Electroweak phase transition and gravitational waves

Gabriel Lourenço

February 10, 2022

Ciências ULisboa

ULISBOA UNIVERSIDADE DE LISBOA

Outline

I. Introduction

II. Electroweak symmetry breaking
- Abelian Higgs model
- SM symmetry breaking

III. Cosmological PTs and GWs
- First-order PTs
- Bubble nucleation
- Gravitational wave spectra

Introduction

- About 10^{-11} s after the Big-Bang the temperature dropps bellow ≈160 GeV and the electroweak symmetry is broken.

- The $W^{\pm}$ and Z bosons (and fermions) acquire mass after spontaneous symmetry breaking - Higgs mechanism.

- Symmetry breaking leads to a phase transition that if first-order proceeds by nucleation and generates a stochastic background of GWs.

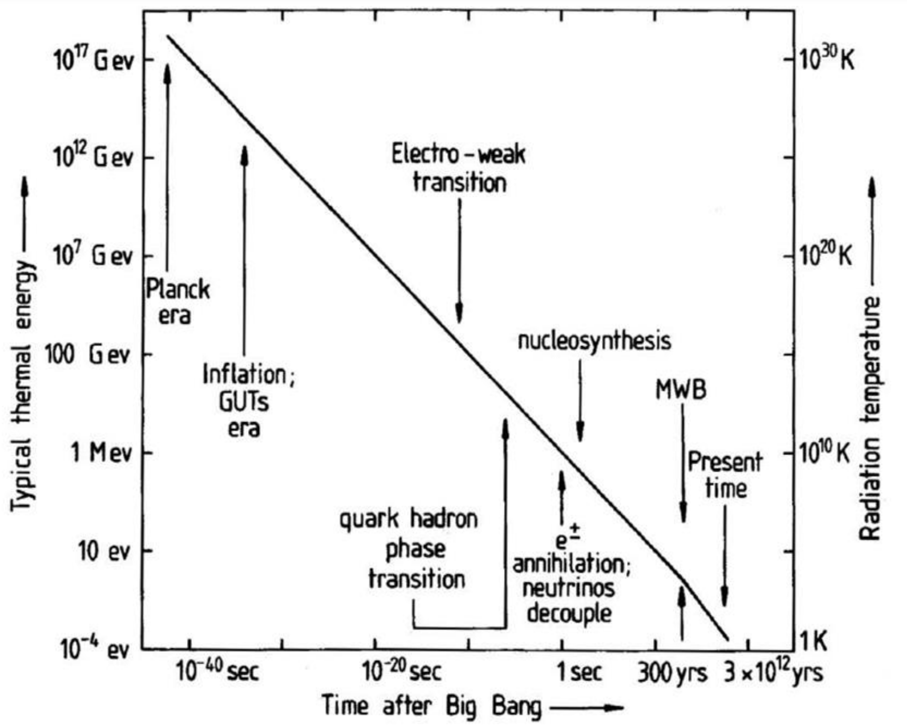


Fig.1: Thermal evolution of the Universe. **[1]**

- Consider a theory with a complex scalar field ϕ with charge $q = -e$ and a gauge field A_μ.

The Lagrangian is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left| D_\mu \phi \right|^2 - V(\phi)$$

with $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ and $D_\mu = \partial_\mu - ieA_\mu$ the covariant derivative.

- Gauge and *U(1)* symmetry imply

$$A_\mu(x) \longrightarrow A'_\mu = A_\mu(x) - \partial_\mu \eta(x)$$
$$\phi(x) \longrightarrow \phi'(x) = \mathrm{e}^{-ie\eta(x)} \phi(x)$$

- Adding a mass term violates local gauge invariance.

$$\tfrac{1}{2} M_A^2 A_\mu A^\mu \longrightarrow \tfrac{1}{2} M_A^2 \left(A_\mu - \partial_\mu \eta \right) \left(A^\mu - \partial^\mu \eta \right) \neq \tfrac{1}{2} M_A^2 A_\mu A^\mu$$

The potential is

$$V(\phi) = \mu^2 \phi^* \phi + \lambda \left(\phi^* \phi\right)^2$$

with $\lambda > 0$.

