 + R_3) =$$

$$= 330 \,||\, (220 + 3300) = 330 \,||\, 3520$$

$$= \frac{330 \times 3520}{(330 + 3520)}\ \Omega \simeq 302\ \Omega$$



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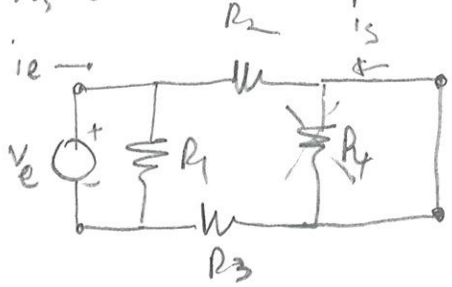
$$\begin{cases} v_e = h_{11} i_e + h_{12} v_s \\ i_s = h_{21} i_e + h_{22} v_s \end{cases}$$

CONDICIÓN $v_s = 0$

$$h_{11} = \left.\frac{v_e}{i_e}\right|_{v_s=0} \quad ; \quad h_{21} = \left.\frac{i_s}{i_e}\right|_{v_s=0}$$



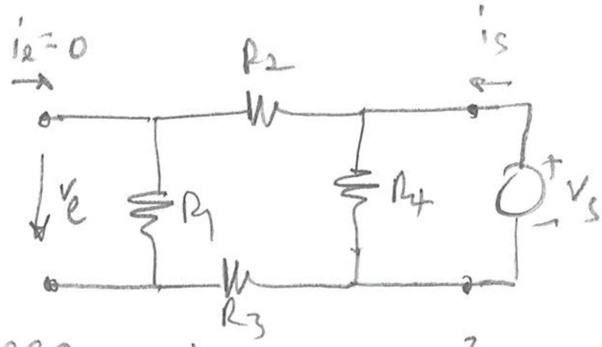
$$h_{11} = R_1 \,\|\, (R_2 + R_3) =$$

$$= 470 \,\|\, (220 + 3300) =$$

$$= \frac{470 \times 3520}{(470 + 3520)}\ \Omega \simeq 415\ \Omega$$

$$i_s = -\frac{R_1}{R_1 + R_2 + R_3}\, i_e \Rightarrow h_{21} = -\frac{R_1}{(R_1+R_2+R_3)} = -\frac{470}{470 + 220 + 3300} \simeq -0.12$$

CONDICIÓN $i_e = 0$



$$h_{12} = \left.\frac{v_e}{v_s}\right|_{i_e=0} \quad ; \quad h_{22} = \left.\frac{i_s}{v_s}\right|_{i_e=0}$$

$$h_{22} = \frac{1}{R_4 \,\|\, \underbrace{(R_2 + R_1 + R_3)}} = \frac{330 + 3990}{330 \times 3990}\ \Omega^{-1} \simeq 3.3 \times 10^{-3}\ \Omega^{-1}$$

$$v_e = \frac{R_1}{R_1 + R_2 + R_3}\, v_s \Rightarrow h_{12} = \frac{R_1}{(R_1 + R_2 + R_3)} = \frac{470}{470 + 220 + 3300} \simeq$$

$$\simeq 0.12$$

$$H \equiv \begin{bmatrix} 415\ \Omega & 0.12 \\ -0.12 & 3.3 \times 10^{-3}\ \Omega^{-1} \end{bmatrix}$$

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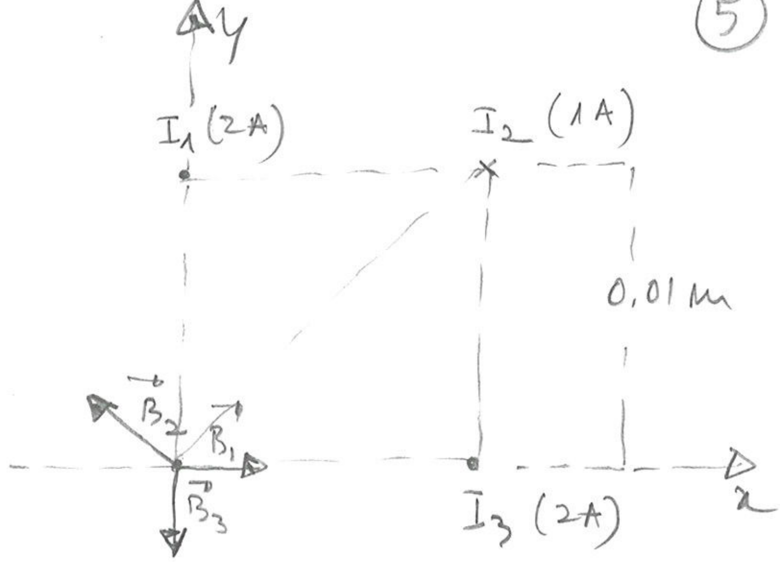


a) $|\vec{B_1}| = \dfrac{\mu_0 I_1}{2\pi \times 0.01} = \dfrac{2\mu_0}{2\pi\times 0.01} =$

