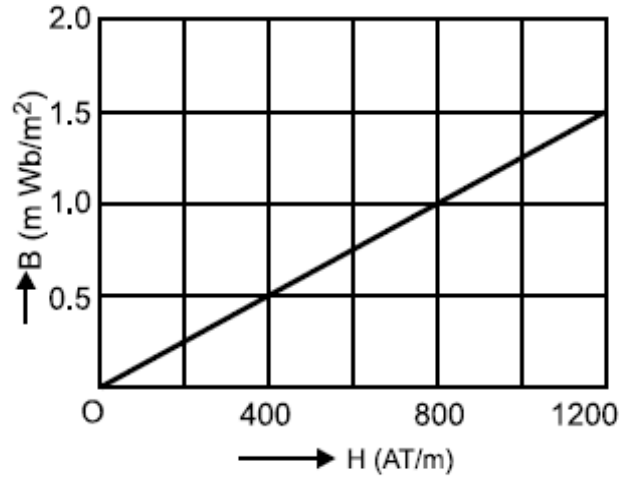


# Circuitos electro- magnéticos

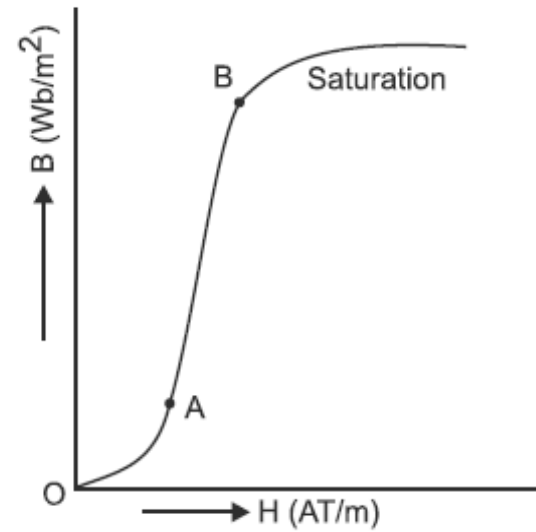
Para um material não magnético

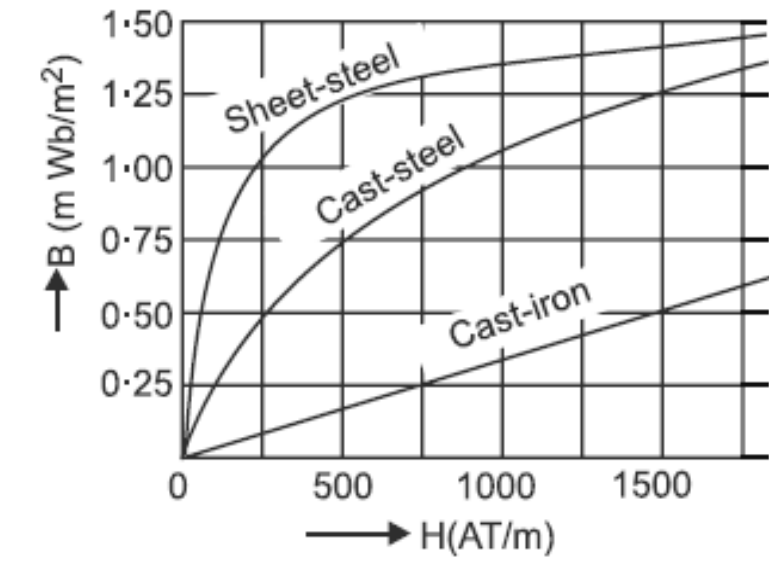
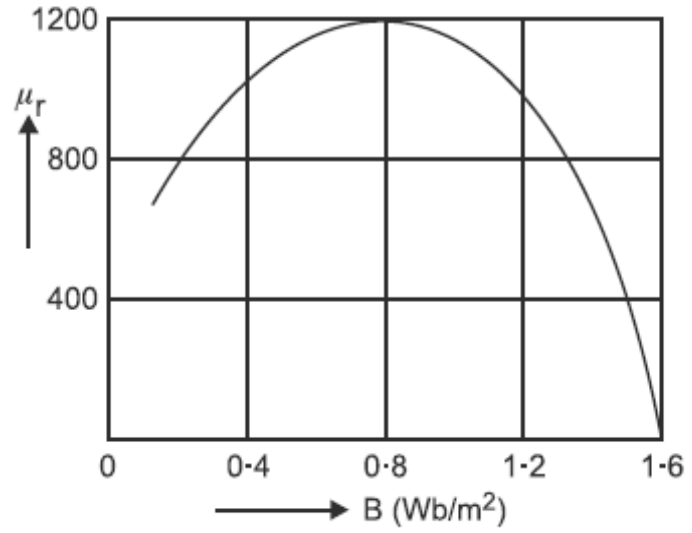


