BJT Transistor Modeling

A model is an equivalent circuit that represents the AC characteristics of the transistor. It uses circuit elements that approximate the behavior of the transistor.

There are 2 models commonly used in small signal AC analysis of a transistor:

* hybrid equivalent model
**Important Parameters**

$Z_i, Z_o, A_v, A_i$ are important parameters for the analysis of the AC characteristics of a transistor circuit.

![Two-port System Diagram](image)

**Input Impedance, $Z_i$**

To determine $I_i$: insert a “sensing resistor”

$$Z_i = \frac{V_i}{I_i}$$

![Sensing Resistor Diagram](image)

then calculate $I_i$:

$$I_i = \frac{V_z - V_i}{R_{\text{sense}}}$$
**Output Impedance, \( Z_o \)**

To determine \( I_o \): insert a “sensing resistor”

\[
Z_o = \frac{V_o}{I_o}
\]

then calculate \( I_o \):

\[
I_o = \frac{V - V_o}{R_{sense}}
\]

**Voltage Gain, \( A_v \)**

\[
A_v = \frac{V_o}{V_i}
\]

For an amplifier with no load:

\[
A_{VNL} = \frac{V_o}{V_i |R_{source} = \infty|}
\]

*Note:* the no-load voltage gain \((A_{VNL})\) is always greater than the loaded voltage gain \((A_v)\).
Current Gain, $A_i$

$$A_i = \frac{I_o}{I_i}$$

The current gain ($A_i$) can also be calculated using the voltage gain ($A_v$):

$$A_i = -A_v \frac{Z_i}{R_L}$$

Phase Relationship

The phase relationship between input and output depends on the amplifier configuration circuit.

- Common – Emitter: 180 degrees
- Common - Base: 0 degrees
- Common – Collector: 0 degrees
Hybrid Equivalent Model

The hybrid parameters: \( h_{ie}, h_{re}, h_{fe}, h_{oe} \) are developed and used to model the transistor. These parameters can be found in a specification sheet for a transistor.

- \( h_i \) = input resistance
- \( h_r \) = reverse transfer voltage ratio \( (V_i/V_o) \)
- \( h_f \) = forward transfer current ratio \( (I_o/I_i) \)
- \( h_o \) = output conductance

General h-Parameters for any Transistor Configuration

\[ h_i = \text{input resistance} \]
\[ h_r = \text{reverse transfer voltage ratio} \ (V_i/V_o) \]
\[ h_f = \text{forward transfer current ratio} \ (I_o/I_i) \]
\[ h_o = \text{output conductance} \]
Simplified General h-Parameter Model

The above model can be simplified based on these approximations:

\[ h_r \approx 0 \text{ therefore } h_r V_o = 0 \text{ and } h_o \approx \infty \]

![Diagram of Simplified General h-Parameter Model]

Common-Emitter h-Parameters

\[ h_{\text{ae}} = \frac{25 \text{mV}}{I_{\text{bq}}} \approx \frac{h_\beta 25 \text{mV}}{I_{\text{bq}}} \]

\[ h_{\text{ie}} = \beta_{\text{ae}} \]

![Diagram of Common-Emitter h-Parameters]
Common-Base $h$-Parameters

$$h_{ib} = \frac{25 \text{mV}}{I_{EQ}}$$

$$h_{fb} = -\alpha_{ac} \approx -1$$

BJT Small-Signal Analysis
Common-Emitter (CE) Fixed-Bias Configuration

The input ($V_i$) is applied to the base and the output ($V_o$) is from the collector.

The Common-Emitter is characterized as having high input impedance and low output impedance with a high voltage and current gain.