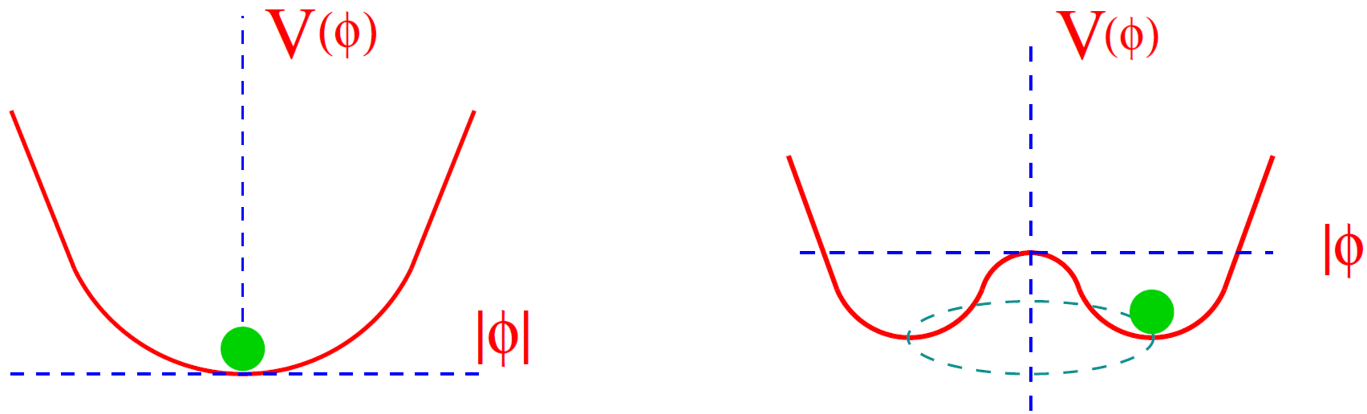


Fig.2: Shape of the scalar potential *V(ϕ)*for $\mu^2 > 0$ (left) and $\mu^2 < 0$ (right). In the second case there is a continuous set of degenerate vacua. **[2]** (adapted)

- If $\mu^2 > 0$ the potential has only the trivial minimum $\phi_0 = 0$.
- If $\mu^2 < 0$ there is an infinite number of states with minimum energy $\phi_0 = \sqrt{\frac{-\mu^2}{2\lambda}}\, e^{iQ\theta} = \frac{v}{\sqrt{2}} e^{iQ\theta}$.

The choice of ground state is arbitrary. For $\theta = 0$, the vacum expectation value (vev) is $\langle \phi \rangle = \frac{v}{\sqrt{2}}$.

- The symmetry is spontaneously broken.

The field can be parametrised as

$$\phi(x) = \frac{1}{\sqrt{2}}(v+h)\mathrm{e}^{\frac{i\chi}{v}}$$

where h and χ are two real fields with no vev.

The Lagrangian is now given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - evA_\mu\partial^\mu\chi + \frac{e^2v^2}{2}A_\mu A^\mu + \frac{1}{2}\left(\partial_\mu h\partial^\mu h + 2\mu^2h^2\right) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + (h,\chi\text{interactions})$$

From the bilinear terms,

- $M_A = ev$
- $M_h = \sqrt{2\lambda}v$
- $M_\chi = 0$

The field can be parametrised as

$$\phi(x) = \frac{1}{\sqrt{2}}(v+h)\mathrm{e}^{\frac{i\chi}{v}}$$

where h and χ are two real fields with no vev.

The Lagrangian is now given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - evA_\mu\partial^\mu\chi + \frac{e^2v^2}{2}A_\mu A^\mu + \frac{1}{2}\left(\partial_\mu h\partial^\mu h + 2\mu^2h^2\right) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + (h,\chi\text{interactions})$$

From the bilinear terms,

- $M_A = ev$
- $M_h = \sqrt{2\lambda}v$
- $M_\chi = 0$

Gauge freedom allows for the transformation

$$A_\mu \longrightarrow A'_\mu = A_\mu - \frac{1}{ev}\partial_\mu\chi$$

that cancels the terms dependent on χ.

- The Lagrangian is equivalent to the QED Lagrangian with a massless photon and a charged scalar field ϕ with mass μ.