$$B = \mu_0 H$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

Para um material magnético





$(\mu_r = B/\mu_0 H)$

Permeabilidade relativa

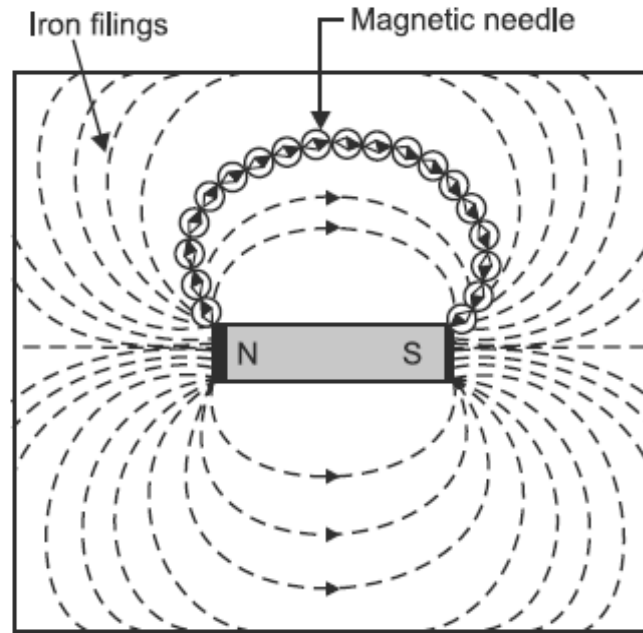


Fig. 1.1 Magnetic field around a bar magnet

- (i) The direction of magnetic lines of force is from N-pole to the S-pole outside the magnet. But inside the magnet their direction is from S-pole to N-pole.
- (ii) They form a closed loop.
- (iii) Their tendency is to follow the least reluctance path.
- (iv) They act like stretched cords, always trying to shorten themselves.
- (v) They never intersect each other.
- (vi) They repel each other when they are parallel and are in the same direction.
- (vii) They remain unaffected by non-magnetic materials.

Consideremos um solenoide com N voltas,  
enrolado num núcleo de ferro

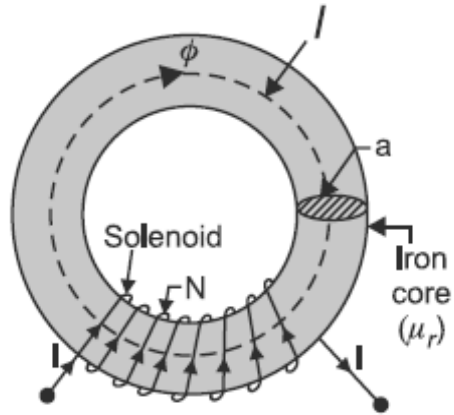


Fig. 1.2 Magnetic circuit

$l$  = mean length of magnetic circuit in m;  
 $a$  = area of cross-section of core in  $m^2$ ;  
 $\mu_r$  = relative permeability of core material.

O fluxo magnético será

$$B = \frac{\phi}{a} \text{ Wb/m}^2$$

E o campo magnético

$$H = \frac{B}{\mu_0 \mu_r} = \frac{\phi}{a \mu_0 \mu_r} \text{ AT/m}$$

Pelo teorema de Ampère

$$Hl = NI \quad \text{or} \quad \frac{\phi}{a \mu_0 \mu_r} \times l = NI \quad \text{or} \quad \phi = \frac{NI}{(l / a \mu_0 \mu_r)} \text{ Wb}$$

O fluxo no núcleo é:

directly proportional to  $N$  and  $I$  i.e.,  $NI$ ,



Força magnetomotriz mmf

inversely proportional to  $l/a \mu_0 \mu_r$



Relutância magnética

$$\phi = \frac{NI}{(l/a \mu_0 \mu_r)} \text{ Wb}$$

$$\text{Ou seja o fluxo} = \frac{\text{mmf}}{\text{relutância}}$$

Esta equação é conhecida como lei de Ohm dos circuitos magnéticos

**Magnetic flux ( $\phi$ ):** The amount of magnetic lines of force set-up in a magnetic circuit is called *magnetic flux*. Its unit is weber (Wb). It is analogous to *electric current I* in electric circuit.

*The magnetic flux density at a point is the flux per unit area at right angles to the flux at that point.*

It is, generally, represented by letter 'B'. Its unit is Wb/m<sup>2</sup> or Tesla, i.e.,

$$B = \frac{\phi}{A} \text{ Wb / m}^2 \quad \text{or} \quad \text{T (1 Wb/m}^2 = 1 \times 10^4 \text{ Wb/cm}^2)$$

**Permeability:** *The ability of a material to conduct magnetic lines of force through it is called the permeability of that material.*

**Relative permeability:** The absolute (or actual) permeability  $\mu$  of a magnetic material is much greater than absolute permeability of air  $\mu_0$ . The relative permeability of a magnetic material is given in comparison with air or vacuum.

