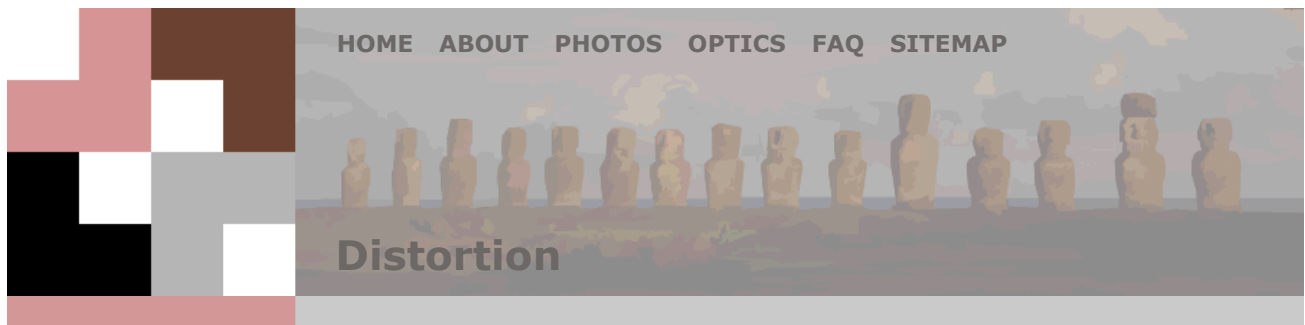




The last of the Seidel aberrations, distortion is the most easily recognized aberration as it deforms the image as whole. Since straight lines in the object space are rendered as curved lines on the sensor, the name curvilinear distortion is frequently encountered. Fig. 1 shows the two fundamental manifestations of the aberration, barrel and pincushion distortion. Straight lines in the undistorted subject (left grid) bulge in the characteristic barrel fashion (middle grid) or bend inward in the pincushion representation (right grid). Straight lines running through the image center remain straight and a circle concentric with the image center remains a circle, although its radius is affected.

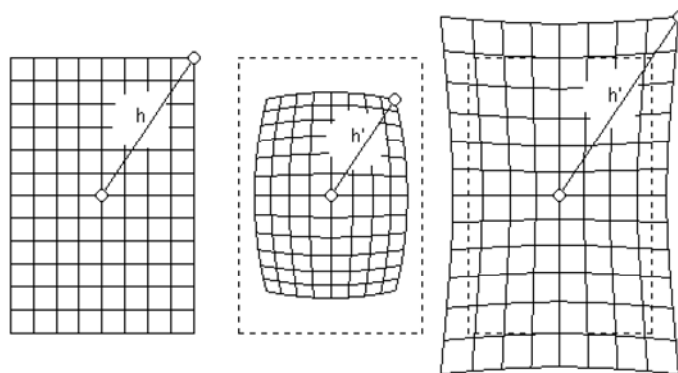


Figure 1. Distortion of a rectangular grid. Left: undistorted grid. Middle grid: barrel distortion. Right grid: pincushion distortion.

### Origin

A common cause of distortion is the introduction of a stop in a system of (thin) lenses [\[1,2\]](#), e.g. to reduce spherical aberration or astigmatism. The position of such a stop determines the amount and the sign of the distortion, as illustrated in Fig. 2 for a simple positive lens.

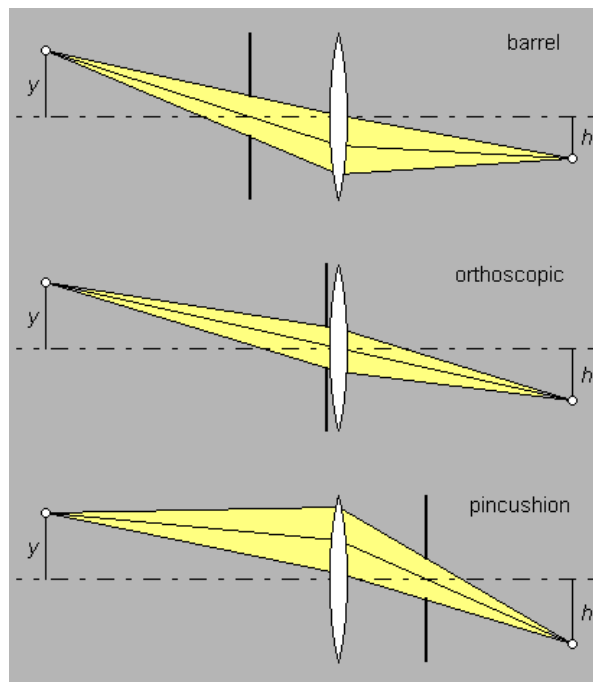


Figure 2. The influence of stop position on distortion. Note that the image size  $h$  differs for a constant object size  $y$ .

