BJT Transistor Modeling

Transistor Modeling

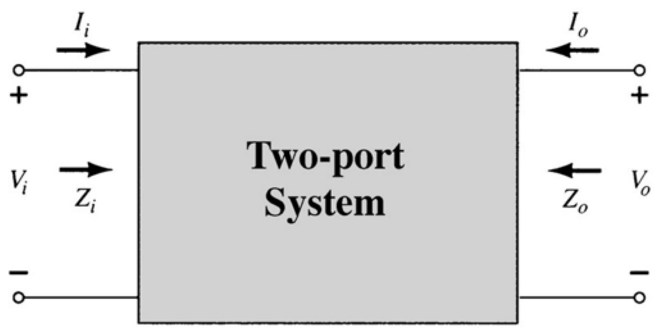
A model is an equivalent circuit that represents the AC characteristics of the transistor. It uses circuit elements that approximate the behavior of the transistor.

There are 2 models commonly used in small signal AC analysis of a transistor:

- hybrid equivalent model

Important Parameters

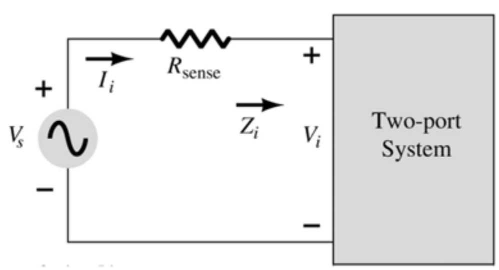
Z_i, Z_o, A_v, A_i are important parameters for the analysis of the AC characteristics of a transistor circuit.



Input Impedance, Z_i

To determine I_i: insert a "sensing resistor"

$$Z_i = \frac{V_i}{I_i}$$

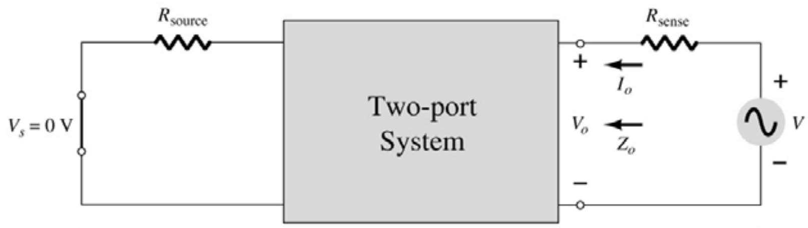


then calculate I_i: $$I_i = \frac{V_s - V_i}{R_{sense}}$$

Output Impedance, Z_o

To determine I_o: insert a "sensing resistor"

$$Z_o = \frac{V_o}{I_o}$$

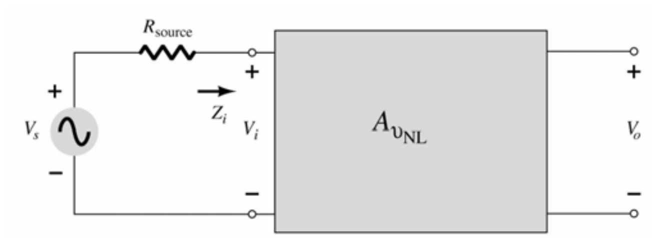


then calculate I_o: $$I_o = \frac{V - V_o}{R_{sense}}$$

Voltage Gain, A_v

$$A_v = \frac{V_o}{V_i}$$

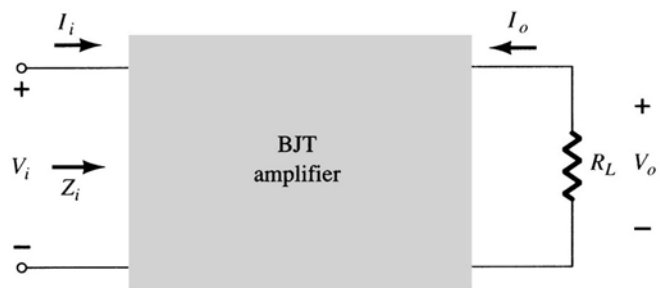
For an amplifier with no load: $$A_{VNL} = \left.\frac{V_o}{V_i}\right|_{R_L = \infty\,\Omega \text{(opencircuit)}}$$



Note: the no-load voltage gain (A_{VNL}) is always greater than the loaded voltage gain (A_V).

Current Gain, A_i

$$A_i = \frac{I_o}{I_i}$$



The current gain (A_i) also be calculated using the voltage gain (A_v):

$$A_i = -A_v \frac{Z_i}{R_L}$$

Phase Relationship

The phase relationship between input and output depends on the amplifier configuration circuit.