Before symmetry breaking:

- 1 gauge field, A – 2 polarization states
- 1 scalar field, ϕ – charge, mass

2+1+1=4 d.o.f.

After symmetry breaking:

- 1 gauge field, A – 2 polarization states, mass
- 1 scalar field, h – mass

3+1=4 d.o.f.

Higgs mechanism

The SSB of a gauge theory by a non-zero vev results in the disappearance of a Goldstone boson and its transformation into the longitudinal component of a massive gauge boson.

- The Glashow–Weinberg–Salam electroweak theory is based on the symmetry group $SU(2)_L \otimes U(1)_Y$.

The Lagrangian of the SM without the strong interaction part is

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{L}_j i\not{D} L_j + \bar{e}_{R_j} i\not{D} e_{R_j} + \bar{Q}_j i\not{D} Q_j + \bar{u}_{R_j} i\not{D} u_{R_j} + \bar{d}_{R_j} i\not{D} d_{R_j}$$

EW symmetry breaking

- The Glashow–Weinberg–Salam electroweak theory is based on the symmetry group $SU(2)_L \otimes U(1)_Y$.

The Lagrangian of the SM without the strong interaction part is

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{L}_j i\not{D} L_j + \bar{e}_{R_j} i\not{D} e_{R_j} + \bar{Q}_j i\not{D} Q_j + \bar{u}_{R_j} i\not{D} u_{R_j} + \bar{d}_{R_j} i\not{D} d_{R_j}$$

- $\bar{L}_j i\not{D} L_j$: Left-handed doublet electron fields
- $\bar{e}_{R_j} i\not{D} e_{R_j}$: Right-handed singlet electron fields
- $\bar{Q}_j i\not{D} Q_j$: Left-handed doublet quark fields
- $\bar{u}_{R_j} i\not{D} u_{R_j} + \bar{d}_{R_j} i\not{D} d_{R_j}$: Right-handed singlet up and down quark fields
- $-\frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$:

$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ is the $U(1)_Y$ (hypercharge) field strength

$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \varepsilon^{abc} W^b_\mu W^c_\nu$ is the $\mathrm{SU}(2)_L$ field strength

- The Glashow–Weinberg–Salam electroweak theory is based on the symmetry group $SU(2)_L \otimes U(1)_Y$.

The Lagrangian of the SM without the strong interaction part is

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{L}_j i\not{D} L_j + \bar{e}_{R_j} i\not{D} e_{R_j} + \bar{Q}_j i\not{D} Q_j + \bar{u}_{R_j} i\not{D} u_{R_j} + \bar{d}_{R_j} i\not{D} d_{R_j}$$

- The Lagrangian is invariant under G gauge transformations for fermions and gauge fields

$$\psi_L \longrightarrow \psi'_L = \mathrm{e}^{i\alpha_a T^a}\mathrm{e}^{i\beta Y}\psi_L$$
$$\psi_R \longrightarrow \psi'_R = \mathrm{e}^{i\beta Y}\psi_R$$

$$W_\mu \longrightarrow W'_\mu = U_L W_\mu U_L^\dagger - \frac{i}{g_2}\partial_\mu U_L U_L^\dagger$$
$$B_\mu \longrightarrow B'_\mu = B_\mu - \frac{1}{g_1}\partial_\mu \beta$$

With symmetry breaking

- the $W^{\pm}$ and Z bosons must acquire mass.
- the photon must remain massless.
- QED must remain and exact symmetry.

The simplest choice is a complex $SU(2)$ doublet of scalar fields

$$\Phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix} \qquad V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

If $\mu^2 < 0$

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad v = \sqrt{-\frac{\mu^2}{\lambda}}$$

The field can be paramterised as

$$\Phi = e^{i\chi_a T^a} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix}$$

With symmetry breaking

- the $W^{\pm}$ and Z bosons must acquire mass.
- the photon must remain massless.
- QED must remain and exact symmetry.

The simplest choice is a complex $SU(2)$ doublet of scalar fields

$$\Phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix} \qquad V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

If $\mu^2 < 0$

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad v = \sqrt{-\frac{\mu^2}{\lambda}}$$

The field can be paramterised as

$$\Phi \longrightarrow \mathrm{e}^{-i\chi_a T^a} \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

With symmetry breaking

- the $W^\pm$ and Z bosons must acquire mass.
- the photon must remain massless.
- QED must remain and exact symmetry.

