Quark-Gluon Plasma and the QCD Phase Transition

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Primordial Universe

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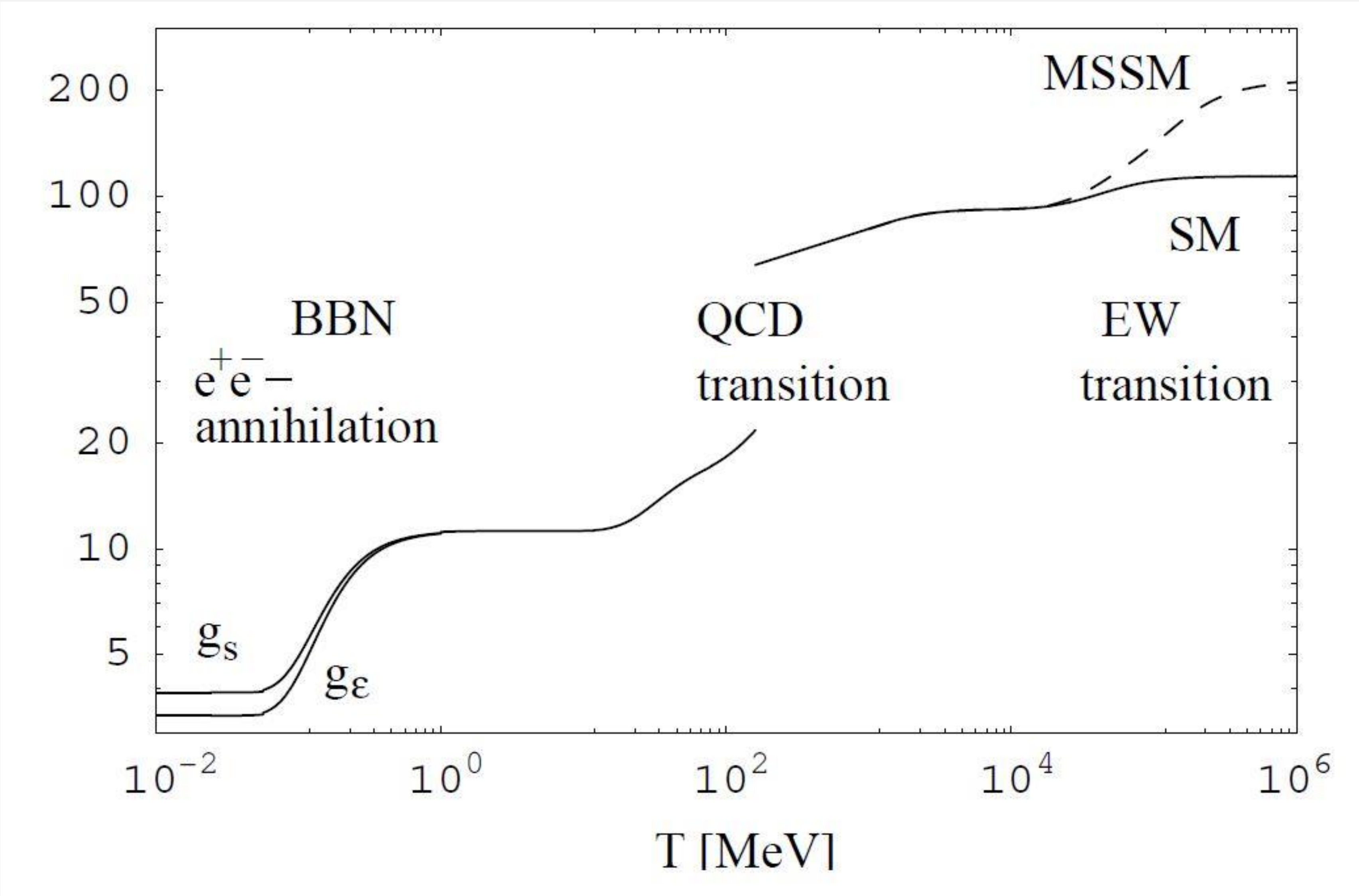
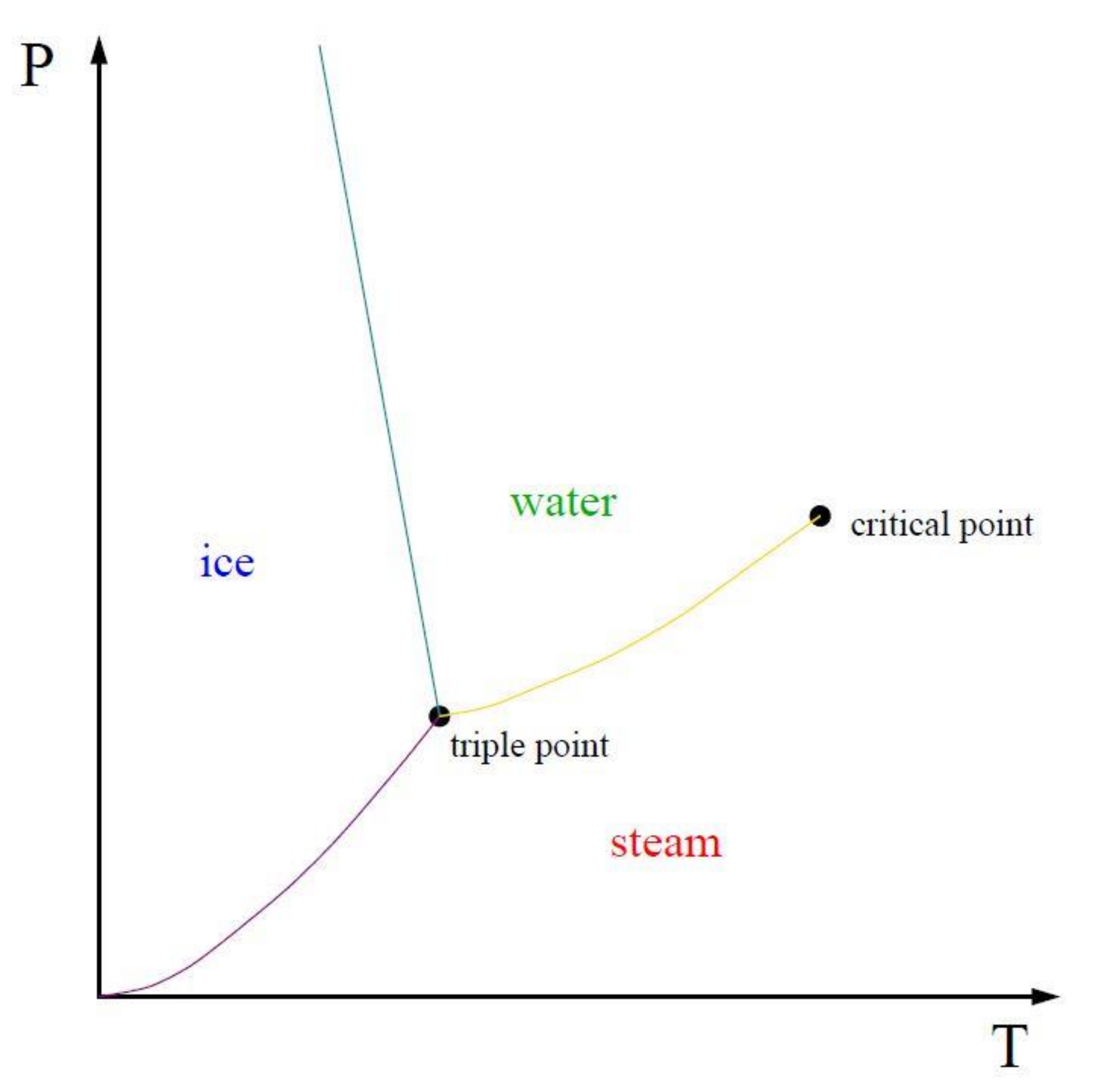
Introduction