Common – Emitter : 180 degrees

Common - Base : 0 degrees

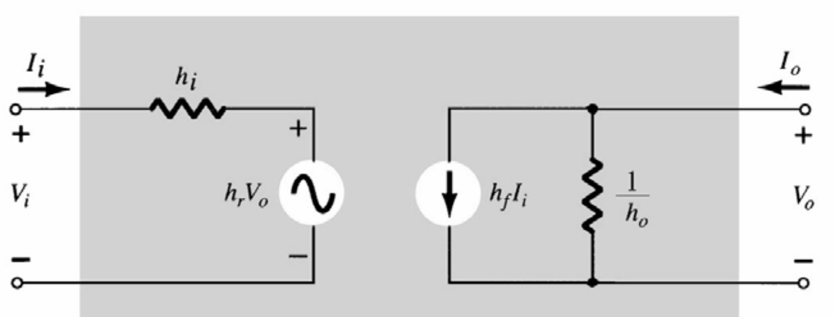
Common – Collector: 0 degrees

Hybrid Equivalent Model

The hybrid parameters: h_{ie}, h_{re}, h_{fe}, h_{oe} are developed and used to model the transistor. These parameters can be found in a specification sheet for a transistor.

- h_i = input resistance
- h_r = reverse transfer voltage ratio (V_i/V_o)
- h_f = forward transfer current ratio (I_o/I_i)
- h_o = output conductance

General h-Parameters for any Transistor Configuration

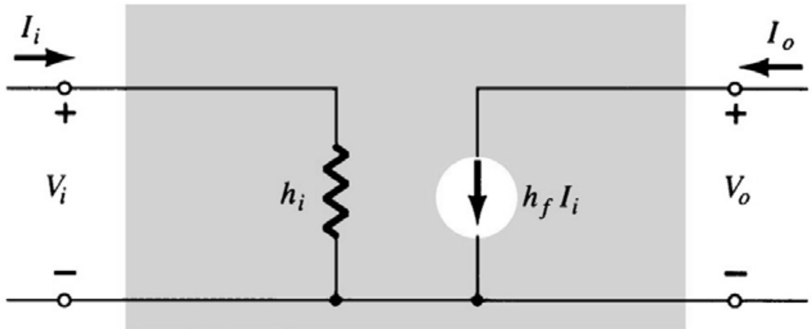


h_i = input resistance
h_r = reverse transfer voltage ratio (V_i/V_o)
h_f = forward transfer current ratio (I_o/I_i)
h_o = output conductance

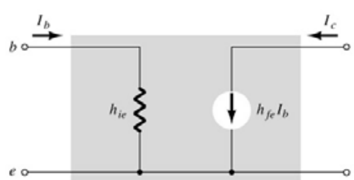
Simplified General h-Parameter Model

The above model can be simplified based on these approximations:

$h_r \cong 0$ therefore $h_r V_o = 0$ and $h_o \cong \infty$



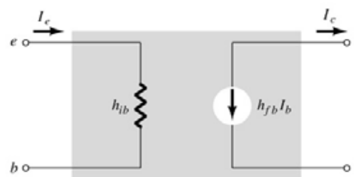
Common-Emitter h-Parameters



$$h_{ie} = \frac{25\text{mV}}{I_{BQ}} \cong \frac{h_{fe} 25\text{mV}}{I_{EQ}}$$

$$h_{fe} = \beta_{ac}$$

Common-Base h-Parameters

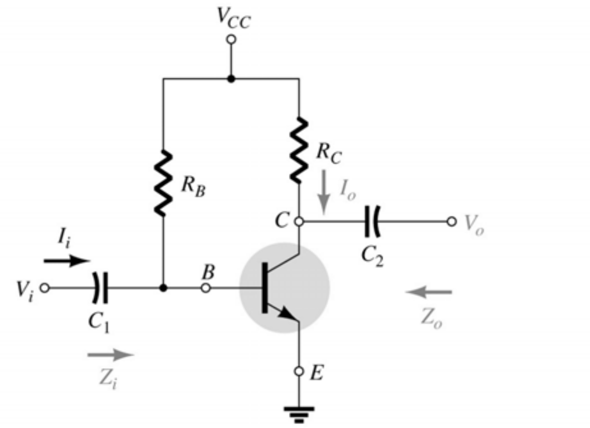


$$h_{ib} = \frac{25\text{mV}}{I_{EQ}}$$

$$h_{fb} = -\alpha_{ac} \cong -1$$

BJT Small-Signal Analysis

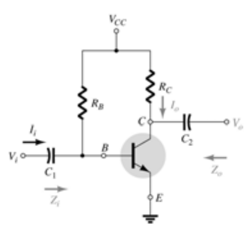
Common-Emitter (CE) Fixed-Bias Configuration



The input (V_i) is applied to the base and the output (V_o) is from the collector.