Removing DC effects of $V_{CC}$ and Capacitors

Hybrid Equivalent Circuit
Hybrid Equivalent Circuit

Determine \( h_{fe}, h_{ie}, \) and \( h_{oe} \):

- \( h_{fe} \) and \( h_{oe} \): look in the specification sheet for the transistor or test the transistor using a curve tracer.
- \( h_{ie} \): calculate \( h_{ie} \) using DC analysis: 
  \[ h_{ie} = \frac{25 \text{mV}}{I_{BQ}} \approx h_{ie} \frac{25 \text{mV}}{I_{EQ}} \]

Impedance Calculations

Input Impedance: 
\[ Z_i = R_B \parallel h_{ie} \]
\[ Z_i \approx h_{ie} \left|_{R_B \geq 10h_{ie}} \right. \]

Output Impedance: 
\[ Z_o = R_C \parallel \frac{1}{h_{oe}} \]
\[ Z_o \approx R_C \left|_{1/h_{oe} \geq 10R_c} \right. \]
Gain Calculations

Voltage Gain \( A_v \): \[
A_v = \frac{v_o}{v_i} = -\frac{h_{fe}(R_c || 1/h_{ce})}{h_{ie}}
\]

Current Gain \( A_i \): \[
A_i = \frac{i_o}{i_i} = \frac{h_{fe}R_b(1/h_{ce})}{(1/h_{ce} + R_C)(R_b + h_{ie})}
\]

Current Gain from Voltage Gain: \[
A_i = -A_v \frac{Z_i}{R_C}
\]

Phase Relationship

The phase relationship between input and output is 180 degrees. The negative sign used in the voltage gain formulas indicates the inversion.
CE – Voltage-Divider Bias Configuration

You still need to determine $h_{ie}$, $h_{oe}$, and $h_{re}$.

Impedance Calculations

Input Impedance: \[ Z_i = R' || h_{ie} \]

\[ R' = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} \]

Output Impedance: \[ Z_o = R_c || \frac{1}{h_{oe}} \]

\[ Z_o \approx R_c \left( \frac{1}{h_{oe}} \right) \leq 10 R_c \]
Gain Calculations

### Voltage Gain (A_v):

\[
A_v = \frac{v_o}{v_i} = -\frac{h_{re}}{h_{ie}} \frac{R_c || 1/h_{re}}{1/h_{ie}}
\]

### Current Gain (A_i):

\[
A_i = \frac{i_o}{i_i} = \frac{h_{re}R'(1/h_{re})}{(1/h_{re} + R_c)(R' + h_{ie})}
\]

Current Gain from Voltage Gain:

\[
A_i = -A_v \frac{Z_i}{R_c}
\]

Phase Relationship

A CE amplifier configuration will always have a phase relationship between input and output is 180 degrees. This is independent of the DC bias.
CE Emitter-Bias Configuration

Unbypassed $R_E$

Again you need to determine $h_{fe}$, $h_{ie}$.

Impedance Calculations

Input Impedance: $Z_i = R_B \parallel Z_b$

$Z_b = h_{ie} + (h_{fe} + 1)R_E$

$Z_b \approx h_{fe}R_E\left|\frac{1}{(h_{be} + 1)R_E}\right| >> b_u, h_{ie} >> 1$

Output Impedance: $Z_o = R_C$
Voltage Gain ($A_v$): $A_v = \frac{v_o}{v_i} = -\frac{h_{fe}R_C}{Z_b}$

Current Gain ($A_i$): $A_i = \frac{i_o}{i_i} = \frac{-h_{fe}R_B}{R_B + Z_b}$

Current Gain from Voltage Gain: $A_i = -A_v \frac{Z_i}{R_C}$

**Phase Relationship**

A CE amplifier configuration will always have a phase relationship between input and output is 180 degrees. This is independent of the DC bias.
Emitter-Follower Configuration

You may recognize this as the Common-Collector configuration. Indeed they are the same circuit.

Note the input is on the base and the output is from the emitter.