- Quantum Chromodynamics studies interactions due to the strong force;
- Strong force is responsible for the confinement of quarks and gluons;
- For high enough temperatures, the confinement is broken;
- Confinement associated with chiral symmetry breaking.



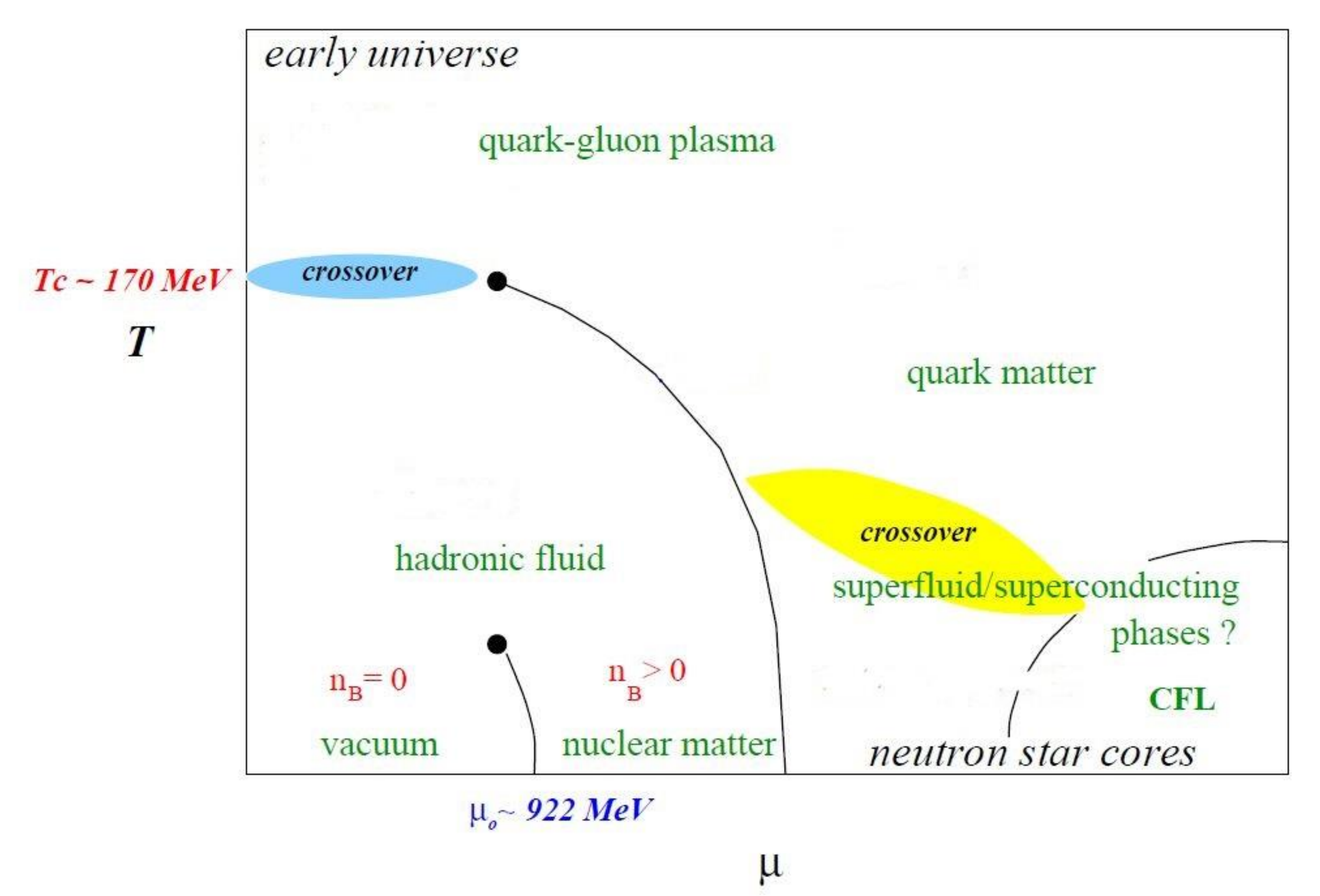
QCD Phase Diagram

- P and T are control parameters;
- Lines are coexistence curves;
- Triple point where all three phases coexist;
- Critical point where water and steam become indistinguishable.



QCD Phase Diagram

- μ and T are control parameters;
- For $T = 0$, increasing μ there are three different phases: vacuum, nuclear matter and superconducting phase;
- At the nuclear matter phase, the density is always smaller than $n_0 = 0{,}17\ \mathrm{fm}^{-3}$;
- Forμ $= 0$, increasing T there is a hadronic fluid until a crossover region at $T_C \approx 170$ MeV and a QGP afterwards.



The Bag Model

- Parameterization of p, s and ρ;
- Massless quarks and gluons;
- Constant negative pressure due to confinement;

$$\begin{cases} \text{QGP, if } T > T_C \\ \text{Both, if } T = T_C \\ \text{HG, if } T < T_C \end{cases}$$