*Hence, the ratio of the permeability of material  $\mu$  to the permeability of air or vacuum  $\mu_0$  is called the relative permeability  $\mu_r$  of the material.*

i.e.,

$$\mu_r = \frac{\mu}{\mu_0} \quad \text{or} \quad \mu = \mu_0 \mu_r$$

**Magnetic field intensity:**  $H = \frac{\text{m.m.f}}{\text{length of magnetic path}} = \frac{NI}{l} \text{ AT / m}$

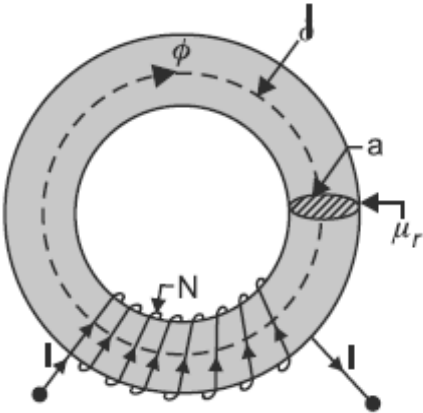
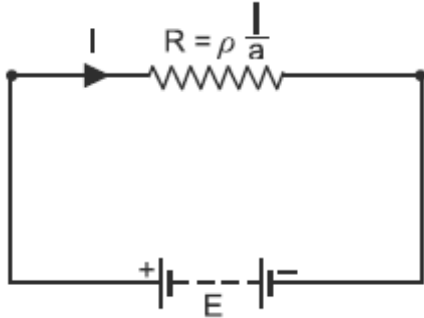
**Magnetomotive force (mmf):**  $\text{mmf} = NI \text{ ampere-turns (or AT)}$

É o análogo à força electromotriz nos circuitos eléctricos

**Reluctance** “resistência magnética”  $S = \frac{l}{a \mu_0 \mu_r}$

Representa a oposição oferecida por um circuito magnético ao fluxo magnético



<b>Magnetic Circuits</b>	<b>Electrical Circuits</b>
 <p data-bbox="690 605 1059 639"><b>Fig. 1.3</b> Magnetic circuit</p>	 <p data-bbox="1454 605 1798 639"><b>Fig. 1.4</b> Electric circuit</p>
<b>Similarities</b>	
<ol style="list-style-type: none"> <li>1. The closed path for magnetic flux is called magnetic circuit.</li> <li>2. Flux = mmf/reluctance</li> <li>3. Flux, <math>\phi</math> in Wb</li> <li>4. mmf in AT</li> <li>5. Reluctance, <math>S = \frac{l}{a\mu} = \frac{l}{a\mu_0 \mu_r}</math> AT/Wb</li> <li>6. Permeance = 1/reluctance</li> <li>7. Permeability, <math>\mu</math></li> <li>8. Reluctivity</li> <li>9. Flux density, <math>B = \frac{\phi}{a}</math> Wb/m<sup>2</sup></li> <li>10. Magnetic intensity, <math>H = NI/l</math></li> </ol>	<ol style="list-style-type: none"> <li>1. The closed path for electric current is called electric circuit.</li> <li>2. Current = emf/resistance</li> <li>3. Current, <math>I</math> in ampere</li> <li>4. emf in V</li> <li>5. Resistance, <math>R = \rho \frac{l}{a} \Omega</math> or <math>R = \frac{1}{\sigma} \frac{l}{a} \Omega</math></li> <li>6. Conductance = 1/resistance</li> <li>7. Conductivity, <math>\sigma = 1/\rho</math></li> <li>8. Resistivity</li> <li>9. Current density, <math>J = \frac{I}{a}</math> A/m<sup>2</sup></li> <li>10. Electric intensity, <math>E = V/d</math></li> </ol>

### *Dissimilarities*

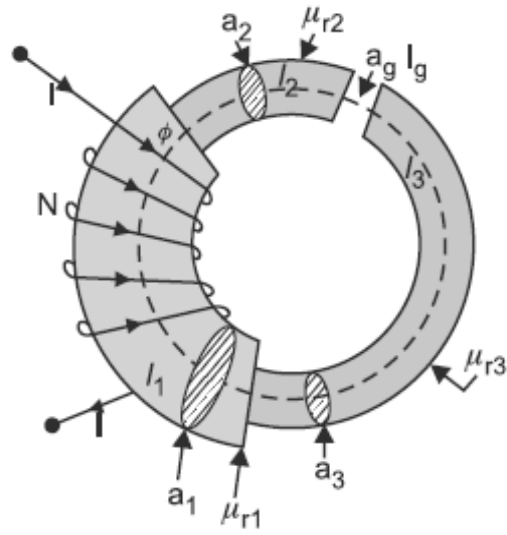
- |   |   |
|---|---|
| <ol style="list-style-type: none"><li>1. In fact, the magnetic flux does not flow but it sets-up in the magnetic circuit (basically molecular poles are aligned).</li><li>2. For magnetic flux, there is no perfect insulator. It can be set-up even in the non-magnetic materials like air, rubber, glass etc. with reasonable mmf</li><li>3. The reluctance (<math>\mathcal{S}</math>) of a magnetic circuit is not constant rather it varies with the value of <math>B</math>. It is because the value of <math>\mu_r</math> changes considerably with the change in <math>B</math>.</li><li>4. Once the magnetic flux is set-up in a magnetic circuit, no energy is expended. However, a small amount of energy is required at the start to create flux in the circuit.</li></ol> | <ol style="list-style-type: none"><li>1. The electric current (electrons) actually flows in an electric circuit.</li><li>2. For electric current, there are large number of perfect insulators like glass, air, rubber, etc., which do not allow it to follow through them under normal conditions.</li><li>3. The resistance (<math>R</math>) of an electric circuit is almost constant as its value depends upon the value of <math>\rho</math> which is almost constant. However, the value of <math>\rho</math> and <math>R</math> may vary slightly if temperature changes.</li><li>4. Energy is expended continuously, so long as the current flows through an electric circuit. This energy is dissipated in the form of heat.</li></ol> |
|---|---|