The position of the image point is given by the intersection of the chief ray (solid line) and the image plane. The chief ray is the ray that passes through the center of the stop, and is characteristic of the light cones in front of and behind the stop. When the stop is located at the lens, the chief ray is the principal ray (i.e., it passes through the optical center) and leaves the lens at the same angle at which it entered. Such a system does not distort the image and is called orthoscopic. With the stop in front or in rear, the chief ray is refracted. The image and object distances, measured along the chief ray, differ from the orthoscopic case. The ratio of these distances determines the image magnification  $h/y$ , which is smaller with the stop in front and larger with a rear stop. Straight lines will be distorted when the magnification  $h/y$  is a function of the off-axis distance  $y$ . The top configuration leads to a situation where  $h/y$  decreases toward the image corners: barrel distortion. The bottom configuration leads to a situation where  $h/y$  increases toward the image corners: pincushion distortion.

The size of the stop has no effect on the distortion, as the chief ray does not alter its route when the aperture is made smaller or larger. To be sure, Fig. 2 cannot be understood in the context of a paraxial theory. In the absence of the stop, the lens in Fig. 2 suffers from spherical aberration, coma, and astigmatism, which result in a blurred image patch for each point in object space.

A complex lens such as a retrofocus wideangle design tends to exhibit barrel distortion as the front group of elements acts as an aperture stop for the positive rear group [3]. Telephoto lenses have a negative rear group and give rise to pincushion distortion. Distortion is difficult to correct for in zoom lenses, which usually go from barrel at the wide end to pincushion at the tele end. The aberration is minimized by a symmetrical lens design, which is (near) orthoscopic. The aperture set by the variable leaf diaphragm has no effect on distortion, but is instrumental in controlling other aberrations. When a lens is turned around with a reversal ring for macro photography, distortion changes sign. Barrel distortion becomes pincushion and vice versa. When a distorted slide is projected with the original picture taking lens in reverse, curvilinear distortion is absent from the projected image. Like other aberrations, distortion also depends on the object distance. Infinity focus and close focus may yield different amounts of distortion with the same lens.

### How to read a distortion graph

Distortion is measured as the relative change of  $h$  compared to its value predicted by the paraxial theory. In Fig. 1 the distance from the image center to the corner of the grid is indicated by  $h$  for the orthoscopic case, and with  $h'$  for the distorted grids. The relative distortion  $D$  is then given by  $(h'-h)/h$ . The corner points of the three grids in Fig. 1 are distorted by, from left to right, 0%, -30%, and +30% respectively. Carl Zeiss provide with their lenses a graph of the relative distortion versus the off-axis distance  $h$ . Fig. 3A shows

such a graph for the Distagon 2.8/21.

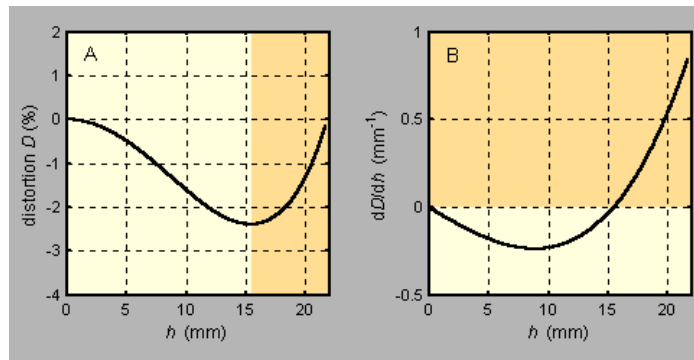


Figure 3. Distortion curve for the Distagon 2.8/21 (A, Zeiss data) and its derivative (B).

It is frequently asserted that negative values of  $D$  correspond to barrel distortion, and positive values to pincushion distortion. For simple distortion curves this is true, but a few lenses with a more complex distortion curve do not follow this simple rule. As a case in point I consider the above Distagon 2.8/21. Fig. 4 shows a grid distorted according to the data in Fig. 3. Although curve A is negative everywhere, the grid reveals significant pincushion distortion toward the corners. Indeed, it is not the sign of  $D$  which determines the type of distortion, but the slope of the curve. A negative slope (yellow part) of  $D$  implies barrel distortion, a positive slope (orange part) pincushion distortion. The steeper the slope, the more pronounced the distortion.

Fig. 3A should be read as follows. Up to 15 mm from the image center the slope is negative, which corresponds to barrel distortion. From 15 mm to the far image corner the slope is positive: pincushion distortion. The latter slope is steeper than the former, so that the pincushion distortion in the image periphery is more pronounced than the barrel distortion in the more central regions. Thus, it is not the magnitude of  $D$  which is important for line straightness, but the slope of the curve. In this light Fig. 3B is a better guide toward line straightness than 3A. Graph B shows the derivative  $dD/dh$ , i.e. the slope of  $D$ . The sign of this curve is uniquely coupled to the distortion type and its amplitude to the degree of line deformation.

