# Cosmologia Física

**Homework 1** due 13 March 2026 (iatereno@fc.ul.pt)

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### Exercise 1: Redshift

The detected frequency of an electromagnetic wave emitted by a cosmological source is redshifted and it is thus different from the emitted frequency.

a) Derive the relation between the cosmological redshift and the scale factor of the Universe at emission time.
Hint: Solved in the slides!

### Exercise 2: Distances

Consider two identical galaxy clusters (meaning they have similar intrinsic sizes and luminosities) in a flat Universe. The clusters are at different redshifts, $z_1$ and $z_2$, with $z_1 < z_2$.

In this Universe, the comoving distance $D_c(z)$ from the observer at $z = 0$ to an object at z is given by

$$D_c = 2\frac{c}{H_0}\left[1 - (1+z)^{-1/2}\right].$$

a) Determine which of the 2 clusters has a larger flux.

b) Determine which of the 2 clusters appears the largest to the observer.

c) Plot the luminosity distance as function of redshift.

d) Plot the angular diameter distance as function of redshift.

e) Which one of the distances, luminosity distance or angular diameter distance, is a better time indicator of the evolution of the universe? Justify your answer.

### Exercise 3: Olbers' Paradox

Consider a sample of N galaxies, with identical luminosity and constant comoving number density $n_0$, contained in a comoving volume $V_C$.

a) Explain that the comoving volume element may be written as

$$dV_C = D_A^2(1+z)^2\, d\Omega\, dD_C,$$

where $d\Omega$ is the solid angle, $dD_C$ is the comoving distance element (it can also be written as $\mathrm{d}\chi$), and $D_A$ is the angular diameter distance.

b) Derive a similar expression using the redshift as the radial element, i.e., an expression for $dV_C/dz$, and show that the number of galaxies per redshift and solid angle is given by

$$\frac{dN}{d\Omega dz} = n_0 D_A^2 (1+z)^2 \frac{c}{H(z)} \, .$$

c) Inserting the luminosity distance, show that the flux function is given by

$$\frac{dN}{d\Omega dF} = -\frac{n_0}{2} \left(\frac{L}{4\pi}\right)^{3/2} F^{-5/2} \frac{c}{H(z)} \frac{1}{(1+z)^2} \frac{dz}{dD_L},$$

where $F$ is the flux.

d) Show that in the local universe ($z \ll 1$) this expression reduces to

$$\frac{dN}{d\Omega dF} = -\frac{n_0}{2} \left(\frac{L}{4\pi}\right)^{3/2} F^{-5/2} \, .$$

Hint: Consider the local Hubble function and a flat model.

Note: This expression is called the Euclidean limit of the flux function, while the remaining factor is a cosmological correction:

$$\frac{c}{H(z)} \frac{1}{(1+z)^2} \frac{dz}{dD_L} \, .$$

e) Using the Euclidean flux function compute the total flux in the volume, including sources with fluxes down to a minimum $F_0$, and recover Olbers' paradox.

f) Consider now Milne's Universe, which is an open (i.e. with negative curvature) cosmological model where the Hubble function is $H(z) = H_0\,(1+z)$. Show that in this case the flux cosmological correction is given by $(1+z)^{-4}$.

Hint: Use $D_L = (1+z)\,D_M$.

g) Compute the total flux in Milne's Universe, showing that Olbers' paradox is solved in this Universe

Hint: It will be useful to write the cosmological correction as function of flux.

### Exercise 4: Quiz - The Homogeneous Universe

A quiz in Kahoot to be done in small teams in class on March 13th.