Rewriting the kintetic term of the scalar Lagrangian gives

$$(D^\mu\Phi)^\dagger(D_\mu\Phi) = \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{1}{8}g_2^2(v+h)^2\left|W_\mu^1 + iW_\mu^2\right|^2 + \frac{1}{8}(v+h)^2\left|g_2W_\mu^3 - g_1B_\mu\right|^2$$

Defining the new fields

$$W^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right) \qquad Z_\mu = \frac{g_2W_\mu^3 - g_1B_\mu}{\sqrt{g_2^2+g_1^2}} \qquad A_\mu = \frac{g_2W_\mu^3 + g_1B_\mu}{\sqrt{g_2^2+g_1^2}}$$

the Lagrangian becomes

$$\mathcal{L}_S = ... + M_W^2W_\mu^+W^{-\mu} + \frac{1}{2}M_Z^2Z_\mu Z^\mu + \frac{1}{2}M_A^2A_\mu A^\mu + ...$$

with

$$M_W = \frac{1}{2}vg_2 \qquad M_Z = \frac{1}{2}v\sqrt{g_2^2+g_1^2} \qquad M_A = 0$$

With symmetry breaking

- the $W^{\pm}$ and Z bosons must acquire mass.
- the photon must remain massless.
- QED must remain and exact symmetry.

The $W^{\pm}$ and Z bosons acquire mass while the photon e kept massless

$$M_W = \frac{1}{2} v g_2 \qquad M_Z = \frac{1}{2} v \sqrt{g_2^2 + g_1^2} \qquad M_A = 0$$

- $M_W = 80.379 \pm 0.012$ GeV
- $M_Z = 91{,}1876 \pm 0.0021$ GeV
- $M_A < 1 \times 10^{-18}$ eV
- $M_h = 125.10 \pm 0.14$ GeV

The original symmetry is broken

$$SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_Q$$

and the electroweak interaction splits into the weak and electromagnetic interactions.

First-order phase transition

FOPTs involve a latent heat. Two global minima of the scalar potential coexist for a range of temperatures. There is a potential barrier that is overcome by quantum tunnelling. The transition is performed via nucleation of the new phase within the old one. The EW PT in the SM is not first-order.

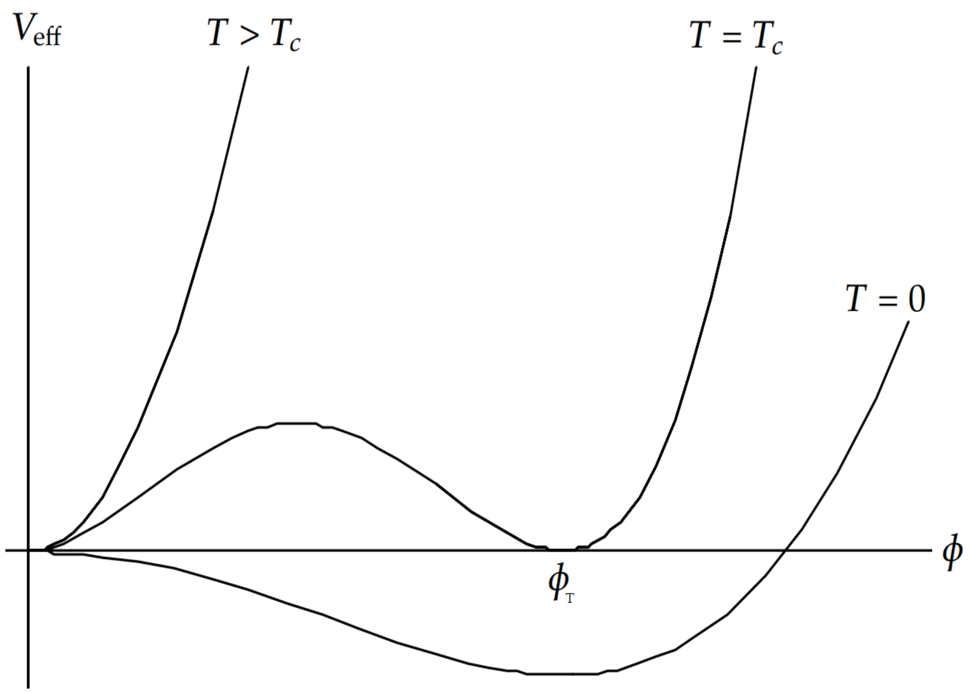


Fig.3: Plot of the effective potential $V_{eff}(\phi,T) = \lambda\phi^2(\phi - v)^2 - aT\phi^3 + \lambda bT\phi^4$. **[3].**