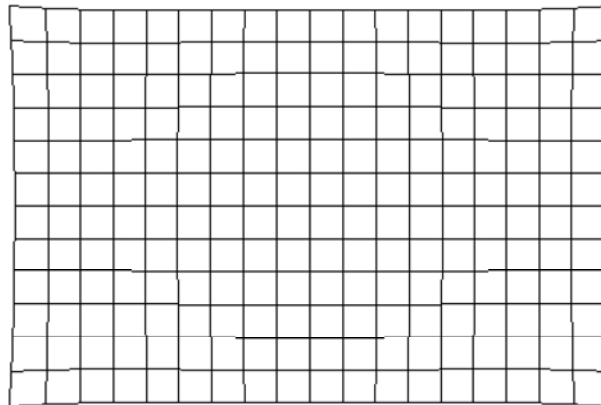


Figure 4. Distortion by the Distagon 2.8/21 according to the data in Fig. 3.

Distortion varies with the off-axis distance as  $h'-h = ah^3 + bh^5 + \dots$ . Relatively,  $D$  varies as  $(h'-h)/h = ah^2 + bh^4$ . The coefficient  $a$  is positive for pincushion and negative for barrel distortion. Usually the quadratic term outweighs the higher-order terms and  $D$  varies with  $h$  squared, getting progressively worse toward the image corners. However, in certain wideangle designs the quartic term is large enough to overcome the quadratic term [4]. The Distagon 2.8/21 exemplifies such behavior, with the curve forced upward in the image corner. This leads to a characteristic behavior that is sometimes referred to as moustache or waveform distortion.

## Distortion in practice

Fig. 5 presents an image captured with the Distagon 2.8/21. The series of windows and red ridge along the roof compare well to the corresponding curves of the theoretical grid in Fig. 4. But there is more. During exposure, a dark ruler was held close to the lens near the bottom of the image. Although somewhat blurred, the ruler clearly shows that distortion depends on the subject distance. The rooftop at a distance of  $\sim 10$  m is subject to moderate moustache distortion, whereas the ruler at a distance of a few centimeters reveals strong barrel distortion.



Figure 5. Distortion by the Distagon 2.8/21 in practice.

### Perspective

The curvilinear distortion treated above is also known as optical distortion. It should not be confused with other effects sometimes referred to as distortion.

The perspective of a photograph is determined by the viewpoint that the photographer takes in relation to the subject. Nearby subjects are rendered larger than faraway subjects of the same size, which can lead to a sense of depth or convergence of lines that are parallel in object space. A well-known example of a perspective effect are the converging verticals that occur when a building is photographed with a tilted camera (Fig. 6). Perspective is not affected by the lens focal length: it depends on the viewpoint only.



Figure 6. Converging verticals. (18-mm lens; 24×36 mm format)

The designation 'perspective distortion' is frequently encountered to describe converging verticals and other manifestations of the perspective, but this is questionable as it unjustly suggests a misrepresentation of the reality. There is actually nothing that is distorted in Fig. 6, the photograph reveals a natural perspective. A perspective control lens, which is a device that can be used to 'correct' converging verticals, creates an illusion of a different perspective, but if anything it's the perspective control lens that presents an unnatural perspective, not the normal picture taking lens. (Strictly speaking a PC lens does not correct anything. Its large image circle and mechanical design merely allow a shift of the image to put the subject into the film frame. The prerequisite for nonconverging verticals is that the film plane is kept parallel to the building.)

Converging verticals often encounter psychological resistance, while photographs with converging horizontals such as a road that narrows toward the horizon are readily accepted or even considered as perspective art. Apart from the appreciation by the audience, however, there is no fundamental difference between the two cases.

Another so-called form of distortion is 'geometric distortion', which arises when a three-dimensional object is projected on a plane such as a digital sensor. For instance, a sphere in the image center is rendered as a round disk on the film, while a sphere in the image periphery of a lens with a large angle of view is elliptically elongated. People in the corners of a wideangle image are deformed (Fig. 7) and may occasionally be saddled with the typical 'egg face'. This geometric effect is a natural consequence of the projection obliquity; the use of the word distortion wrongly suggests an imaging anomaly.



Figure 7. Geometric distortion: the photograph does not flatter the person at the left. (18-mm lens on a 24×36 mm slide)

In contrast to optical distortion, perspective and geometric distortions are no lens aberrations. The apparent anomaly is emphasized by a wrong viewpoint for the image. Ideally each photograph should be viewed from a viewpoint that corresponds to the viewpoint of the photographer in relation to the captured scene. A telephoto image should be viewed from far and a wideangle photograph from close. A photograph with converging verticals looks more natural from a low viewpoint, and, similarly, egg faces improve when the photograph is viewed from close. Unfortunately the human eye does not cover a sufficiently large angle to view the entire image of a wideangle lens from nearby, so in practice the viewer takes a distance, notices the deformation, and speaks of distortion.

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