In a magnetic circuit, flux produced,

$$\phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{l/a\mu_0\mu_r}$$

or AT required,  $NI = \frac{\phi l}{a\mu_0\mu_r} = \frac{B}{\mu_0\mu_r} l = Hl$

## Circuitos magnéticos em série



Total reluctance of the magnetic circuit,

$$S = S_1 + S_2 + S_3 + S_g$$

$$= \frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0}$$

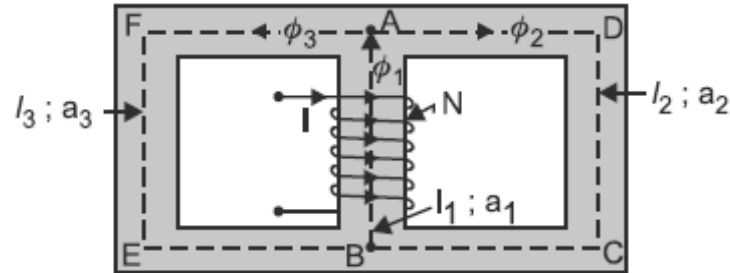
Total mmf =  $\phi S$

$$= \phi \left( \frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0} \right)$$

$$= \frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_3 l_3}{\mu_0 \mu_{r3}} + \frac{B_g l_g}{\mu_0}$$

$$= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

## Circuitos magnéticos em paralelo



It is clear that  $\phi_1 = \phi_2 + \phi_3$

The two magnetic paths  $ADCB$  and  $AFEBC$  are in parallel. The ATs required for this parallel circuit is equal to the ATs required for any one of the paths.

If  $S_1 =$  reluctance of path  $BA$  i.e.,  $l_1/a_1 \mu_0 \mu_{r1}$

$S_2 =$  reluctance of path  $ADCB$  i.e.,  $l_2/a_2 \mu_0 \mu_{r2}$

$S_3 =$  reluctance of path  $AFEBC$  i.e.,  $l_3/a_3 \mu_0 \mu_{r3}$

$\therefore$  Total mmf required = mmf required for path  $BA$  + mmf required path  $ADCB$  or path  $AFEBC$ .

i.e., Total mmf or  $AT = \phi_1 S_1 + \phi_2 S_2 = \phi_1 S_1 + \phi_3 S_3$

An iron ring of 400 cm mean circumference is made from round iron of cross-section 20 cm<sup>2</sup>. Its permeability is 500. If it is wound with 400 turns, what current would be required to produce a flux of 0.001 Wb?

**Solution:**

The magnetic circuit is shown in Fig. 1.8.

Mean length of magnetic path,  $l_m = 400 \text{ cm} = 4 \text{ m}$

Area of X-section of iron ring,  $a = 20 \times 10^{-4} \text{ m}^2$

Absolute permeability,  $\mu_0 = 4\pi \times 10^{-7}$

Now mmf = flux  $\times$  reluctance

$$NI = \phi \times \frac{l_m}{a\mu_0\mu_r}$$

$$400 \times I = 0.001 \times \frac{4}{20 \times 10^{-4} \times 4\pi \times 10^{-7} \times 500}$$

$$\therefore \text{Current, } I = \frac{0.001 \times 4}{20 \times 10^{-4} \times 4\pi \times 10^{-7} \times 500 \times 400} = 7.958 \text{ (Ans.)}$$