$$V_{eff}(\phi,T) = V_0(\phi) + \Delta V(\phi,T)$$

- $T > T_c$: $\phi_0 = 0$
- $T = T_c$: $\phi_0 = 0$ and $\phi_0 = \phi_T$
- $T < T_c$: $\phi_0 = \phi_T$

Bubbles of the true vacum nucleate spontaneously in the false vacuum.

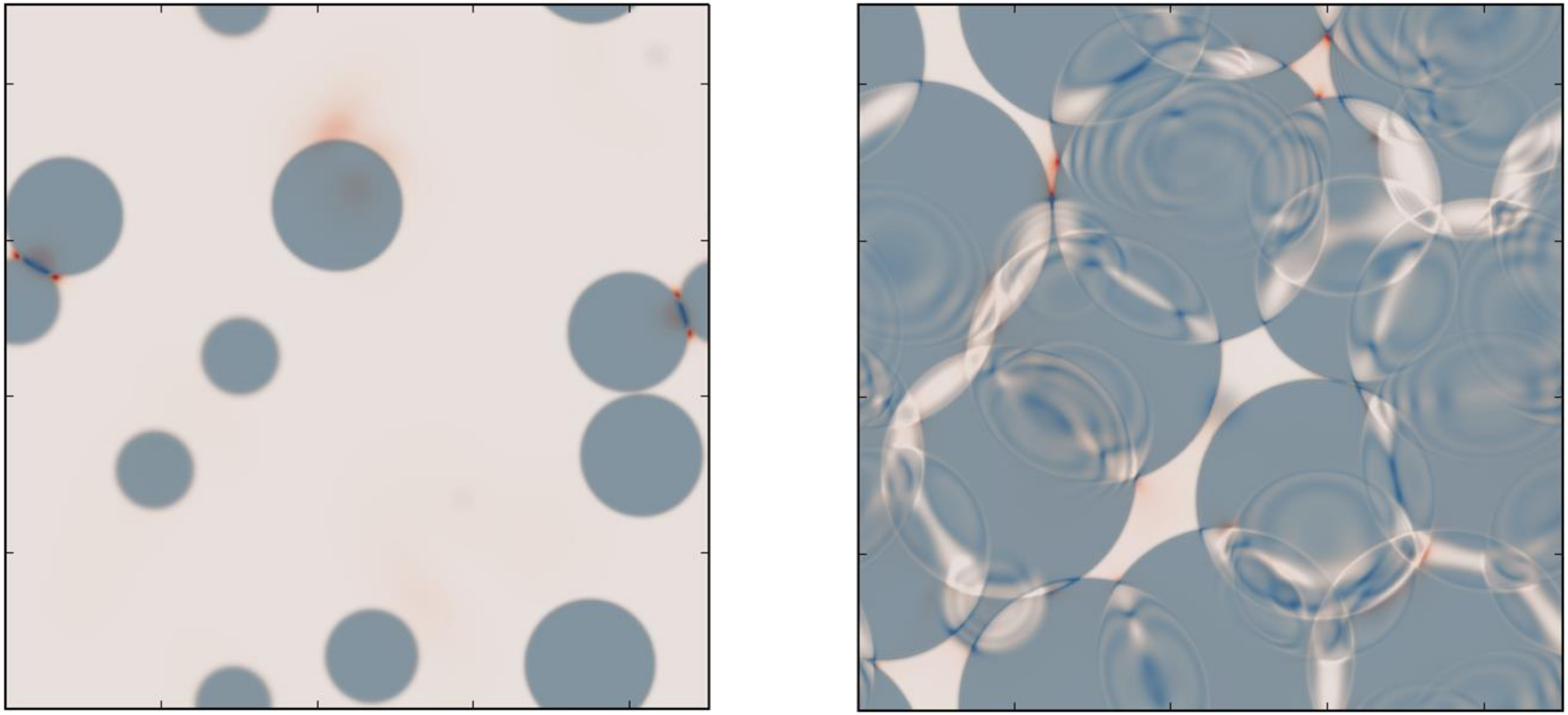
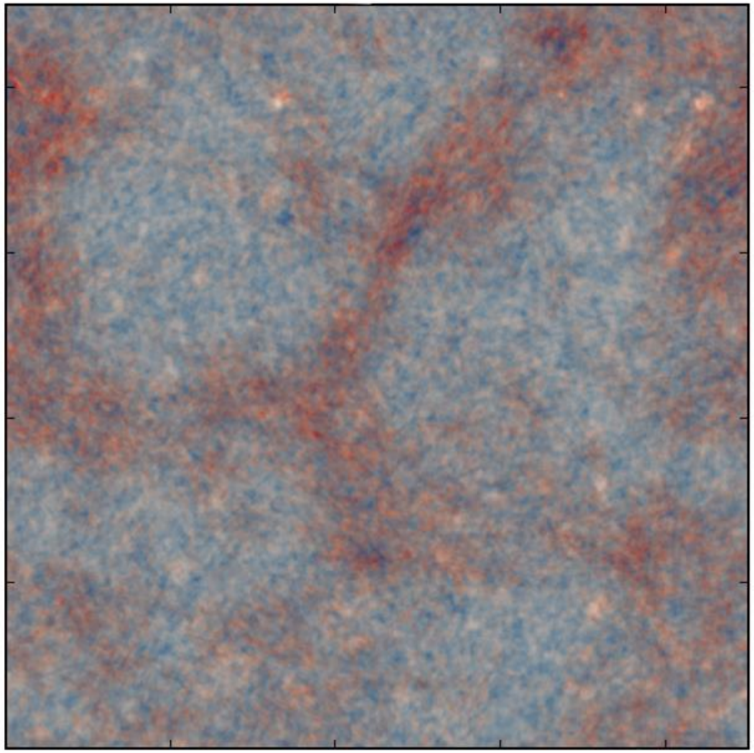