The Bag Model

QGP

$$p_{QGP}(T) = \frac{\pi^2}{90} g_{QGP} T^4 - B$$

$$\rho_{QGP}(T) = \frac{\pi^2}{30} g_{QGP} T^4 - B$$

$$s_{QGP}(T) = \frac{2\pi^2}{45} g_{QGP} T^3$$

HG

$$p_{HG}(T) = \frac{\pi^2}{90} g_{HG} T^4 \longrightarrow B = \frac{\pi^2}{90}\left(g_{QGP} - g_{HG}\right) T_C^4$$

$$\rho_{HG}(T) = \frac{\pi^2}{30} g_{HG} T^4 \longrightarrow \rho = T\frac{dp}{dT} - p$$

$$s_{HG}(T) = \frac{2\pi^2}{45} g_{HG} T^3 \longrightarrow s = \frac{dp}{dT}$$

$$l = T_C \Delta s = 4B$$

Quark-Gluon Plasma

$$p_{QGP}(T_C) = p_{HG}(T_C)$$

$$\Leftrightarrow \frac{\pi^2}{90} g_{QGP} T_C^4 - B = \frac{\pi^2}{90} g_{HG} T_C^4$$

$$g_{p\bar{p}} + g_g = \frac{7}{4} * 2 * 2 * 3 + 2 * 8 = 37$$

$$g_{mesons} = 3$$

$$T_C \approx 144 \text{ MeV}$$

Quark-Gluon Plasma

- Grand canonical partition function:

$$Z(T,\mu) = \mathrm{Tr}\left[\exp\left(-\frac{\hat{H}+\mu\hat{N}}{T}\right)\right]$$

$$\xrightarrow[\mu = 0]{}$$

- Grand canonical potential:

$$\Omega(T,V) = T\ln\big(Z(T,V)\big) - \Omega_0$$

$$p = \frac{1}{V}\Omega(T,V)$$

$$\rho = \frac{T^2}{V}\frac{\partial}{\partial T}\left(\frac{\Omega(T,V)}{T}\right)$$

$$\frac{\Theta^{\mu\mu}}{T^4} = \frac{\rho - 3p}{T^4} = T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right)$$

First-order Phase Transition - Homogeneous

- Change of the free energy due to the creation of one bubble:

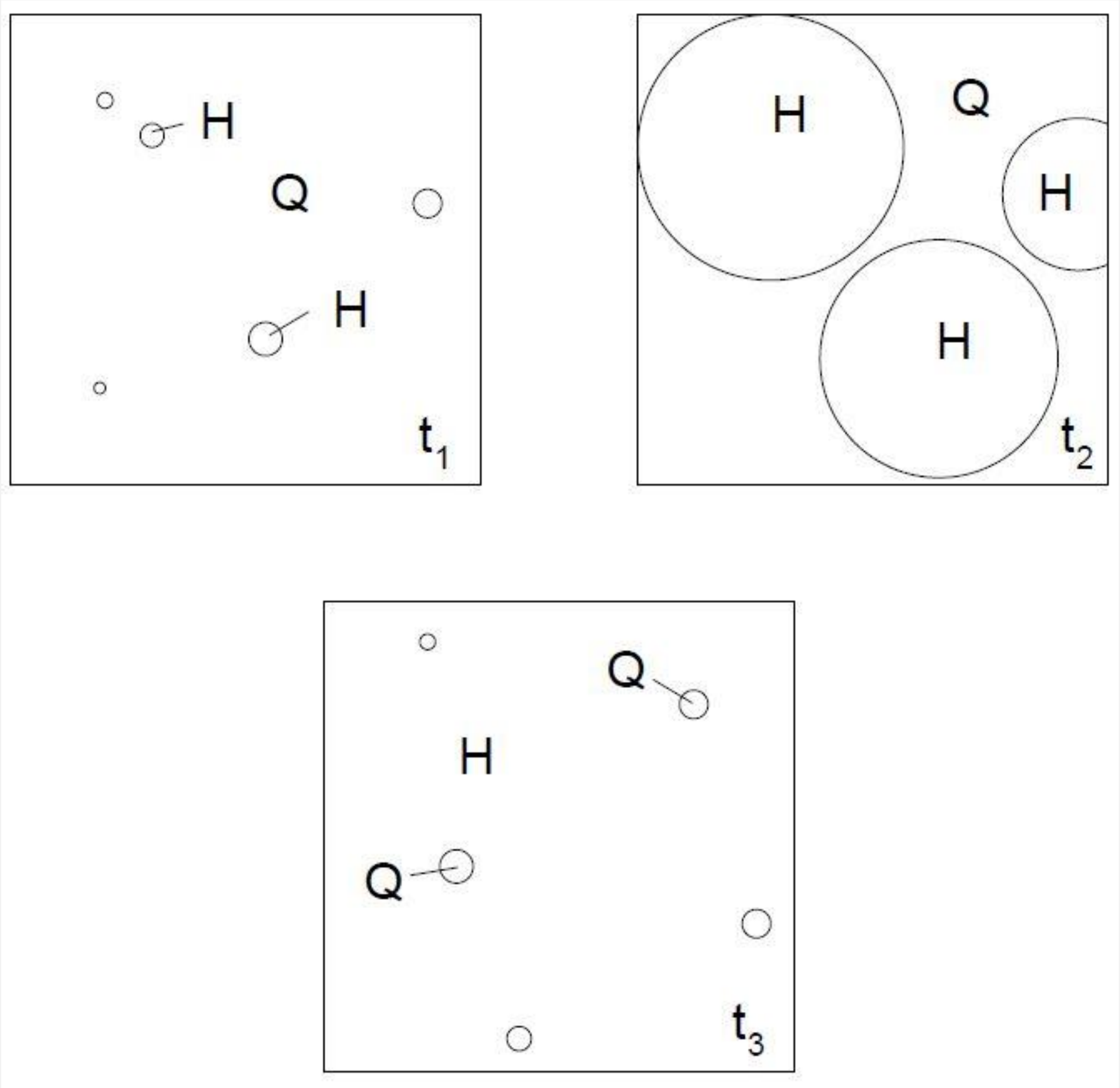
$$\Delta F = \frac{4}{3}\pi R^3 \left(p_{QGP} - p_{HG}\right) + 4\pi\sigma R^2$$