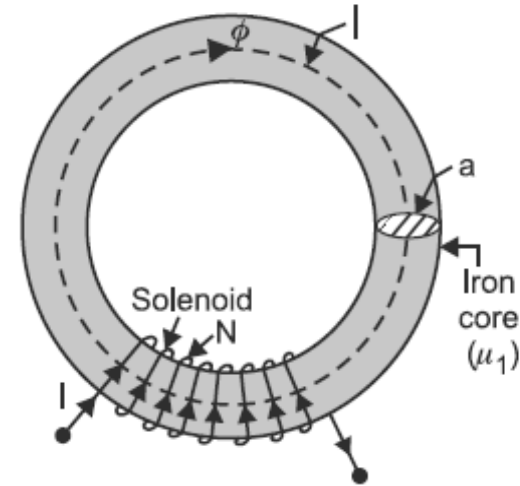
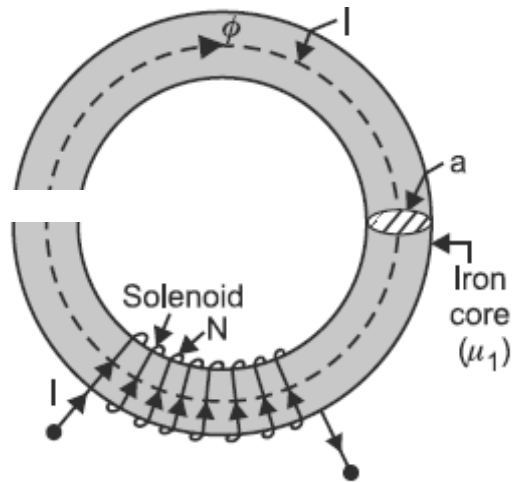


Fig. 1.8 Magnetic circuit

What are the similarities between electrical circuits and magnetic circuits? An iron ring of mean length 50 cm and relative permeability 300 has an air gap of 1 mm. If the ring is provided with winding of 200 turns and a current of 1 A is allowed to flow through, find the flux density across the airgap.



Here,  $l_i = 50 \text{ cm} = 0.5 \text{ m}$ ;  $\mu_r = 300$ ;  $l_g = 1 \text{ mm} = 0.001 \text{ m}$ ;  $N = 200 \text{ turns}$ ;  $I = 1 \text{ A}$

$$\text{Ampere-turns required for air gap} = \frac{B}{\mu_0} l_g$$

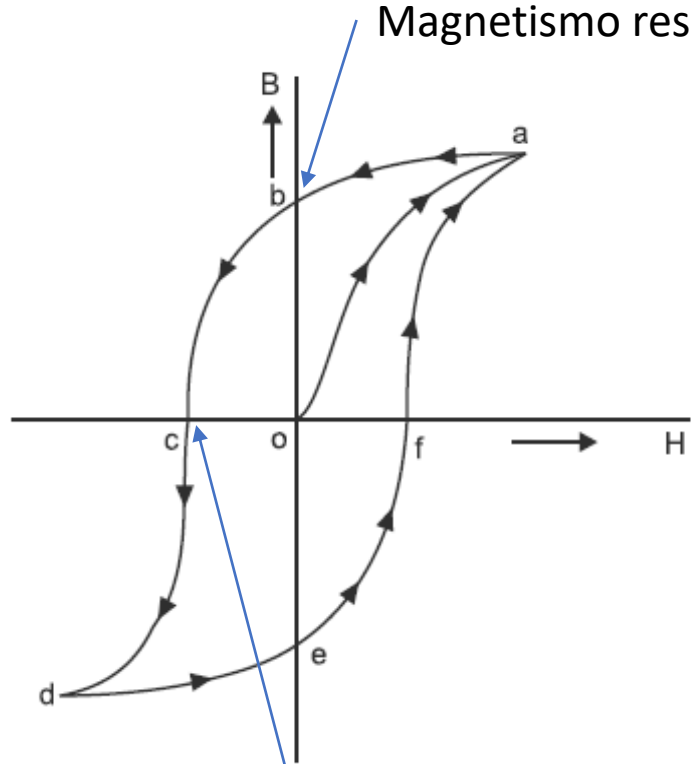
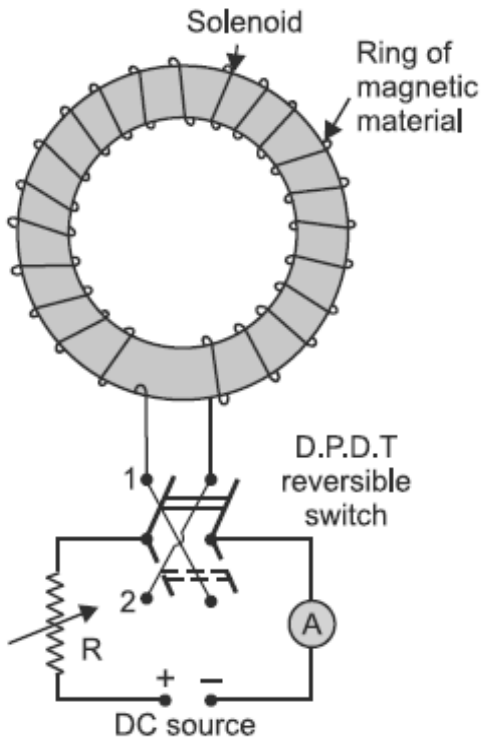
$$\text{Ampere-turns required for iron ring} = \frac{B}{\mu_0 \mu_r} l_i$$

$$\text{Total ampere-turns required} = \frac{B}{\mu_0} l_g + \frac{B}{\mu_0 \mu_r} l_i \quad N I = 200 \times 1 = 200$$

$$200 = \frac{B}{\mu_0} \left( l_g + \frac{l_i}{\mu_r} \right)$$

$$\text{Flux density, } B = \frac{200 \times \mu_0}{0.00267} = \frac{200 \times 4\pi \times 10^{-7}}{0.00267} = 0.09425 \text{ T}$$