Fig.4: Slices through a simultaneous nucleation simulation showing the expansion (left) and collision (right) phase of the scalar field. The scalar field value is shown in blue, and the gravitational wave energy density is shown in red. **[4]**



- The net result of the many bubble collisions is a uniform stochastic background of gravitational radiation.
- The main sources of gravitational waves are:

  Collisions between bubble walls

  Sound waves in the plasma

  Magnetohydrodynamic turbulence in the plasma

$$h^2\Omega_{GW} \approx h^2\Omega_{\phi} + h^2\Omega_{sw} + h^2\Omega_{turb}$$

Fig.5: Slice through a simultaneous nucleation simulation showing the oscillatory phase of the scalar field. The scalar field value is shown in blue, and the gravitational wave energy density is shown in red. **[4]**

- The GW spectrum is modeled based on numerical simulations.

For the acoustic contribution,

$$h^2\Omega_{sw}(f) = 1.67 \times 10^{-5} \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\kappa\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \left(\frac{0.11 v_w^3}{0.42 + v_w^2}\right) S(f)$$

with

$$\frac{\beta}{H_*} = T_* \left.\frac{dS}{dT}\right|_{T=T_*}$$

- Inverse time duration of the phase transition

$$\alpha = \frac{\rho_{vac}}{\rho_r^*}$$

- Amount of energy released in the transition (transition strength)

$$\kappa$$

- Fraction of latent heat transformed into the kinetic energy of the scalar field

