UNIVERSO PRIMITIVO

Mestrado em Física Astronomia 2026-2027

Exercise Sheet 2

1. Convert kilogram (kg), second (s), meter (m) and kelvin (K) into the appropriate powers of giga - electron volts (GeV), using natural units $c = \hbar = k_B = 1$. Use your findings to express your body mass, age, height and body temperature in these units.

2. Derive separately the number density, energy density and pressure for an ultra-relativistic gas of bosons and for an ultra-relativistic gas of fermions, each in thermal equilibrium with zero chemical potential.

3. Show that, for nonrelativistic particles with zero chemical potential, the energy density and pressure are $\rho = (m + \frac{3}{2}T)\, n$ and $P = nT$, respectively. Explain why, in these conditions, one has $P \ll \rho$.

4. Consider an ideal gas of ultra-relativistic particles in thermal equilibrium at temperature T and chemical potential μ. Derive the leading-order expressions for the number density, energy density and pressure in each of the following limits:

   4.1. Degenerate fermions with $\mu \gg T$.

   4.2. Either bosons or fermions with $\mu < 0$ and $|\mu| \gg T$.

   [Hint for 4.1: in the strongly degenerate regime $T \ll E_F \simeq \mu$, where E_F is the Fermi energy, thermal smearing of the Fermi surface is negligible. One may therefore treat fermion states with $E < \mu$ occupied, and those with $E > \mu$ as empty.]

5. Consider the nonrelativistic limit for particles and antiparticles with nonzero chemical potential μ. Assume that pair creation and annihilation are in chemical equilibrium, so that $\bar{\mu} = -\mu$. The overbar denotes antiparticle densities (which need not to equal the corresponding particle densities). Derive the following expressions in the dilute-gas limit:

   5.1. $n = g\left(\frac{mT}{2\pi}\right)^{3/2} \exp(-\frac{m-\mu}{T})$

   5.2. $n - \bar{n} = 2g\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{T}\right) \sinh\left(\frac{\mu}{T}\right)$

   5.3. $\rho + \bar{\rho} = 2gm\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{T}\right) \cosh\left(\frac{\mu}{T}\right)$

   [Hint: In the dilute-gas limit, the phase-space mean occupation number is much less than one. For both particles and antiparticles, this is ensured by $m - |\mu| \gg T$.]