$$\simeq \frac{\cancel{2}\times 4\cancel{\pi}\times 10^{-7}}{\cancel{2}\cancel{\pi}\times 0.01} T = 4\times 10^{-5} T$$

$$|\vec{B_3}| = |\vec{B_1}| = 4\times 10^{-5} T$$

$$|\vec{B_2}| = \frac{\mu_0 \times I_2}{2\pi \times \sqrt{(0.01)^2+(0.01)^2}} = \frac{4\cancel{\pi}\times 10^{-7}}{2\sqrt{2}\times\cancel{\pi}\times 0.01} T = \frac{1}{2\sqrt{2}}\times 10^{-5} T \simeq$$

$$\simeq 0.35\times 10^{-5} T$$

$$\vec{B_1} = 4\times 10^{-5}\,\vec{e_x} \quad (T)$$

$$\vec{B_3} = -4\times 10^{-5}\,\vec{e_y} \quad (T)$$

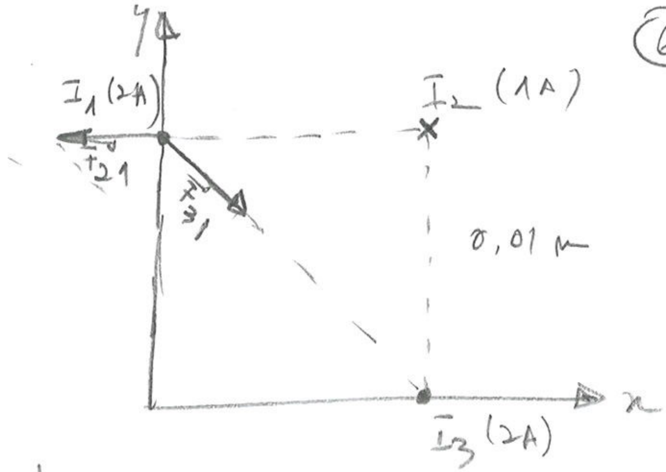
$$\vec{B_2} = -0.35\times 10^{-5}\times \cos(45^\circ)\,\vec{e_x} + 0.35\times 10^{-5}\times \text{sen}(45^\circ)\,\vec{e_y} =$$

$$= -2.5\times 10^{-6}\,\vec{e_x} + 2.5\times 10^{-6}\,\vec{e_y} \quad (T)$$

$$\vec{B} = (4\times 10^{-5} - 2.5\times 10^{-6})\,\vec{e_x} + (2.5\times 10^{-6} - 4\times 10^{-5})\,\vec{e_y}$$

$$\vec{B} = 3.75\times 10^{-5}\,\vec{e_x} - 3.75\times 10^{-5}\,\vec{e_y} \quad (T)$$

b)



$$\begin{cases} |\vec{F}_{21}| = \dfrac{\mu_0 \times 2 \times 1}{2\pi \times 0{,}01} \ \mathrm{N\,m^{-1}} \\[2ex] |\vec{F}_{31}| = \dfrac{\mu_0 \times 2 \times 2}{2\pi \times \sqrt{2} \times 0{,}01} \ \mathrm{N\,m^{-1}} \end{cases}$$

$$\begin{cases} |\vec{F}_{21}| = \dfrac{\cancel{4\pi} \times 10^{-7} \times \cancel{2} \times 1}{\cancel{2\pi} \times 0{,}01} \simeq 4 \times 10^{-5} \ \mathrm{N\,m^{-1}} \\[2ex] |\vec{F}_{31}| = \dfrac{4\cancel{\pi} \times 10^{-7} \times 2 \times \cancel{2}}{\cancel{2\pi} \times \sqrt{2} \times 0{,}01} \simeq 5{,}6 \times 10^{-5} \ \mathrm{N\,m^{-1}} \end{cases}$$

$$\begin{cases} \vec{F}_{21} = -4 \times 10^{-5}\, \vec{e}_x \ (\mathrm{N\,m^{-1}}) \\[2ex] \vec{F}_{31} = \underbrace{5{,}6 \times 10^{-5} \cos(45^\circ)}_{4 \times 10^{-5}}\, \vec{e}_x - \underbrace{5{,}6 \times 10^{-5}\, \mathrm{sen}(45^\circ)}_{4 \times 10^{-5}}\, \vec{e}_y = \\[2ex] \quad = 4 \times 10^{-5}\, \vec{e}_x - 4 \times 10^{-5}\, \vec{e}_y \end{cases}$$

Logo,

$$\vec{F} = \vec{F}_{21} + \vec{F}_{31} = -4 \times 10^{-5}\, \vec{e}_y \ (\mathrm{N\,m^{-1}})$$