The Common-Emitter is characterized as having high input impedance and low output impedance with a high voltage and current gain.

Removing DC effects of V_{CC} and Capacitors



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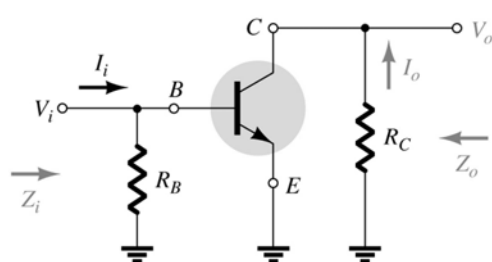
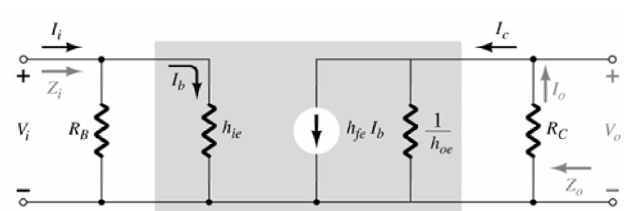
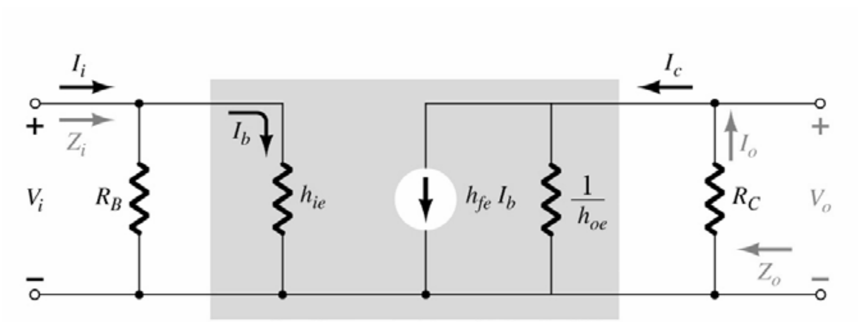


Hybrid Equivalent Circuit



Hybrid Equivalent Circuit

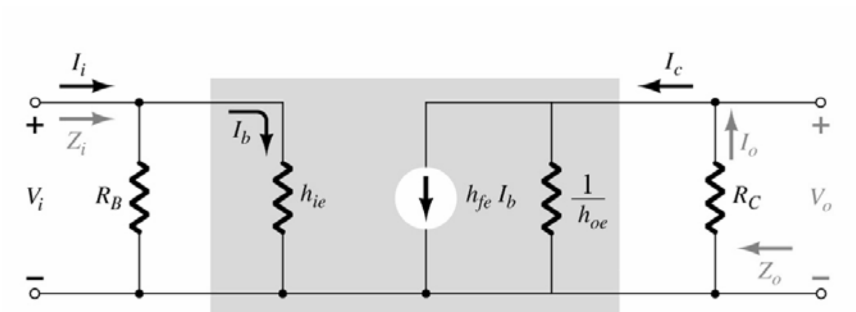


Determine h_{fe}, h_{ie}, and h_{oe}:

h_{fe} and h_{oe}: look in the specification sheet for the transistor or test the transistor using a curve tracer.

h_{ie}: calculate h_{ie} using DC analysis: $h_{ie} = \frac{25mV}{I_{BQ}} \cong h_{fe} \frac{25mV}{I_{EQ}}$

Impedance Calculations



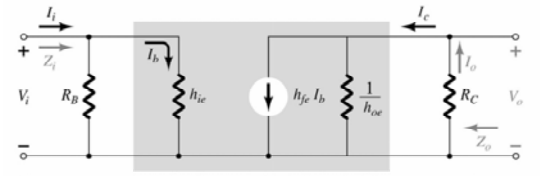
Input Impedance: $\boxed{Z_i = R_B \| h_{ie}}$

$$Z_i \cong h_{ie} \Big|_{R_B \geq 10 h_{ie}}$$

Output Impedance: $\boxed{Z_o = R_C \| \frac{1}{h_{oe}}}$

$$Z_o \cong R_C \Big|_{1/h_{oe} \geq 10 R_C}$$

Gain Calculations



Voltage Gain (A_v):

$$A_v = \frac{v_o}{v_i} = -\frac{h_{fe}(R_C \parallel 1/h_{oe})}{h_{ie}}$$

$$A_v = -\frac{h_{fe} R_C}{h_{ie}} \Bigg|_{1/h_{oe} \geq 10R_C}$$

Current Gain (A_i):