- Critical bubble radius:

$$R_c(T) = \frac{2\sigma}{\left(p_{QGP}(T) - p_{HG}(T)\right)}$$

- Probability of the creation of a bubble with critical radius:

$$I(T) = I_0(T)\exp\left(-\frac{\Delta F_C}{T}\right)$$



First-order Phase Transition - Homogeneous

- The bubbles present at a certain time were created in the preceding time interval:

$$\Delta t_{nuc} = I\left(\frac{dI}{dt}\right)^{-1}$$

- Using:

$$p_{QGP} - p_{HG} \approx \Delta l \qquad \frac{d\Delta}{dt} = \frac{3c_s^2}{t_H} \qquad d_{nuc} \approx 2v_{heat}\Delta t_{nuc}$$

- We get the nucleation distance:

$$\boxed{d_{nuc} \approx \frac{v_{heat}\Delta_{sc}^3 t_H}{3c_s^2 A}} \qquad A = \frac{16\pi\sigma^3}{3l^2T_C}$$

First-order Phase Transition - Heterogeneous

- The nucleation starts near impurities;

- If $n \gg d_{nuc,hom}^{-3}$, then $d_{nuc} = n^{-1/3} \ll d_{nuc,hom}$ ⟶ Heterogeneous nucleation

- If $n \ll d_{nuc,hom}^{-3}$, then $d_{nuc} = d_{nuc,hom}$ ⟶ Homogeneous nucleation

First-order Phase Transition - Heterogeneous

- The sum of probabilities to form a bubble must be 1:

$$1 = n(v_{heat}\Delta t)^3 + (v_{heat}\Delta t_{nuc})^3 \Delta t_{nuc} I(t_{sc})$$

- If $\Delta t/\Delta t_{nuc} \sim 1$:

$$\mathrm{d_{nuc}} \approx \frac{1}{\left(\mathrm{n} + \Delta \mathrm{t_{nuc}} \mathrm{I}(\mathrm{t_{sc}})\right)^{1/3}} \leq \mathrm{d_{nuc,hom}}$$

- If $\Delta t/\Delta t_{nuc} \gg 1$:

$$1 = n(v_{heat}\Delta t)^3 + (v_{heat}\Delta t_{nuc})^3 \Delta t_{nuc} I(t_{sc})(1 - n(v_{heat}\Delta t)^3)$$

Decrease of the QGP volume

$$d_{nuc} \approx \frac{1}{\left(n + \Delta t_{nuc} I(t_{sc})(1 - n(v_{heat}\Delta t)^3)\right)^{1/3}} \approx n^{-1/3}$$