$$S(f)$$

- Spectral shape

$$v_w$$

- Velocity of the bubble wall

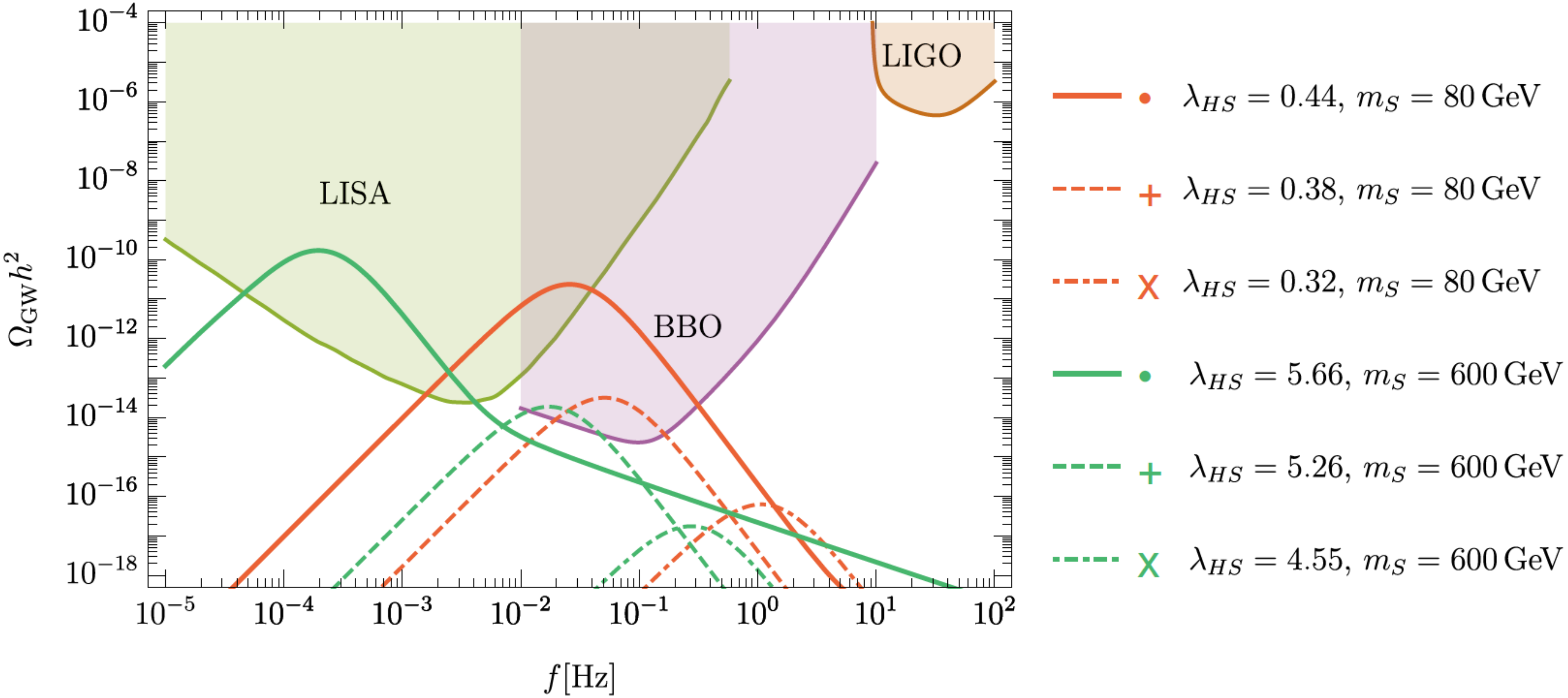


Fig.6: Spectra of gravitational waves from the electroweak phase transition for a few benchmark points. Projected sensitivities of the future based gravitational waves detectors such as LISA and BBO as well as the current sensitivity of LIGO are also shown. Notice that LIGO cannot probe any part of the parameter space. **[5]**

References - Figures

[1] Langer, N. (2012). Nucleosynthesis - lecture notes. Bonn University.

[2] Pich, A. (2012). The Standard Model of Electroweak Interactions.

[3] Hou,W., Kohda, M. (2013). Towards Reviving Electroweak Baryogenesis with a Fourth Generation. Advances in High Energy Physics, 2013, 1-9.

[4] Cutting, D., Hindmarsh, M., Weir, D.J. (2018). Gravitational waves from vacuum first-order phase transitions: From the envelope to the lattice. Physical Review D.

[5] Beniwal, A., Lewicki, M.,Wells, J.D., White, M., Williams, A.G. (2017). Gravitational wave, collider and dark matter signals from a scalar singlet electroweak baryogenesis. Journal of High Energy Physics, 2017, 1-26.