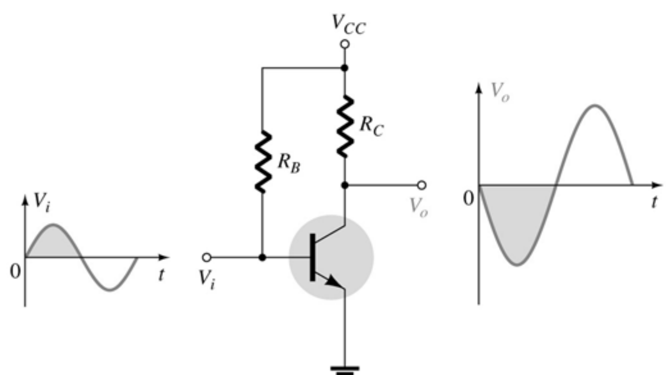
$$A_i = \frac{i_o}{i_i} = \frac{h_{fe} R_B (1/h_{oe})}{(1/h_{oe} + R_C)(R_B + h_{ie})}$$

$$A_i \cong h_{fe} \Big|_{1/h_{oe} \geq 10R_C,\ R_B \geq 10h_{ie}}$$

Current Gain from Voltage Gain:

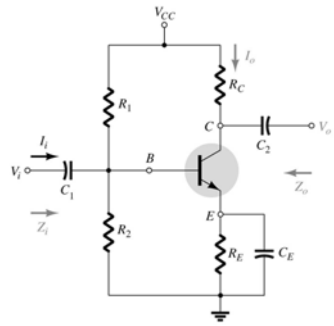
$$A_i = -A_v \frac{Z_i}{R_C}$$

Phase Relationship



The phase relationship between input and output is 180 degrees. The negative sign used in the voltage gain formulas indicates the inversion.

CE – Voltage-Divider Bias Configuration



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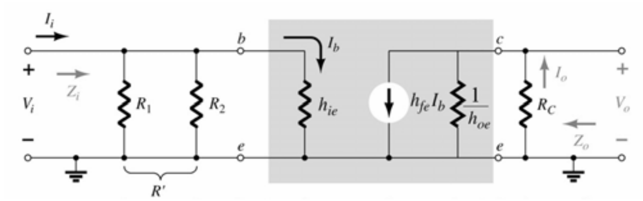
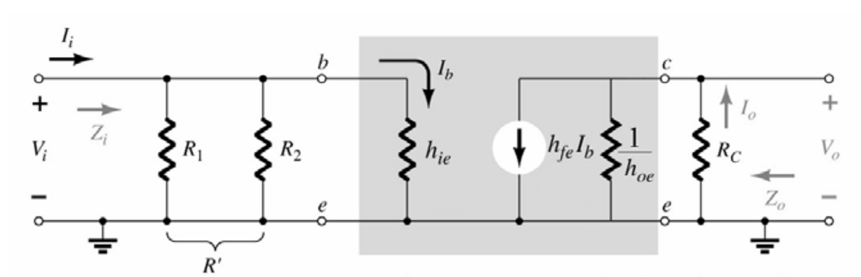


You still need to determine h_{fe}, h_{ie}, and h_{oe}.

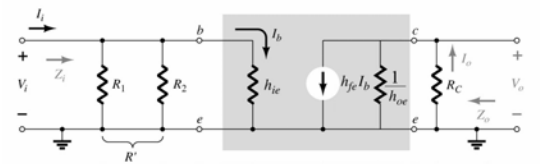
Impedance Calculations



Input Impedance: $\boxed{Z_i = R' \parallel h_{ie}}$ $\quad R' = R_1 \parallel R_2 = \dfrac{R_1 R_2}{R_1 + R_2}$