# Histerese magnética



Magnetismo residual

coercividade

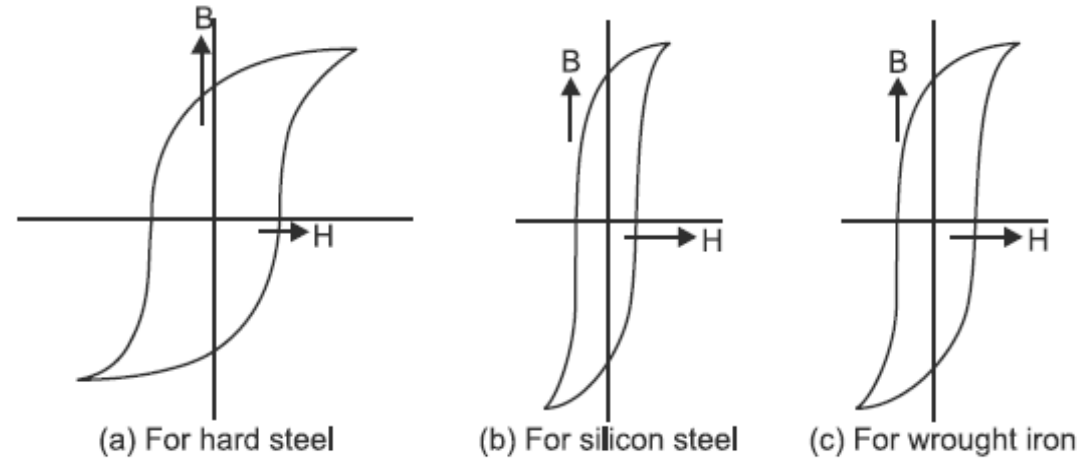
A histerese dá origem a perdas por calor no material magnético

Estas perdas ocorrem sempre que há inversão da magnetização

Uma boa escolha do material magnético é importante para reduzir estas perdas

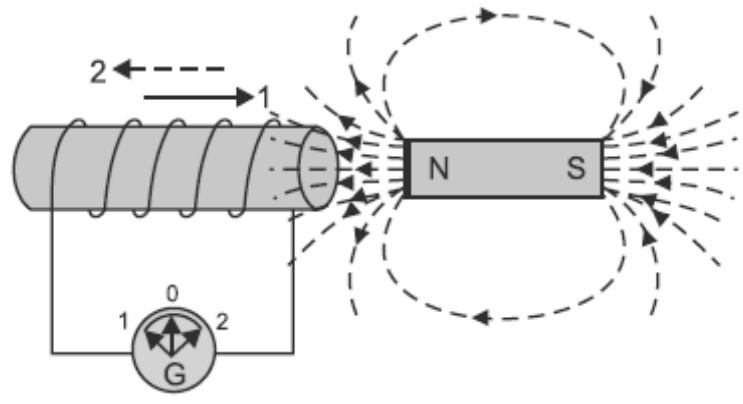
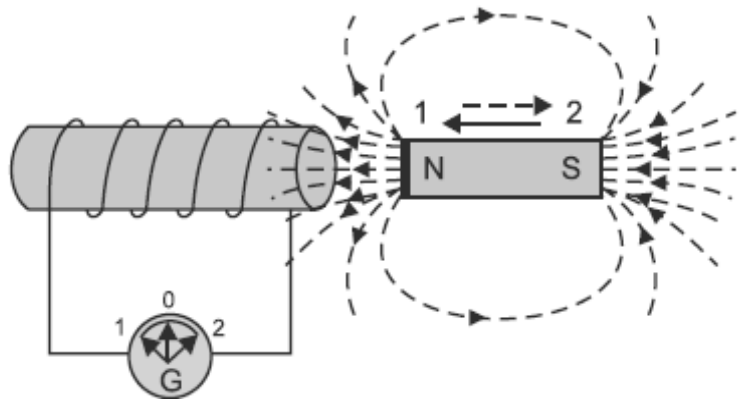