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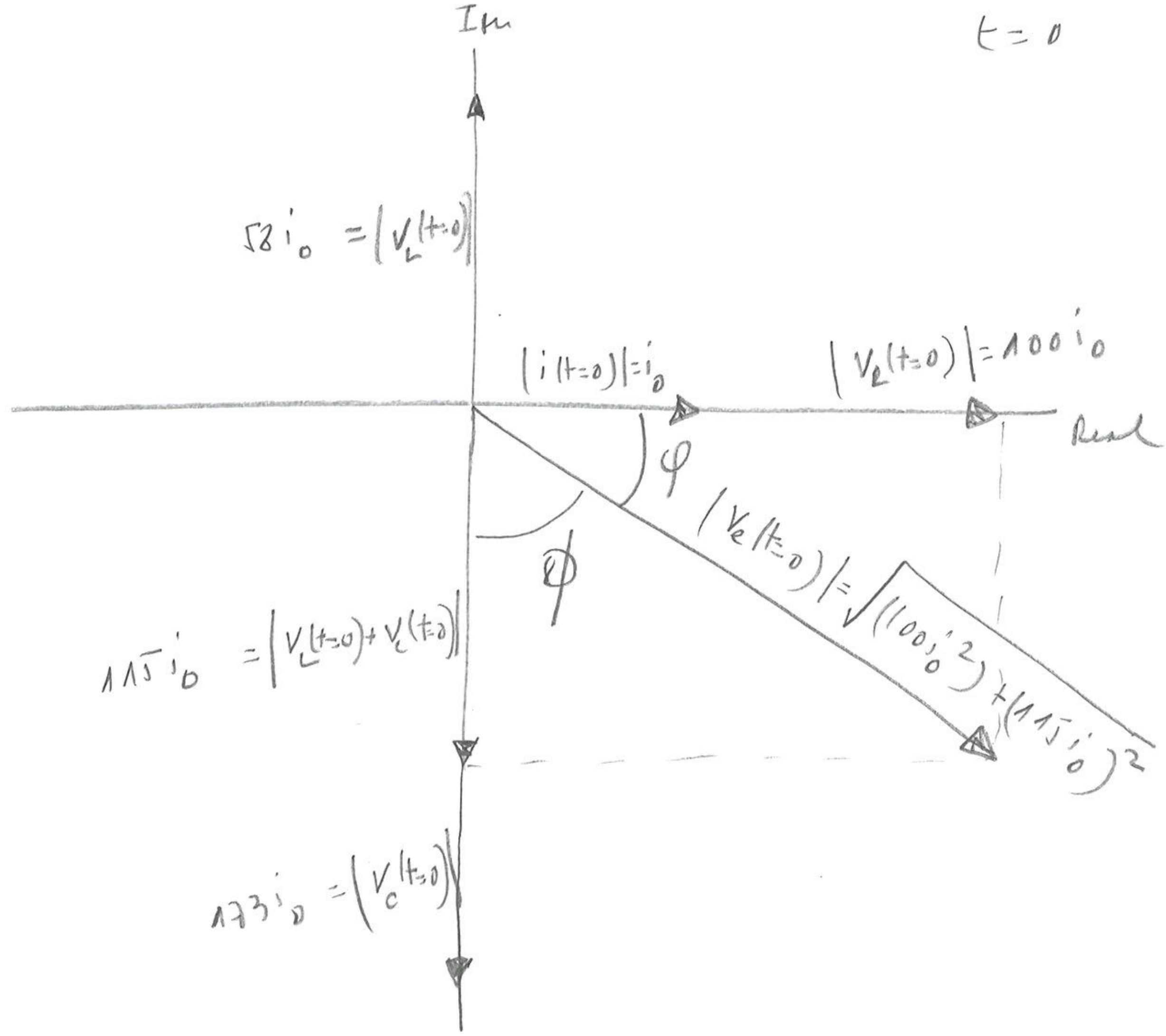
a) $R = 100\ \Omega$
$L = 10\ mH$
$C = 1\ \mu F$
$f = 920\ Hz$

$$X_R = 100\ \Omega$$
$$X_L = 2\pi \times 920 \times 10 \times 10^{-3}\ \Omega \simeq 58\ \Omega$$
$$X_C = \frac{1}{2\pi \times 920 \times 1 \times 10^{-6}}\ \Omega \simeq 173\ \Omega$$



$$i_0 \sqrt{(100)^2 + (115)^2} = |\vec{V_e}| = 5\ V$$

$$i_0 = \frac{5}{\sqrt{(100)^2 + (115)^2}}\ A \simeq 32.8\ mA$$

b)

$$f(\omega) = \frac{|\vec{V}_s(t)|}{|\vec{V}_e(t)|} = \frac{115 \times 32{,}8 \times 10^{-3}}{5} \simeq 0.75$$

$$\phi(\omega) = + tg^{-1}\left(\frac{100 \cancel{\cdot 10}}{115 \cancel{\cdot 10}}\right) \simeq 41^\circ$$

c) $\vec{Z} = \vec{Z}_R + \vec{Z}_L + \vec{Z}_C = X_R + j X_L - j X_C =$

$$= 100 + j(58 - 173) = 100 - 115 j$$

d) $P_{ACTIVA} = \frac{V_{PICO}}{\sqrt{2}} \times \frac{i_{PICO}}{\sqrt{2}} \times \cos(\varphi) =$

$$= \frac{1}{2} \times 5 \times 32.8 \times 10^{-3} \times \cos(90^\circ - 41^\circ) \simeq 53.8\ mW$$