Output Impedance: $Z_o = R_C \parallel \dfrac{1}{h_{oe}}$

$$\boxed{Z_o \cong R_C \Big|_{1/h_{oe} \geq 10R_C}}$$



Gain Calculations

Voltage Gain (A_v): $A_v = \frac{v_o}{v_i} = -h_{fe}\frac{R_C \parallel 1/h_{oe}}{h_{ie}}$

$$A_v \cong -\frac{h_{fe}R_C}{h_{ie}}\Bigg|_{1/h_{oe} \geq 10R_C}$$

Current Gain (A_i): $A_i = \frac{i_o}{i_i} = \frac{h_{fe}R'(1/h_{oe})}{(1/h_{oe} + R_C)(R' + h_{ie})}$

$$A_i \cong \frac{h_{fe}R'}{R' + h_{ie}}\Bigg|_{1/h_{oe} \geq 10R_C}$$

$$A_i \cong h_{fe}\Big|_{1/h_{oe} \geq 10R_C,\ R' \geq 10h_{ie}}$$

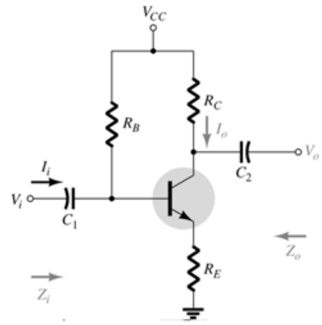
Current Gain from Voltage Gain: $A_i = -A_v\frac{Z_i}{R_C}$

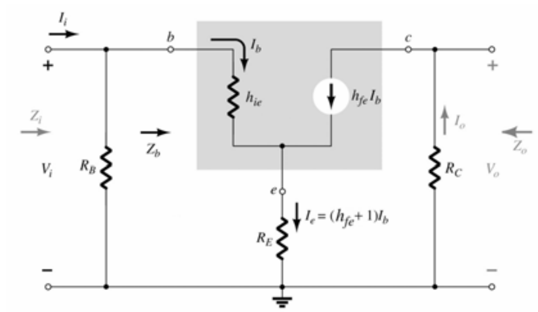
Phase Relationship

A CE amplifier configuration will always have a phase relationship between input and output is 180 degrees. This is independent of the DC bias.

CE Emitter-Bias Configuration

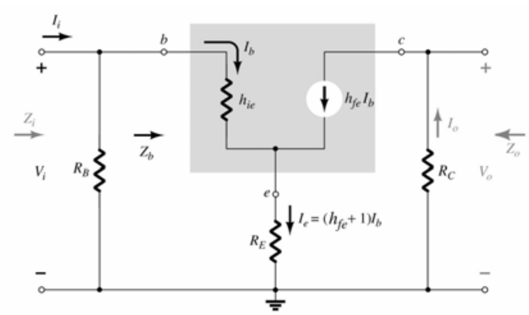
Unbypassed R_E





Again you need to determine h_{fe}, h_{ie}.

Impedance Calculations

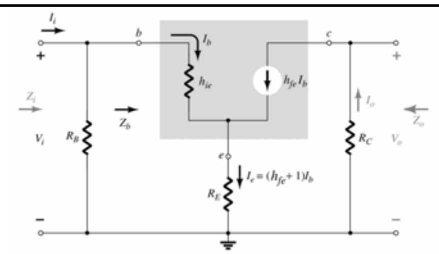


Input Impedance: $\boxed{Z_i = R_B \parallel Z_b}$

$$Z_b = h_{ie} + (h_{fe} + 1)R_E$$

$$Z_b \cong h_{fe} R_E \Big|_{(h_{fe}+1)R_E >> h_{ie},\, h_{fe} >> 1}$$

Output Impedance: $\boxed{Z_o = R_C}$



Gain Calculations

Voltage Gain (A_v): $$A_v = \frac{v_o}{v_i} = -\frac{h_{fe} R_C}{Z_b}$$

$$A_v \cong -\frac{R_C}{R_E}\bigg|_{Z_b \cong h_{fe} R_E}$$

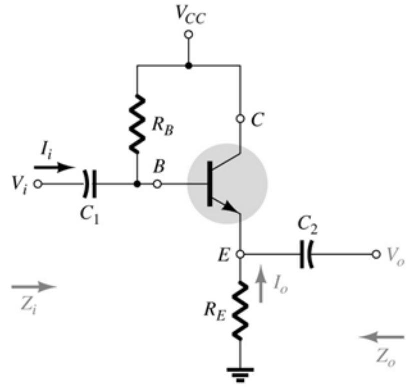
Current Gain (A_i): $$A_i = \frac{i_o}{i_i} = \frac{h_{fe} R_B}{R_B + Z_b}$$

Current Gain from Voltage Gain: $$A_i = -A_v \frac{Z_i}{R_C}$$

Phase Relationship

A CE amplifier configuration will always have a phase relationship between input and output is **180 degrees**. This is independent of the DC bias.