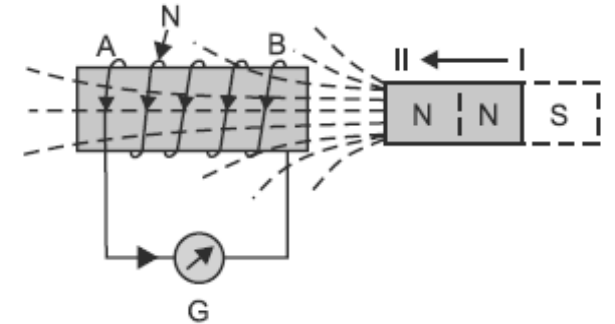
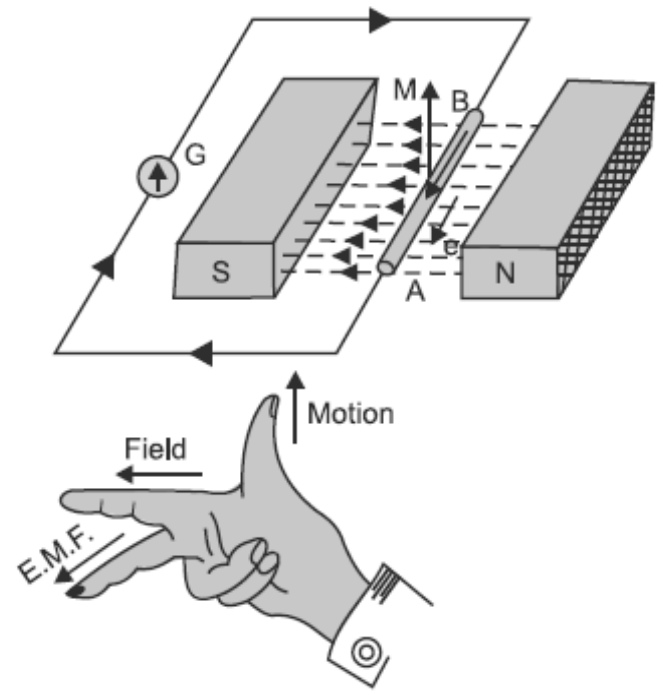
- (i) **Hard steel:** The hysteresis loop for hard steel is shown in Fig. 1.26 (a). This loop has larger area which indicates that this material will have more hysteresis loss. Therefore, it is never used for the construction of machine parts. However, its loop shows that the material has high retentivity and coercivity. Therefore, it is more suitable for making permanent magnets.



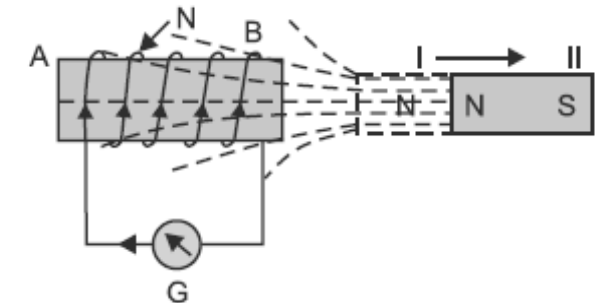
**Fig. 1.26** Hysteresis loop for different magnetic materials

- (ii) **Silicon steel:** The hysteresis loop for silicon steel is shown in Fig. 1.26 (b). This loop has smallest area which indicates that this material will have small hysteresis loss. Therefore, it is most suitable for the construction of those parts of electrical machines in which reversal of magnetisation is very quick e.g., armature of DC machines, transformer core, starter of induction motors etc.
- (iii) **Wrought iron:** Figure 1.26 (c) shows the hysteresis loop for wrought iron. This loop shows that this material has fairly good residual magnetism and coercivity. Therefore, it is best suited for making cores of electromagnets.