Hybrid Equivalent Model

You still need to determine $h_e$ and $h_{ic}$. 
### Impedance Calculations

**Input Impedance:**

\[ Z_i = R_B \parallel Z_b \]

\[ Z_b = h_{ie} + (h_{fe} + 1)R_E \]

\[ Z_b \approx h_{fe}R_E \quad (h_{ie} + 1)R_E \gg h_{ie}, h_{fe} \gg 1 \]

### Impedance Calculations (cont’d)

**Output Impedance:**

\[ Z_o = R_E \parallel \frac{h_{ie}}{h_{fe} + 1} \]

\[ Z_o \approx \frac{h_{ie}}{h_{fe}} \quad (h_{ie} + 1)R_E \gg h_{ie}, h_{fe} \gg 1 \]
Gain Calculations

Voltage Gain (A_v): \( A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + h_{ie}/(h_{fe} + 1)} \)

\( A_v \approx \frac{1}{(h_{be} + 1)R_E} \gg h_{we} \)

Current Gain (A_i): \( A_i \approx i_o/i_i - \frac{h_{fe}R_B}{R_B + Z_b} \)

Current Gain from Voltage Gain: \( A_i = -A_v \frac{Z_i}{R_E} \)

Phase Relationship

A CC amplifier or Emitter Follower configuration has no phase shift between input and output.
The input ($V_i$) is applied to the emitter and the output ($V_o$) is from the collector.

The Common-Base is characterized as having low input impedance and high output impedance with a current gain less than 1 and a very high voltage gain.

You will need to determine $h_{fb}$ and $h_{ib}$.

$$h_{ib} = \frac{h_{fe}}{h_{fe} + 1} = \frac{25 \text{mV}}{I_{EQ}}$$

$$h_{fb} = -\alpha_{ac} \approx -1$$
**Impedance Calculations**

Input Impedance: \( Z_i = R_E \parallel h_{ib} \)

Output Impedance: \( Z_o = R_C \)

**Gain Calculations**

Voltage Gain (\( A_v \)): \( A_v = \frac{v_o}{v_i} = \frac{h_{fb}R_C}{h_{ib}} \approx \frac{R_C}{h_{ib}} \)

Current Gain (\( A_i \)): \( A_i = \frac{i_o}{i_i} = h_{fb} \approx -1 \)
Phase Relationship

A CB amplifier configuration has no phase shift between input and output.

CE Collector Feedback Configuration

This is a variation of the CE Fixed-Bias configuration.
Hybrid Equivalent Model

You will need to determine $h_{fe}$ and $h_{ie}$.

Impedance Calculations

Input Impedance: $Z_i = \frac{h_{ie}}{1 + h_{ie} \frac{R_C}{R_F}}$

Output Impedance: $Z_o \approx R_C \parallel R_F$
Gain Calculations

Voltage Gain ($A_v$): 
\[
A_v = \frac{V_o}{V_i} = -\frac{h_{fe}R_C}{h_{fe}}
\]

Current Gain ($A_i$): 
\[
A_i = \frac{I_o}{I_i} = \frac{h_{fe}R_F}{R_F + h_{fe}R_C}
\]

\[
A_i = \frac{I_o}{I_i} \approx \frac{R_F}{R_C}
\]

Example

According to the figure above
- Perform DC analysis and find the Q-point.
- Evaluate the voltage gain $A_v$ and the current gain $A_i$.
- Sketch $v_o$ on the AC+DC load line graph when
  - $v_s = 100 \text{ sin}(\omega t) \text{ mV}$.
  - $v_s = 900 \text{ sin}(\omega t) \text{ mV}$.

- $h_{fe} = h_{FE} = 200$
- $I_{CO} = 0 \text{ A}$
- $V_{BE} = 0.7 \text{ V}$
- $V_{CEsat} = 0 \text{ V}$

\[
I_{CQ} = 0.39 \text{ mA}
\]
\[
V_{CBQ} = 6.8 \text{ V}
\]

\[
V_{CCQ} = 10 \text{ V}
\]

\[
I_{COL} = 0.39 \text{ mA}
\]
\[
V_{CEQ} = 6.8 \text{ V}
\]