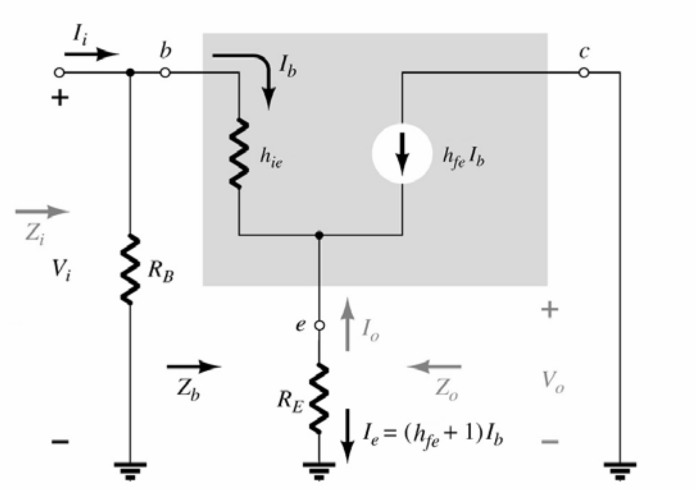
Emitter-Follower Configuration



You may recognize this as the Common-Collector configuration. Indeed they are the same circuit.

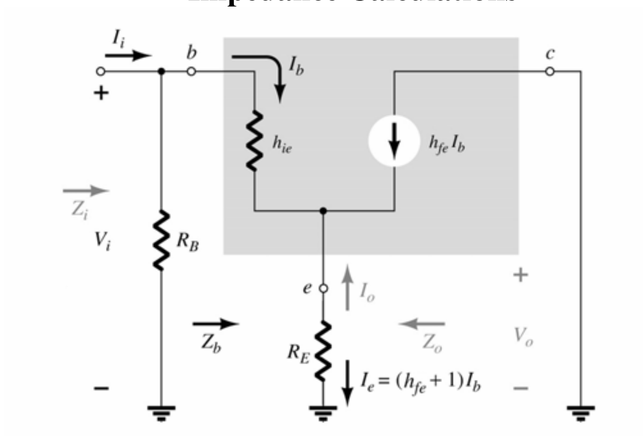
Note the input is on the base and the output is from the emitter.

Hybrid Equivalent Model



You still need to determine h_{fe} and h_{ie}.

Impedance Calculations

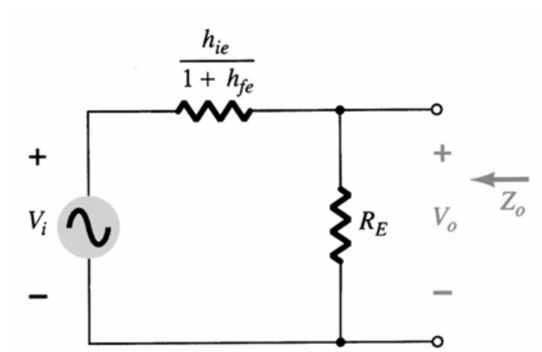


Input Impedance: $\boxed{Z_i = R_B \parallel Z_b}$

$$Z_b = h_{ie} + (h_{fe} + 1)R_E$$

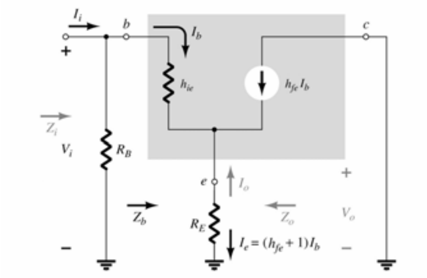
$$Z_b \cong h_{fe} R_E \Big|_{(h_{fe}+1)R_E >> h_{ie},\, h_{fe} >> 1}$$

Impedance Calculations (cont'd)



Output Impedance: $Z_o = R_E \parallel \dfrac{h_{ie}}{h_{fe} + 1}$

$$\boxed{Z_o \cong \frac{h_{ie}}{h_{fe}} \Big|_{(h_{fe}+1)R_E >> h_{ie},\ h_{fe} >> 1}}$$



Gain Calculations

Voltage Gain (A_v): $$A_v = \frac{v_o}{v_i} = \frac{R_E}{R_E + h_{ie}/(h_{fe}+1)}$$

$$A_v \cong 1 \big|_{(h_{fe}+1)R_E >> h_{ie}}$$

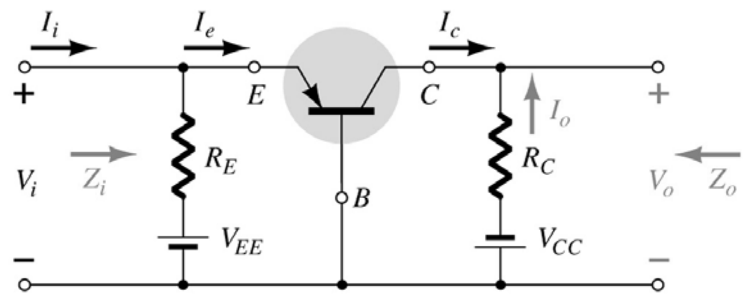
Current Gain (A_i): $$A_i \cong \frac{i_o}{i_i} - \frac{h_{fe}R_B}{R_B + Z_b}$$