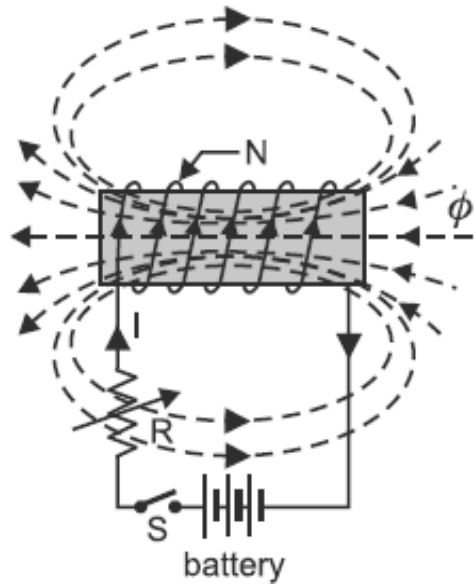




(a) Bar magnet taken near the coil

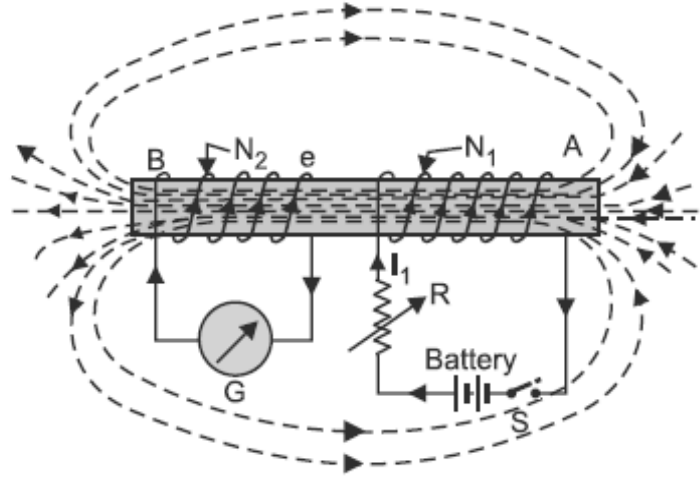


(b) Bar magnet taken away from the coil

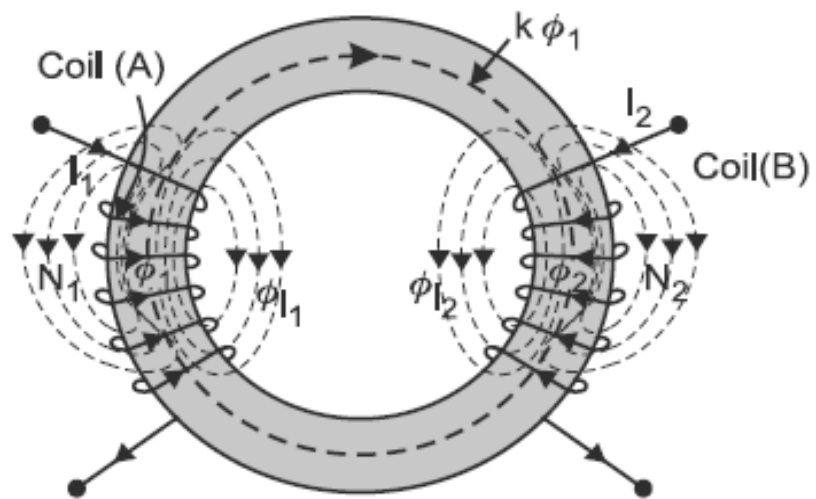


$$e \propto \frac{dI}{dt} \quad \text{or} \quad e = L \frac{dI}{dt}$$

$$\begin{aligned}
 L &= \frac{e}{dI / dt} \left( \text{since } e = L \frac{dI}{dt} \right) \\
 &= \frac{N\phi}{I} \left( \text{since } e = N \frac{d\phi}{dt} = L \frac{dI}{dt} \right) = \frac{N^2}{l / a\mu_0\mu_r} \left( \text{since } \phi = \frac{NI}{l / a\mu_0\mu_r} \right)
 \end{aligned}$$



$$\begin{aligned}
 M &= \frac{e_m}{dI_1 / dt} \left( \text{since } e_m = M \frac{dI_1}{dt} \right) \\
 &= \frac{N_2 \phi_{12}}{I_1} \left( \text{since } e_m = N_2 \frac{d\phi_{12}}{dt} = M \frac{dI_1}{dt} \right) \\
 &= \frac{N_1 N_2}{l / a \mu_0 \mu_r} \left( \text{since } \phi_{12} = \frac{N_1 I_1}{l / a \mu_0 \mu_r} \right)
 \end{aligned}$$

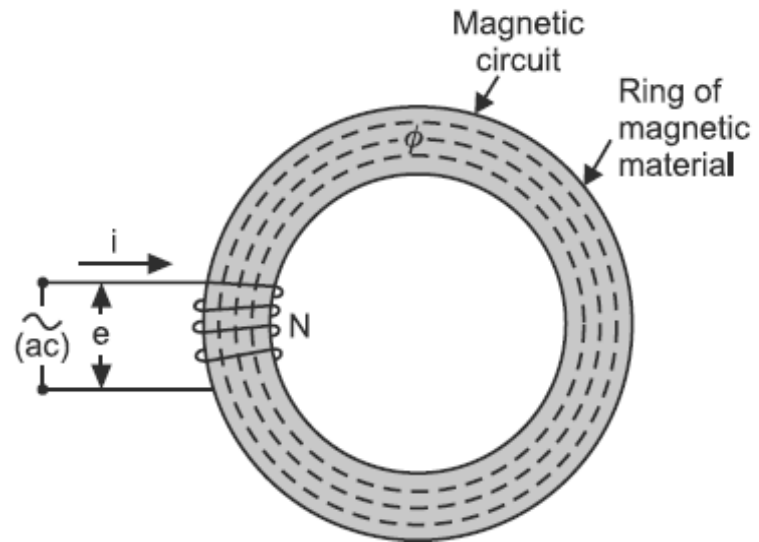


$$L_1 = \frac{N_1 \phi_1}{I_1} \text{ and } M = \frac{N_2 \phi_{12}}{I_1} = \frac{N_2 k \phi_1}{I_1}$$

$$L_2 = \frac{N_2 \phi_2}{I_2} \text{ and } M = \frac{N_1 \phi_{21}}{I_2} = \frac{N_1 k \phi_2}{I_2}$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

## Excitação AC de circuitos magnéticos



$$e = N \frac{d\phi}{dt}$$

$$\phi = \phi_m \sin \omega t = \phi_m \sin 2\pi ft$$