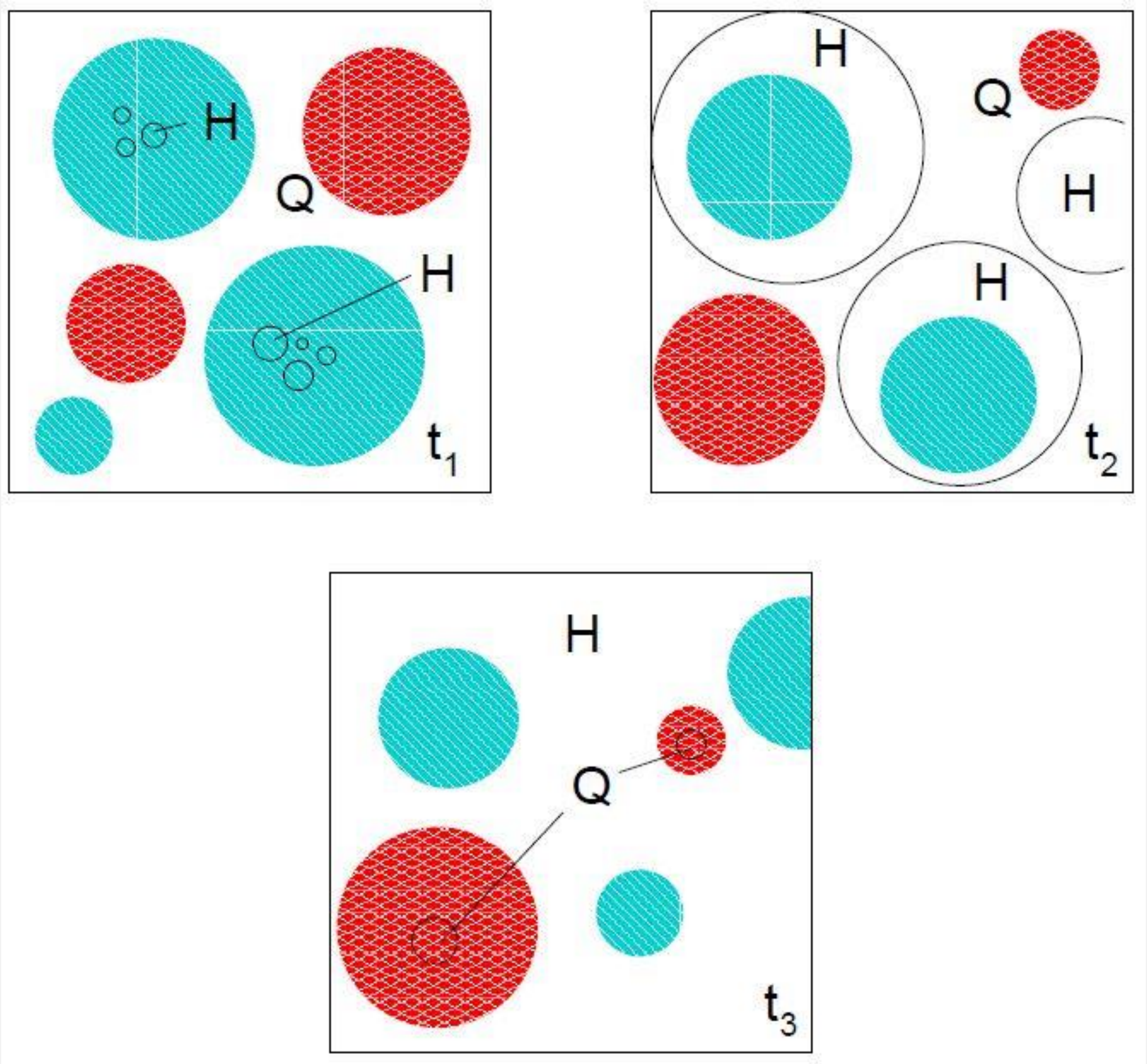
First-order Phase Transition - Inhomogeneous

- Root mean square fluctuation:

$$\Delta_T^{rms} \approx 10^{-4}(3c_s^2)^{3/4}\left(\frac{k}{k_0}\right)^{(n-1)/2}$$

- If $\Delta_{nuc} > \Delta_T^{rms}$, then the nucleation is homogeneous;

- If $\Delta_{nuc} < \Delta_T^{rms}$, then the nucleation is inhomogeneous.



Signatures

- Quark Nuggets - Pauli Principle allows that a larger number of quarks is compact in a fixed volume if there are three flavours;
- Inhomogeneous Nucleosynthesis - a strong first-order QCD phase-transition leads to an inhomogeneous nucleosynthesis with large fluctuations of the baryon number;
- Cold Dark Matter (CDM) Clumps - if the (CDM) doesn't decouple from the radiation fluid at the QCD phase-transition, the CDM spectrum is amplified, leading to the formation of CDM clumps.

Conclusions

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