Current Gain from Voltage Gain: $$A_i = -A_v \frac{Z_i}{R_E}$$

Phase Relationship

A CC amplifier or Emitter Follower configuration has **no phase shift** between input and output.

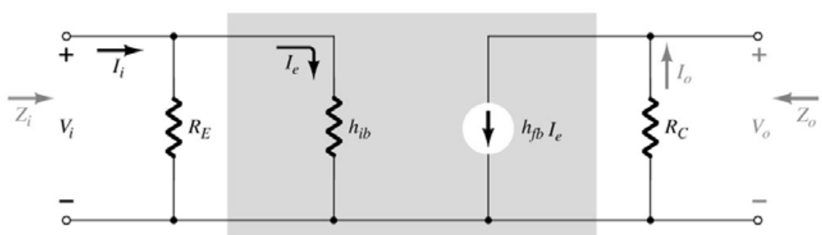
Common-Base (CB) Configuration



The input (V_i) is applied to the emitter and the output (V_o) is from the collector.

The Common-Base is characterized as having low input impedance and high output impedance with a current gain less than 1 and a very high voltage gain.

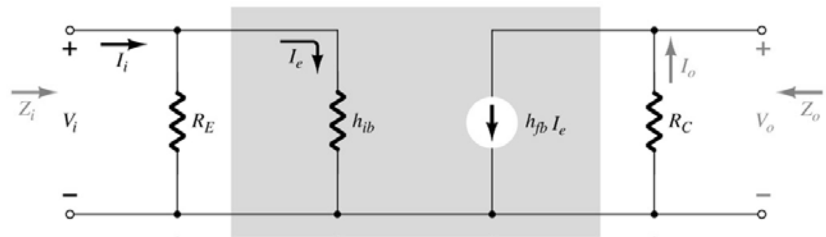
Hybrid Equivalent Model



You will need to determine h_{fb} and h_{ib}.

$$h_{ib} = \frac{h_{ie}}{h_{fe}+1} = \frac{25\text{mV}}{I_{EQ}} \qquad h_{fb} = -\alpha_{ac} \cong -1$$

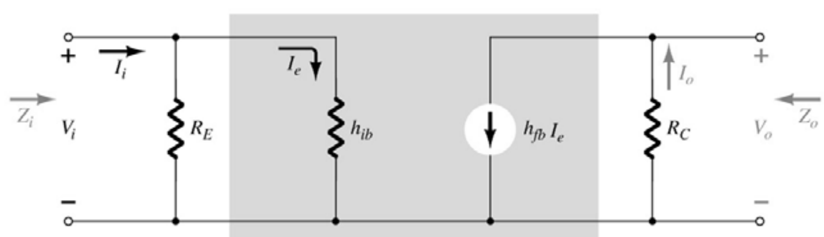
Impedance Calculations



Input Impedance: $Z_i = R_E \parallel h_{ib}$

Output Impedance: $Z_o = R_C$

Gain Calculations



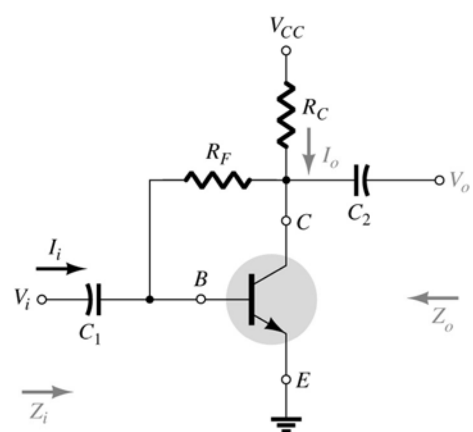
Voltage Gain (A_v): $A_v = \frac{v_o}{v_i} = \frac{h_{fb} R_C}{h_{ib}} \cong \frac{R_C}{h_{ib}}$

Current Gain (A_i): $A_i = \frac{i_o}{i_i} = h_{fb} \cong -1$

Phase Relationship

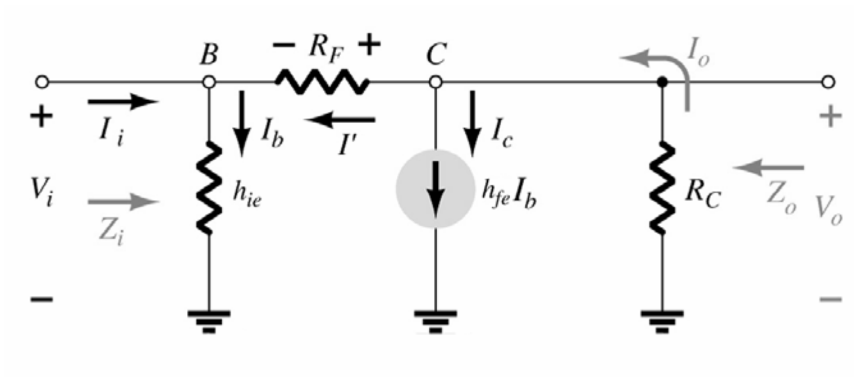
A CB amplifier configuration has **no phase shift** between input and output.

CE Collector Feedback Configuration



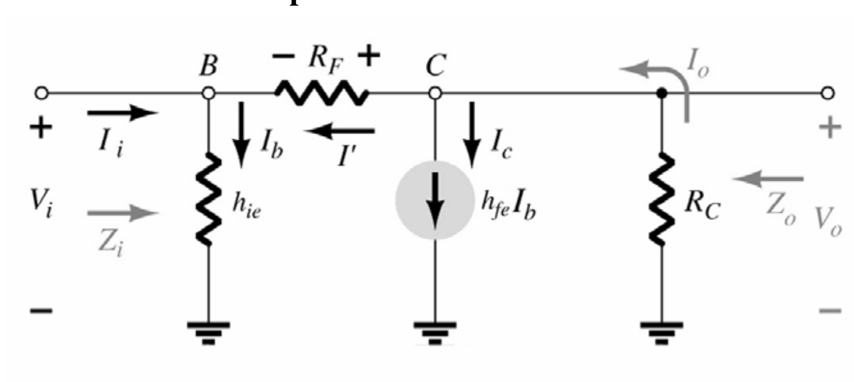
This is a variation of the CE Fixed-Bias configuration.

Hybrid Equivalent Model



You will need to determine h_{fe} and h_{ie}.

Impedance Calculations



Input Impedance: $$Z_i = \frac{h_{ie}}{1 + h_{fe}\dfrac{R_C}{R_F}}$$

Output Impedance: $$Z_o \cong R_C \parallel R_F$$

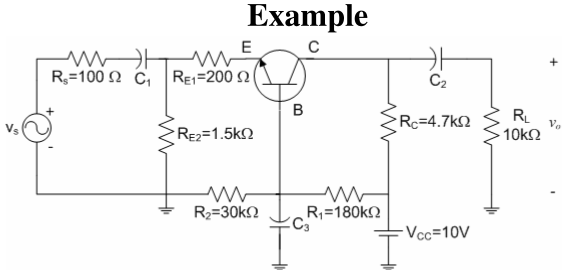
Gain Calculations

Voltage Gain (A_v): $$A_v = \frac{V_o}{V_i} = -\frac{h_{fe} R_C}{h_{ie}}$$

Current Gain (A_i): $$A_i = \frac{I_o}{I_i} = \frac{h_{fe} R_F}{R_F + h_{fe} R_C}$$

$$\boxed{A_i = \frac{I_o}{I_i} \cong \frac{R_F}{R_C}}$$

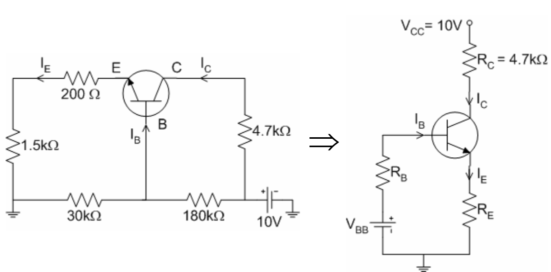
Example



$h_{fe} = h_{FE} = 200$
$I_{CO} = 0$ A
$V_{BE} = 0.7$ V
$V_{CEsat} = 0$ V

According to the figure above

- Perform DC analysis and find the Q-point.
- Evaluate the voltage gain A_{vs} and the current gain A_i.
- Sketch vo on the AC+DC load line graph when
  - vs = 100 sin(wt) mV.
  - vs = 900 sin(wt) mV.



$I_{CQ} = 0.39$ mA

$V_{CBQ} = 6.8$ V